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Hacking PROCESS for Estimation and Probing of Linear Moderation of Quadratic Effects and Quadratic Moderation of Linear Effects

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Abstract

PROCESS model 1, used for estimating, testing, and probing interactions in ordinary least squares regression, constrains focal predictor X's linear effect on outcome variable Y to be linearly moderated by a single moderator W. In this document I describe how to hack PROCESS to get it to estimate a model that includes linear moderation by W of a quadratic effect of X on Y, and quadratic moderation by W of a linear effect of X on Y. Instructions are provided for the implementation of the pick-a-point and Johnson-Neyman techniques for probing interactions in models that combine quadratic nonlinearity and moderation.

In all of the examples of moderation in *Introduction to Moderation, Mediation, and Conditional Process Analysis* (Hayes, 2022), effects are estimated as *linear* effects. In a model of the form $\hat{Y} = i_Y + b_1X + b_2W + b_3XW$, X's effect on Y is constrained to be linear, meaning that a one unit difference in X corresponds to the same estimated difference in Y regardless of where you start on X. Furthermore, this model presumes that any influence of W on X's effect is also linear. That is, as W changes by one unit, X's effect on Y is constrained to change by b_3 units, regardless of where you start on W.

As described in many treatments of regression analysis, in spite of its name, *linear* regression can be used to model effects that are *nonlinear*. For instance, OLS regression can be used to estimate the regression coefficients in a model of the form

$$\hat{Y} = i_Y + b_1X + b_2X^2$$

which is not the equation for a straight line but, rather, the equation for a curve—a parabolic or *quadratic* function. In this model, the steepness of the curve—how much it bends—as well as whether it curves upward (convex) or downward (concave) is determined by b_2. A

Andrew F. Hayes, Haskayne School of Business, University of Calgary, andrew.hayes@ucalgary.ca. Learn more about the use of PROCESS for moderation analysis by taking a class from CCRAM. See the public workshop schedule at haskayne.ucalgary.ca/CCRAM test of significance for b_2 can be used as a test of nonlinearity in the relationship between X and Y.

The two hacks introduced here are not about how to model a nonlinear effect using PROCESS. Rather, this document describes how to use PROCESS to estimate and probe a moderation model in which one variable's nonlinear effect is linearly moderated, or in which one variable's linear effect is nonlinearly moderated. This can be done using PROCESS models 1 or 2 fairly easily, as will be seen. Although the hack is easy to describe, understanding what it is doing and how to interpret the results is considerably more complex, and so I first give some treatment to the required statistical background.

For both hacks presented here, I rely on data from 340 respondents to the 2000 American National Election Study, which is a regular survey of the U.S. public taken prior to each federal election (see *www.electionstudies.org*). The outcome variable in both examples is *political knowledge* (PKNOW), operationalized as the number of questions (out of 22) that a participant answered correctly about various public officials and people running for office (such as their stance on various social issues, or the positions in government they currently occupy). The data also includes a measure of *news use* (NEWS), which is a 3-item index constructed as the average number of days per week respondents reported watching the local or national network news broadcast and reading the newspaper. The analyses also include the respondent's age in years (AGE), his or her sex (SEX, coded 0 for females and 1 for males), and a measure of the respondent's socioeconomic status, operationalized as the average of his or her standardized income and number of years of education (SES).

Estimating and Probing Linear Moderation of a Quadratic Effect

Is there a relationship between political knowledge (Y) and news use (X)? Using Pearson's correlation as an index of association, this correlation is positive ($r = 0.18, p < 0.001$), meaning that those who use these sources of news relatively more frequently are more knowledgeable about various political actors on the national and international stage. Furthermore, controlling for sex, age, and socioeconomic status in a multiple regression analysis, this relationship persists (partial $r = 0.120$, partial regression coefficient $= 0.265$, $p < 0.05$). Holding constant age, sex, and socioeconomic status, each additional day of news use per week translates into a difference of 0.265 units of political knowledge. This is not a particularly big effect in might seem, but it is also not particularly surprising that the effect is positive.

This analysis and its interpretation assumes that the relationship between news use and political knowledge is linear. But maybe it isn't. There are many forms that nonlinearity can take. We focus here on examining if the association can be better characterized with a quadratic function. This is accomplished by adding the square of news use to the model as an additional predictor (X^2). Thus, continuing to use age (W), sex (U_1), and socioeconomic status (U_2) as covariates, the model estimated is

$$Y = i_Y + b_1X + b_2X^2 + b_3W + b_4U_1 + b_5U_2 + e_Y \tag{1}$$

In this model, the relationship between news use and age is modeled as quadratic. In a nonlinear model such as this, X's effect on Y ($\theta_{X \to Y}$) is, in geometric terms, the slope of the line tangent to the function at the point X, also called the instantaneous rate of change of Y. It is calculated as the first derivative of equation 1 with respect to X, which here is

$$\theta_{X \rightarrow Y} = b_1 + 2b_2X \tag{2}$$

So the effect of X on Y, $\theta_{X \rightarrow Y}$, is itself a function of X in this model. Observe from equation 2 that if b_2 is zero, then the effect of X on Y is b_1, and therefore *not* a function of X. This is equivalent to saying that X's effect is linear. If b_2 is positive, this means that the relationship between X and Y is nonlinear and convex, but if b_2 is negative, then the relationship is nonlinear and concave. The larger b_2 in absolute value, the sharper the bend in the curve.

A test of nonlinearity of this form can be conducted by estimating the model and determining whether b_2 is statistically different from zero. This analysis can be accomplished using any OLS regression program. Doing so yields

$$\hat{Y} = 7.168 + 1.372X - 0.156X_2 + 0.022W + 1.720U_1 + 2.472U_2$$

and b_2 is statistically different from zero, $b_2 = -0.156, p < .01$. In terms of improvement in model fit, the multiple correlation increases from $R^2 = 0.320$ when X^2 is not in the model to $R^2 = 0.336$ when X^2 is added. This change in $R^2, \Delta R^2 = 0.016$, is statistically significant, $F(1, 334) = 7.879, p < .01$. But we already knew this, as a hypothesis test for b_2 in the regression analysis is mathematically equivalent to this hypothesis test for ΔR^2.

A picture almost always helps and rarely hinders interpretation. A visual representation of this model can be found Figure 1 panel A, which sets all the covariates to their sample means. As can be seen, the relationship between news use and political knowledge is concave, with a peak in knowledge among those more moderate in their news use. Those who use the news relatively little or relatively frequently are lower in knowledge, at least according to this model.

Moderation of the Curvilinearity

The prior analysis is all background for understanding linear moderation of a quadratic effect. If this quadratic relationship between X and Y is moderated by some variable W, that means that the curve linking X to Y depends on W. More specifically, does the extent of the bend in the function—its steepness or direction (convex or concave) depend linearly on W? In this example, W will be age, and I will illustrate the estimation of the linear moderation by age of the curvilinear effect of news use on political knowledge revealed in the prior analysis.

Assuming W is either dichotomous or continuous, the standard approach described in various books on regression analysis that tackle such a complex modeling process (e.g., Aiken & West, 1991; Cohen, Cohen, West, & Aiken, 2003) is to estimate Y from X, X^2, W, XW, and X^2W. Additional predictors could be included in the model as covariates. In this example, W is no longer a covariate as before but is now a moderator variable, but sex and socioeconomic status are still covariates. The model we estimate to test moderation of the curvilinear relationship between news use and political knowledge by age is

$$\hat{Y} = i_Y + b_1X + b_2X^2 + b_3W + b_4XW + b_5X^2W + b_6U_1 + b_7U_2 \tag{3}$$

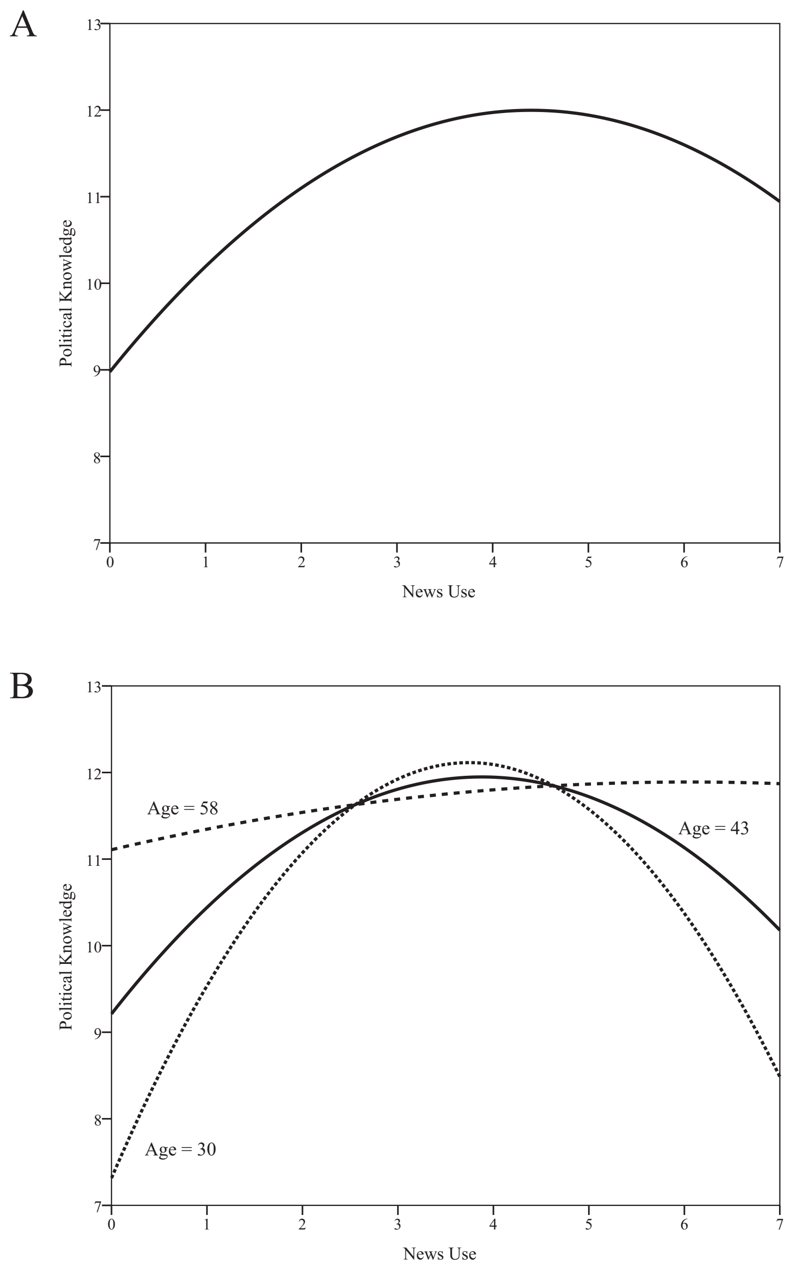

Figure 1. A visual representation of the curvilinear association between news use and political knowledge (A) and how that curvilinearity is moderated by age (B).

or, in equivalent form,

$$\hat{Y} = i_Y + b_1X + \theta_{X^2\to Y}X^2 + b_3W + b_4XW + b_6U_1 + b_7U_2$$

where

$$\theta_{X^2\to Y} = b_2 + b_5W. \tag{4}$$

Expressed in this way, it can be seen that the curvilinear component of the effect of X on Y, $\theta_{X^2\to Y}$, is a function of W with the influence of W on that curve determined by the size of b_5. If $b_5 = 0$, then W is unrelated to the extent of curvilinearity, but if $b_5 \neq 0$ then the curvilinearity depends on W. So W can be deemed a moderator of the curvilinearity of X's effect on Y if b_5 is statistically different from zero.

Any OLS regression program can estimate this model. In SPSS, for example, try

```
compute newsage=news*age.
compute news2=news*news.
compute news2age=news2*age.
regression/dep=pknow/method=enter news news2 age newsage news2age sex ses.
```

or in SAS, use

```
data politics;set politics;news2=news*news;
newsage=news*age;news2age=news2*age;run;
proc reg data=politics;model pknow=news news2 age newsage news2age sex ses;run;
```

The resulting output (from SPSS) can be found in Figure 2. Substituting the regression coefficients estimated from the data into equation 3 yields

$$\hat{Y} = 2.600 + 4.945X - 0.674X^2 + 0.129W - 0.079XW + 0.011X^2W + 1.681U_1 + 2.478U_2$$

In this model, b_5 is the regression coefficient for X^2W or "news2age." As can be seen, $b_5 = 0.011, p < 0.01$. Therefore, age moderates the curvilinearity of the association between news use and political knowledge. Rephrased in terms of improvement in model fit, a version of the model in equation 3 that excludes X^2W does not fit as well ($R^2 = 0.336$) as one that includes it ($R^2 = 0.351$). This difference in R^2 of $\Delta R^2 = 0.015$ is statistically significant, $F(1, 332) = 7.395, p < 01$. This equivalent to concluding that the regression weight for X^2W is different from zero. Mathematically, these are the same test.

This complex model is depicted graphically in Figure 1 panel B, constructed by setting all covariates to their sample mean and generating estimates of political knowledge from various values of news use and age using the estimated model. I used values of age corresponding to the 16th percentile (30 years), the median (age = 43) and the 84th percentile (age = 58) of the distribution of age in the sample. As can be seen, the curvilinearity in the association is most pronounced among those relatively younger (30 years old), with the curve flattening as age increases. Among the older (58 years old), there is little bend to the relationship at all. That is, the link between news use and political knowledge appears more linear among the older than it is among the younger.

ModelS ummary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.592[a]	.351	.337	3.561

a. Predictors: (Constant), ses, news2, Sex (1 = Male, 0 = female), Age, news, news2age, newsage

ANOVA[a]

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	2272.650	7	324.664	25.603	.000[b]
	Residual	4209.923	332	12.680		
	Total	6482.574	339			

a. Dependent Variable: Political Knowledge

b. Predictors: (Constant), ses, news2, Sex (1 = Male, 0 = female), Age, news, news2age, newsage

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients		
		B	Std. Error	Beta	t	Sig.
1	(Constant)	2.600	2.304		1.129	.260
	news	4.945	1.426	2.071	3.468	.001
	news2	-.674	.197	-2.066	-3.429	.001
	Age	.129	.056	.431	2.301	.022
	newsage	-.079	.031	-2.149	-2.514	.012
	news2age	.011	.004	2.013	2.719	.007
	Sex (1 = Male, 0 = female)	1.681	.398	.192	4.218	.000
	ses	2.478	.245	.462	10.130	.000

a. Dependent Variable: Political Knowledge

Figure 2. SPSS output corresponding to the model represented by equation 3

.

Estimation and Probing using PROCESS

Figure 2 is a holistic representation of the moderation of the curvilinearity of news use's effect on political knowledge by age. Most investigators would follow up this analysis by then probing this moderation. One might use the pick-a-point approach to estimate the conditional curvilinearity of news use's effect at various values of age. This would not be impossible to do without PROCESS. Aiken and West (1991) and Cohen et al. (2003) describe how, although doing so is quite tedious, not particularly fun, and mistakes are easily made along the way. PROCESS makes it easy with this hack.

Alternatively, as age is a continuous moderator, one could employ the Johnson-Neyman technique to determine the regions of significance for the curvilinearity. That is, what values of age define the points of transition between statistically significant and nonsignificant curvilinearity in the relationship between news use and political knowledge? This can be *quite* difficult without PROCESS, but using PROCESS it is super easy.

PROCESS model 1 can be used. Model 1 is a simple moderation model, and even though this model is anything but simple, model 1 works just fine. In Model 1, W is estimated as linearly moderating the effect of X on Y. The key to this hack is to think of X in Model 1 not as news use, but as the square of news use, X^2, while specifying age the moderator W, as always. But it is not quite as simple as this, as you have to also construct a few products and include them as covariates in the PROCESS command because PROCESS won't construct all the necessary products for you. It will, consistent with its programming, produce X^2W, but it won't produce XW. You have to manually construct it, as well as X^2 prior to executing PROCESS.

In SPSS, the model in equation 3 is estimated by PROCESS version 3 with the command below. This command also automatically implements the pick-a-point approach to probing the moderation and requests output for determining regions of significance.

```
compute newsage=news*age.
compute news2=news*news.
process x=news2/w=age/y=pknow/cov=sex ses news newsage/jn=1/model=1.
```

In SAS, the equivalent PROCESS command is

```
data politics;set politics;
newsage=news*age;news2=news*news;run;
%process (data=politics,x=news2,w=age,y=pknow,cov=sex ses news newsage,jn=1,model=1);
```

Compare the PROCESS output from SPSS in Figure 3 to the output in Figure 2. More specifically, look at the regression coefficients, their tests of significance, R^2, and the F ratio and p-value for the model. They are the same. PROCESS is estimating the same model as SPSS and SAS's regression procedure. Furthermore, PROCESS calculates the increase in R^2 attributable to the moderation of X^2 by age in the line that reads "Test(s) of highest order unconditional interaction(s)" and, most importantly, it generates additional information pertinent to probing of the moderation of the curvilinearity.

In this model, the extent of the curvature in relationship between news use and political knowledge is expressed by the function in equation 4 or, in terms of the estimated model coefficients,

$$\theta_{X^2 \rightarrow Y} = -0.677 + 0.011W$$

In calculus terms, $\theta_{X^2 \rightarrow Y}$ is the instantaneous rate of change in the *curvilinearity* in the effect of X on Y. Moderation of the curvilinearity of X's effect by W means that this rate of change depends on W, or that the steepness of the bend in the relationship between X and Y depends on W. Plugging in values of W into this equation gives a measure of the curvature of the relationship between X and Y at that value of W. With this value estimated, one can test whether it is statistically different from zero. If so, then the relationship between X and Y is curvilinear conditioned on that value of W. But if not, then one can claim that there is no evidence of curvilinearity in the association between X and Y at that value of W.

Doing so is the essence of the pick-a-point approach to probing the moderation of the curvilinearity in this model, and PROCESS does this automatically and displays it in the section of output in Figure 3 labeled "Conditional effects of the focal predictor at values of the moderator(s):". Do *not* interpret this section of output as providing the conditional effect of X as it is labeled, because remember that X^2 is specified as the argument in **x=**, not X itself. Rather, the values provided here are $\theta_{X^2 \rightarrow Y}$ estimated at values of W corresponding to the 16th percentile (30 years old), the median age (43 years old), and

```
***************** PROCESS Procedure for SPSS Release 3.00 ****************

          Written by Andrew F. Hayes, Ph.D.       www.afhayes.com
    Documentation available in Hayes (2018) www.guilford.com/p/hayes3

**************************************************************************
Model  : 1
    Y  : pknow
    X  : news2
    W  : age

Covariates:
 sex      ses      news     newsage

Sample
Size:  340

**************************************************************************
OUTCOME VARIABLE:
 pknow

Model Summary
          R       R-sq        MSE          F        df1        df2          p
      .5921      .3506    12.6805    25.6035     7.0000   332.0000      .0000

Model
              coeff         se          t          p       LLCI       ULCI
constant     2.6002     2.3039     1.1286      .2599    -1.9320     7.1323
news2        -.6744      .1967    -3.4288      .0007    -1.0614     -.2875
age           .1292      .0561     2.3014      .0220      .0188      .2396
Int_1         .0111      .0041     2.7194      .0069      .0031      .0191
sex          1.6806      .3984     4.2177      .0000      .8968     2.4644
ses          2.4777      .2446    10.1299      .0000     1.9966     2.9589
news         4.9455     1.4259     3.4682      .0006     2.1405     7.7505
newsage      -.0788      .0314    -2.5144      .0124     -.1405     -.0172

Product terms key:
 Int_1    :        news2    x        age

Test(s) of highest order unconditional interaction(s):
       R2-chng          F        df1        df2          p
X*W      .0145     7.3953     1.0000   332.0000      .0069
----------
    Focal predict: news2    (X)
          Mod var: age      (W)

Conditional effects of the focal predictor at values of the moderator(s):

        age     Effect         se          t          p       LLCI       ULCI
    30.0000     -.3423      .0881    -3.8843      .0001     -.5156     -.1689
    43.0000     -.1984      .0603    -3.2920      .0011     -.3169     -.0798
    58.0000     -.0323      .0763     -.4230      .6725     -.1824      .1178
```

Test of linear moderation of the quadratic effect of news use by age

(*continued*)

Figure 3. PROCESS output for a model estimating and probing the moderation of the quadratic effect of news use on political knowledge by age.

```
Moderator value(s) defining Johnson-Neyman significance region(s):
      Value    % below    % above
    50.0604    65.0000    35.0000

Conditional effect of focal predictor at values of the moderator:
        age     Effect         se          t          p       LLCI       ULCI
    18.0000     -.4752      .1287    -3.6929      .0003     -.7283     -.2220
    21.6000     -.4353      .1158    -3.7580      .0002     -.6632     -.2074
    25.2000     -.3954      .1035    -3.8213      .0002     -.5990     -.1919
    28.8000     -.3556      .0918    -3.8729      .0001     -.5362     -.1750
    32.4000     -.3157      .0811    -3.8920      .0001     -.4753     -.1561
    36.0000     -.2759      .0718    -3.8400      .0001     -.4172     -.1345
    39.6000     -.2360      .0646    -3.6542      .0003     -.3630     -.1090
    43.2000     -.1961      .0601    -3.2642      .0012     -.3143     -.0779
    46.8000     -.1563      .0590    -2.6492      .0085     -.2723     -.0402
    50.0604     -.1202      .0611    -1.9671      .0500     -.2404      .0000
    50.4000     -.1164      .0615    -1.8938      .0591     -.2374      .0045
    54.0000     -.0766      .0671    -1.1403      .2550     -.2086      .0555
    57.6000     -.0367      .0753     -.4876      .6261     -.1848      .1114
    61.2000      .0032      .0852      .0370      .9705     -.1644      .1707
    64.8000      .0430      .0963      .4467      .6554     -.1464      .2324
    68.4000      .0829      .1083      .7655      .4445     -.1301      .2958
    72.0000      .1227      .1208     1.0158      .3105     -.1149      .3604
    75.6000      .1626      .1338     1.2151      .2252     -.1006      .4258
    79.2000      .2024      .1471     1.3762      .1697     -.0869      .4918
    82.8000      .2423      .1606     1.5083      .1324     -.0737      .5583
    86.4000      .2822      .1744     1.6183      .1066     -.0608      .6252
    90.0000      .3220      .1882     1.7109      .0880     -.0482      .6923

*********************** ANALYSIS NOTES AND ERRORS ************************

Level of confidence for all confidence intervals in output:
  95.0000

W values in conditional tables are the 16th, 50th, and 84th percentiles.
```

FIGURE 3 continued.

the 84th percentile (58 years old). These values, listed under "Effect", are −0.342, −0.198, and −0.032, respectively. They quantify the steepness of the bend in the lines in Figure 1 panel B and correspond to what you can see visually. Among those relatively younger, the curvilinearity is much steeper or stronger, but the curve flattens out with increasing age. Furthermore, we can say that among those "relatively older" (age = 58), there is no statistically significant evidence of curvilinearity in the association between news use and political knowledge. The null hypothesis that $\theta_{X^2 \rightarrow Y} = 0$ cannot be rejected when $M = 58$, $\theta_{X^2 \rightarrow Y|M=59} = -0.032$, $t(332) = -0.423, p = .673$. But among those moderate in age and relatively young, the relationship is curvilinear (and concave) to a statistically significant degree: $\theta_{X^2 \rightarrow Y|M=43} = -0.198$, $t(332) = -3.292, p = .001$; $\theta_{X^2 \rightarrow Y|M=30} = -0.342$, $t(332) = -3.884, p < .001$.

By default, PROCESS selects values of W corresponding to the 16th, 50th, and 84th percentile of the distribution whenever W is quantitative. If W happened to be dichotomous, PROCESS would automatically produce the conditional curvilinear effects for the two groups coded by W. The **`wmodval`** option, described in the PROCESS documentation (Appendix A of *Introduction to Mediation, Moderation, and Conditional Process Analysis*, could also be used if you wanted to choose your own value of W and estimate the curvilinearity of the relationship between news use and political knowledge at that value.

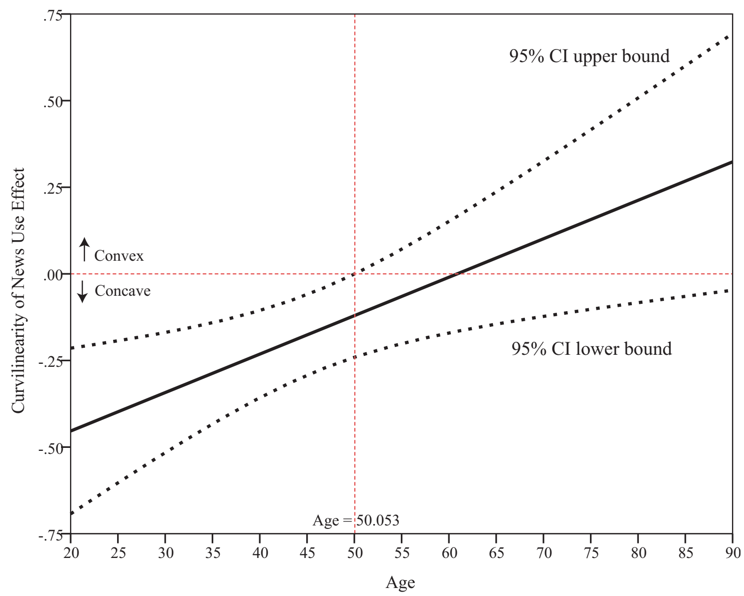

Figure 4. The curvilinearity of the quadratic effect of news use on political knowledge as a function of age.

Most often, implementation of the pick-a-point approach when W is a quantitative variable involves the arbitrary selection of values of the moderator, and different choices can result in different interpretations. As discussed in Chapter 7 of *Introduction to Mediation, Moderation, and Conditional Process Analysis* (Hayes, 2022), the Johnson-Neyman technique gets us around this problem. In this context, the Johnson-Neyman technique ascertains the values of the moderator at which point the curvilinearity of the association between X and Y transitions between statistically significant and not at some chosen level of significance. The `jn=1` specification in the PROCESS line implements this. The results of this implementation can be found at the end of the output in Figure 3. As can be seen, the only point of transition is age = 50.060. Below this age, the relationship between news use and political knowledge is concave (as $\theta_{X^2 \to Y}$ is significantly negative with a p-value of less than 0.05 when age $\leq$ 50.060). Above this age, one cannot conclude that the relationship between news use and age is curvilinear, as $\theta_{X^2 \to Y}$ is not significantly different from zero when age $\geq$ 50.060.

Figure 4 plots $\theta_{X^2 \to Y}$ as a function of W along with upper and lower bounds of the confidence interval for conditional curvilinearity. This plot is another way of illustrating the relationship between age and the curvilinearity in the effect of news use. As can be seen, the confidence interval for the conditional curvilinearity is entirely below 0 (remember that 0 corresponds to the absence of curvilinearity in the association between X and Y) when age is less than about 50, but is straddles zero when age is above 50.

Estimating and Probing Quadratic Moderation of a Linear Effect

The prior example illustrates the mathematics and estimation mechanics of examining linear moderation of a quadratic effect, and that PROCESS can be used to estimate models that combine nonlinear associations with moderation of nonlinearity. In that example, the

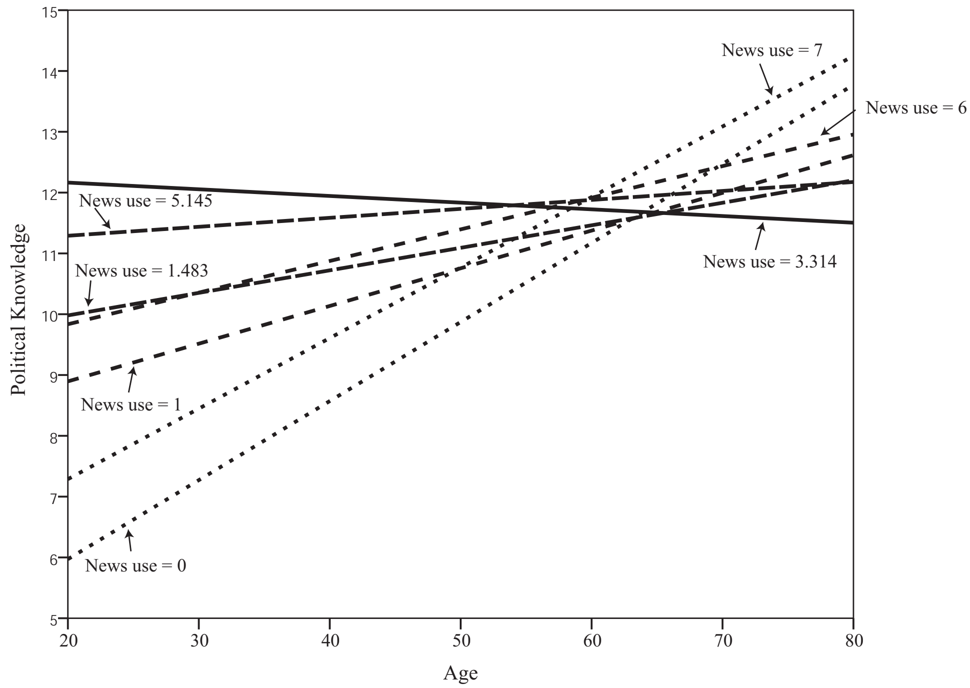

Figure 5. The effect of age on political knowledge at various values of news use.

moderation of X's quadratic effect on Y was estimated as linear in W. But PROCESS can also be used to estimate models in which a *linear* effect of X is *nonlinearly*—quadratically to be more precise—moderated by W. In fact, as will be seen, these two forms of moderation are identical mathematically, in that they are based on the same linear model. But to estimate and probe the quadratic moderation of a linear effect using PROCESS, care must be taken. It is not simply a matter of employing the prior hack while reversing the roles of X and W.

I illustrate by relying on the same data. In the prior example, Y was political knowledge, and we modeled moderation of the the curvilinearity in the association between news use (X) and political knowledge by age (W). Figure 1 panel B shows that the nonlinearity in the association was much more pronounced among the relatively younger. Among the older, the relationship between news use and knowledge was linear or nearly so. But a close look this figure reveals another way of interpreting the findings. Notice that among moderate news users (those in the middle of the scale), there appears to be little difference between people of different ages in their level of political knowledge. Whether young or old, moderate news users appear to know about the same amount. Age differences in knowledge, however, appear quite pronounced among the relatively light and the relatively heavier users of news. Among heavier and lighter users, age seems to be positively associated with knowledge.

If this is not apparent, perhaps Figure 5 will help. This conveys the model estimated in the prior hack and depicted in panel B of Figure 1, but putting age rather than news use on the X axis and using different lines depicting the association between age and knowledge among people at different values of news use. Depicted in this way, with news use as the moderator, it is apparent that the relationship between age and knowledge varies with news use. Although this plot on first glance appears to depict a standard linear by linear interaction, look again. Observe that the slope relating age to knowledge is positive and steeper among people relatively more *extreme* in their news use, whether extremely low *or* extremely high. Among the more moderate, the relationship between news use and knowledge appears much less pronounced. And the change in the slope as news use changes is fairly small in the range of moderate news use, but accelerates as you move toward the extremes of news use.

Both Figures 1 panel B and Figure 5 are visual representations of a model of the form

$$\hat{Y} = i_Y + b_1X + b_2X^2 + b_3W + b_4XW + b_5X^2W + b_6U_1 + b_7U_2 \quad (5)$$

or, more specifically in terms of the estimated model coefficients,

$$\hat{Y} = 2.600 + 4.946X - 0.674X^2 + 0.129W - 0.079XW + 0.011X^2W + 1.681U_1 + 2.478U_2$$

They differ in how they direct the viewer's attention to one variable as moderator and one as focal predictor. When news use was focal predictor X and age was moderator W, it was seen in the prior hack that curvilinearity in the relationship between news use and political knowledge ($\theta_{X^2 \rightarrow Y}$) could be expressed as a function of age: $\theta_{X^2 \rightarrow Y} = b_2 + b_5W = -0.677 + 0.011W$. But with a little algebra, this model can be written equivalently as

$$\hat{Y} = i_Y + b_1X + b_2X^2 + \theta_{W \rightarrow Y}W + b_6U_1 + b_7U_2$$

where

$$\theta_{W \rightarrow Y} = b_3 + b_4X + b_5X^2 \quad (6)$$

which shows that W effect on Y is a quadratic function of X. Rephrased, two people that differ by one year in age (W) are estimated to differ by $b_3 + b_4X + b_5X^2$ units in political knowledge. So the effect of a one year difference in age on political knowledge is a nonlinear function of news use in this model.

We already know from the analysis in the prior example that $b_5 = 0.011$ and is statistically different from zero, so there is statistically significant nonlinearity, at least expressed in this quadratic form, in how much age's effect on knowledge depends on news use. Given this, a natural next question is to probe this nonlinear moderation of a linear effect. In principle, the pick-a-point approach as described in Chapter 7 of *Introduction to Mediation, Moderation, and Conditional Process Analysis* could be used. As will be seen, that model is similar to one that allows for nonlinear moderation of a curvilinear effect of X on Y. The Johnson-Neyman technique could also be used in principle, but doing so by hand would be next to impossible, and it can't be done in PROCESS. Given that this document is dedicated to using PROCESS, I focus on how to estimate this model and probe it using the pick-a-point approach only.

Estimating and Probing the Nonlinear Moderation using PROCESS

We saw in the prior example that PROCESS model 1 can be used to estimate this model, specifying X^2 as X in the `x=` line of the PROCESS command, constructing some products, and using those products as covariates. Although that procedure estimates the model correctly, it cannot be used to probe nonlinear moderation of a linear effect like it can to probe linear moderation of a quadratic effect. A different procedure must be used.

The trick to understanding this hack is the recognition that PROCESS does have a model preprogrammed to estimate *additive linear moderation* of X's *linear* effect on Y by two moderators W and Z. This is PROCESS model 2, and it is used to estimate the coefficients in

$$Y = i_Y + b_1X + b_2W + b_3Z + b_4XW + b_5XZ$$

and produce estimates of the conditional effect of X, defined as

$$\theta_{X \rightarrow Y} = b_1 + b_3W + b_4Z \tag{7}$$

as discussed in Chapter 9 of Hayes (2022). Equation 7 is similar to but not the same as equation 6. They differ in two important respects other than the arbitrary subscripts for the regression coefficients. First, equation 7 has Z as a variable rather than X^2. Second, whereas equation 7 is a model of X's effect on Y, equation 6 is a model of W's effect on Y.

This second difference is easily eliminated merely by being consistent in our labeling of focal predictor and moderator. In the prior hack, age was the moderator and labeled W whereas news use was the focal predictor and labeled X. But in this application, age is the focal predictor and news use is the moderator. So to be consistent, let's just now label age as X and news use as W.

The first difference is also easily eliminated. Let's just think of Z as X^2 rather than as a separate variable. PROCESS model 2 was designed with the idea that W and Z would be different variables each serving as moderator of X's effect. But the mathematics don't care *to a point*. It turns out that using X^2 for Z in model 2 will have some shortcomings in terms of output that PROCESS produces, but we can get around that.

With all this in mind, we use PROCESS model 2 to estimate a model which includes quadratic moderation by W (news use) of X's (age) linear effect on Y (political knowledge). As in the prior hack, we are also going to have to create a new variable that is W^2 prior to executing PROCESS because PROCESS won't do it for us. This new variable will be assigned the role of Z in model 2.

In SPSS, the commands below estimate the model:

```
compute news2=news*news.
process x=age/w=news/z=news2/y=pknow/cov=sex ses/model=2.
```

The equivalent code in SAS is

```
data politics;set politics;news2=news*news;run;
%process (data=politics,x=age,w=news,z=news2,y=pknow,cov=sex sex,model=2);
```

Figure 6 contains the resulting output. As can be seen, it estimates the model represented in equations 3 and 5, and the regression coefficients are the same. There are slight differences in the section of output under the heading "R-squared increase due to interaction(s)" because PROCESS model 1, as far as PROCESS knows, contains only one interaction whereas PROCESS model 2 is designed to estimate two interactions and so produces output for each. The incremental increase in R^2 due to allowing the moderation of X's effect by W to be quadratic in W rather than linear is in the row labeled "X*Z" $R^2 = 0.015, F(1, 332) = 7.395, p < .01$. This is the same as the inference when framed instead as a test of (1) the increase due to the interaction in PROCESS model 1 when the square or news use was the focal predictor and age was the moderator, or as (2) the regression coefficient for X^2W in either model. Mathematically these are all the same test.

Thus far, we've learned nothing new relative to what was learned in the prior example. We see from the hypothesis test for b_5 that the linear effect of age on political knowledge is quadratically moderated by news use. The next step, and what this hack is ultimately about, is to probe this nonlinear moderation. We will do so using the pick-a-point approach.

PROCESS model 2 is designed to implement the pick-a-point approach automatically. The output it generates at the bottom of Figure 6 in the section titled "Conditional effects of the focal predictor at values of the moderator(s):" ordinarily would be used to test whether the conditional effect of X on Y is different from zero for certain combinations of moderators W and Z. PROCESS sees that both the W and Z specified in the command are quantitative variables and so it generates the conditional effect of X for various combinations of "low," "moderate," and "high" on the moderators—the 16th, 50th, and 84th percentiles of the distribution.

There is no point in looking long at this section of the output, however, because it is completely meaningless when PROCESS is hacked in this fashion. Because Z is actually W^2, it is not sensible to fix W and vary W^2 when estimating the conditional effect of X because W determines W^2 completely. PROCESS doesn't realize this, of course. It does not know that Z is W^2. Rather, it thinks Z as just a separate moderator distinct from W. It is simply doing what it is programmed to do. ***So just ignore this part of the output.***

To estimate the conditional effect of age on political knowledge at various values of news use, we need to specify a value of news use at which to condition PROCESS's computations, thereby overriding what PROCESS will do by default. This is done with the **wmodval** and **zmodval** options. For example, suppose we want to estimate the conditional effect of age on political knowledge among people average in news use. In the data, the sample *mean* of news use is 3.314. So add **wmodval=3.314** and **zmodval=10.9826** to the PROCESS command. It is important that the value specified in the **zmodval** option is the square of the value listed in the **wmodval** argument. So in SPSS, the code below will estimate the model and produce the conditional effect of age on poltical knowledge among those average in news use:

```
compute news2=news*news.
process x=age/w=news/z=news2/y=pknow/cov=sex ses/model=2/wmodval=3.314/
      zmodval=10.9826.
```

or in SAS,

```
***************** PROCESS Procedure for SPSS Release 3.00 ****************

          Written by Andrew F. Hayes, Ph.D.       www.afhayes.com
    Documentation available in Hayes (2018) www.guilford.com/p/hayes3

**************************************************************************
Model  : 2
    Y  : pknow
    X  : age
    W  : news
    Z  : news2

Covariates:
 sex      ses

Sample
Size:  340

**************************************************************************
OUTCOME VARIABLE:
 pknow

Model Summary
          R       R-sq        MSE          F        df1        df2          p
      .5921      .3506    12.6805    25.6035     7.0000   332.0000      .0000

Model
             coeff         se          t          p       LLCI       ULCI
constant    2.6002     2.3039     1.1286      .2599    -1.9320     7.1323
age          .1292      .0561     2.3014      .0220      .0188      .2396
news        4.9455     1.4259     3.4682      .0006     2.1405     7.7505
Int_1       -.0788      .0314    -2.5144      .0124     -.1405     -.0172
news2       -.6744      .1967    -3.4288      .0007    -1.0614     -.2875
Int_2        .0111      .0041     2.7194      .0069      .0031      .0191
sex         1.6806      .3984     4.2177      .0000      .8968     2.4644
ses         2.4777      .2446    10.1299      .0000     1.9966     2.9589

Product terms key:
 Int_1    :        age      x        news
 Int_2    :        age      x        news2

Test(s) of highest order unconditional interaction(s):
       R2-chng          F        df1        df2          p
X*W      .0124     6.3220     1.0000   332.0000      .0124
X*Z      .0145     7.3953     1.0000   332.0000      .0069
----------
    Focal predict: age      (X)
          Mod var: news     (W)
          Mod var: news2    (Z)

Conditional effects of the focal predictor at values of the moderator(s):

      news      news2     Effect         se          t          p       LLCI       ULCI
    1.5000     2.2500      .0358      .0254     1.4124      .1588     -.0141      .0857
    1.5000     9.4928      .1160      .0444     2.6142      .0094      .0287      .2033
    1.5000    28.4089      .3254      .1168     2.7871      .0056      .0957      .5552
    3.0800     2.2500     -.0888      .0430    -2.0625      .0399     -.1734     -.0041
    3.0800     9.4928     -.0086      .0190     -.4498      .6532     -.0460      .0289
    3.0800    28.4089      .2009      .0684     2.9376      .0035      .0664      .3354
    5.3300     2.2500     -.2662      .1098    -2.4250      .0158     -.4821     -.0502
    5.3300     9.4928     -.1860      .0809    -2.2981      .0222     -.3452     -.0268
    5.3300    28.4089      .0235      .0198     1.1869      .2361     -.0154      .0624

********************** ANALYSIS NOTES AND ERRORS ************************

Level of confidence for all confidence intervals in output:
  95.0000

W values in conditional tables are the 16th, 50th, and 84th percentiles.

Z values in conditional tables are the 16th, 50th, and 84th percentiles.

------ END MATRIX -----
```

Test of quadratic moderation of the linear effect of age by news use.

Ignore all this. It is nonsense

Figure 6. Model 2 PROCESS output estimating quadratic moderation of the linear effect of age on political knowledge.

```
process ... /model=2/wmodval=0/zmodval=0.

Conditional effects of the focal predictor at values of the moderator(s):
     news     news2    Effect        se         t         p      LLCI      ULCI
    .0000     .0000     .1292     .0561    2.3014     .0220     .0188     .2396

process ... /model=2/wmodval=1/zmodval=1.

Conditional effects of the focal predictor at values of the moderator(s):
     news     news2    Effect        se         t         p      LLCI      ULCI
   1.0000    1.0000     .0614     .0328    1.8713     .0622    -.0031     .1259

process ... /model=2/wmodval=1.4828/zmodval=2.1987.

Conditional effects of the focal predictor at values of the moderator(s):
     news     news2    Effect        se         t         p      LLCI      ULCI
   1.4828    2.1987     .0366     .0256    1.4319     .1531    -.0137     .0869

process ... /model=2/wmodval=3.3140/zmodval=10.9826.

Conditional effects of the focal predictor at values of the moderator(s):
     news     news2    Effect        se         t         p      LLCI      ULCI
   3.3140   10.9826    -.0105     .0191    -.5502     .5826    -.0481     .0271

process ... /model=2/wmodval=5.1451/zmodval=26.4721.

Conditional effects of the focal predictor at values of the moderator(s):
     news     news2    Effect        se         t         p      LLCI      ULCI
   5.1451   26.4721     .0166     .0191     .8692     .3854    -.0210     .0542

process ... /model=2/wmodval=6/zmodval=36.

Conditional effects of the focal predictor at values of the moderator(s):
     news     news2    Effect        se         t         p      LLCI      ULCI
   6.0000   36.0000     .0547     .0255    2.1419     .0329     .0045     .1049

process ... /model=2/wmodval=7/zmodval=49.

Conditional effects of the focal predictor at values of the moderator(s):
     news     news2    Effect        se         t         p      LLCI      ULCI
   7.0000   49.0000     .1198     .0444    2.6987     .0073     .0325     .2071
```

Figure 7. Using the **wmodval** and **zmodval** options in PROCESS model 2 to estimate the conditional effect of age on political knowledge when news use (the moderator W) is equal to 0, 1, 1.4828 (one standard deviation below the sample mean), 3.314 (the sample mean), 4, 5.1451 (one standard deviation above the sample mean), 6, and 7. When using this hack, always set the value in **zmodval** to the square of the value for W in **wmodval**.

```
data politics;set politics;news2=news*news;run;
%process (data=politics,x=age,w=news,z=news2,y=pknow,cov=sex ses,model=2,wmodval=3.314,
      zmodval=10.9826);
```

Seeing as we have been switching symbols around from equation to equation, it is worth at this point to make it clear just what model has been estimated by PROCESS model 2 and why this approach to estimating the conditional effect of age works. Quadratic moderation of the linear effect of X on Y by W while controlling for two covariates U_1 and U_2 is examined by estimating

$$Y = i_Y + b_1X + b_2W + b_3W^2 + b_4XW + b_5XW^2 + b_6U_1 + b_7U_2 + e_Y$$

X's linear effect is quadratically moderated by W by determining if b_5 is statistically different from zero. In this hack we used PROCESS model 2 to estimate the coefficients in this model by specifying W^2 as Z in the PROCESS code. We used PROCESS model 2 rather than model 1 as in the prior hack—for a model that is mathematically the same—because the formula for the conditional linear effect of X on Y as a function of W is different than the formula for the conditional nonlinearity of X's effect as a function of W. For the former, which is what we are doing with this hack

$$\theta_{X\to Y} = b_3 + b_4W + b_5W^2$$

or in terms of the estimated coefficients from the model

$$\theta_{X\to Y} = 0.129 - 0.079W + 0.011W^2 \qquad (8)$$

PROCESS thinks W^2 is Z, so for this reason we specify the square of W in the **`zmodval`** option. Plugging $W = 3.314$ into equation 8 yields $\theta_{X\to Y} = -0.011$. More precisely, if the computations done by hand were done to a higher level of precision (we used only three or four decimals places in the computation), $\theta_{X\to Y} = -0.0105$. This value is generated in the PROCESS output (see Figure 7), along with an estimated standard error, test of significance, and a confidence interval. As can be seen, among people average in news use, there is no statistically significant evidence of a linear association between age and political knowledge.

This process can be repeated as many times as desired, substituting a different value of W into the **`wmodval`** argument, specifying its square in the argument for **`zmodval`**, and rerunning PROCESS. Figure 7 contains the output that results when using $W = 0$, $W = 1$, $W = 2.1987$ (one standard deviation below the sample mean, $W = 3.314$, $W = 5.1451$ (one standard deviation above the sample mean), $W = 6$, and $W = 7$. As can be seen, at the extremes of news use (i.e., $W = 0, 1, 6, 7$), age is positively related to political knowledge by a test of significance or 95% confidence interval. But in the middle of the distribution of news use (within a standard deviation of the mean), the conditional effect of age is not statistically different from zero.

A Caution About the Use of the PLOT option in PROCESS

Visualizing moderation, using graphs or plots such as found in Figures 1 and 5 aides interpretation. PROCESS has a **plot** option to facilitate the visualization of moderation by producing a table of estimates of Y from the model for various combinations of the focal predictor X and moderator W, setting all covariates to their sample means. Figures 1 and 5 were *not* produced with the assistance of the **plot** option but were instead constructed manually without the assistance of PROCESS. Do not use or interpret the table generated with the **plot** option in PROCESS when using these hacks. One of the problems is that these hacks require the manual construction of covariates that involve X or W, which will vary depending on the values of X, W in the table the **plot** option produces. These covariates cannot be set to their sample mean for the production of $\hat{Y}$ while X and W are also varying in the production of $\hat{Y}$ for various combinations of X and W. As a result, the $\hat{Y}$ values that are produced will be inaccurate. Again, ***do not use the plot option or interpret or graph the data in the table it generates when using these hacks***.

References

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