Balancing openness and prioritization in a two-tier Internet: Appendix

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1. Example of Cournot competition represented by the reduced form profit function

Following Tirole (1988) and Levi and Nault (2004), we suppress aggregate output in our reduced form profit function notation. In this way the profit condition can be written as $PR(\theta,x) \equiv PR_X(\theta,x) = r(X)\,x - g(\theta,x)$, where $r(X)$ is the inverse demand (price) function from end users and is decreasing and concave in aggregate output. The cost of production, $g(\theta,x)$, is increasing and convex in output, and depends on the edge provider's bandwidth requirement such that edge providers that consume more bandwidth to produce a unit of output have slightly higher marginal costs, $\partial^2 g(\theta,x)/\partial\theta\partial x \geq 0$. Using this Cournot form, and accounting for the effect of the edge provider's output on aggregate output, each edge provider's first-order condition for profit maximization is

$$\frac{\partial PR(\theta,x)}{\partial x} = \frac{dPR_X(\theta,x)}{dx} = r(X) - \frac{\partial g(\theta,x)}{\partial x} + r'(X)\frac{dX}{dx} = 0.$$

The second-order conditions follow directly from concave price and convex costs in output. Additional technical conditions can be assumed to obtain existence and uniqueness in pure strategies (see Tirole 1988, 224-226).

With the different levels of output in the open and fast-lane Internet, the inverse demand function can be restated to account for output from non-converting and converting edge providers, $r(X;X_c)$ and $r_c(X_c;X)$. The first and second order conditions for profit maximization are the same in either case.

2. Proofs of our Lemmas and Theorems

LEMMA 1. *For edge providers that convert to the fast-lane Internet, output is weakly lower for those edge providers that require greater bandwidth per unit of output, and is decreasing in the usage-based fee.*

Proof: For converting edge providers, assuming an interior solution, the first-order condition by choice of output is

$$\frac{\partial \Pi_c}{\partial x_c} = \frac{\partial PR_c(\theta, x_c)}{\partial x_c} - s\theta = 0 = \psi_c(\theta, x_c, s),$$

where $\psi_c(\theta, x_c, s)$ implicitly defines the optimal value function $x_c(\theta, s)$. From Assumption 4*(a)* we have

$$\frac{\partial \psi_c(\theta, x_c, s)}{\partial \theta} = \frac{\partial^2 PR_c(\theta, x_c)}{\partial x_c \partial \theta} - s \leq 0.$$

The sign of the second-order condition follows directly from Assumption 2 and is

$$\frac{\partial^2 \Pi_c}{\partial x_c^2} = \frac{\partial^2 PR_c(\theta, x_c)}{\partial x_c^2} = \frac{\partial \psi_c(\theta, x_c, s)}{\partial x_c} < 0.$$

Directly from the implicit function rule we have

$$\frac{\partial x_c(\theta, s)}{\partial \theta} = -\frac{\partial \psi_c(\cdot)/\partial \theta}{\partial \psi_c(\cdot)/\partial x_c} \leq 0 \text{ and } \frac{\partial x_c(\theta, s)}{\partial s} = -\frac{-\theta}{\partial \psi_c(\cdot)/\partial x_c} < 0.$$

Q.E.D.

LEMMA 2. *For edge providers that do not convert, output is lower for those edge providers that require greater bandwidth per unit of output, is increasing in the investments in the Internet, and is decreasing in the aggregate bandwidth use of the fast-lane Internet.*

Proof: For non-converting edge providers, assuming an interior solution, the first-order condition by choice of output is

$$\begin{aligned}\frac{\partial \Pi}{\partial x} &= \frac{\partial PR(\theta,x)}{\partial x} - \frac{\partial K(\theta,U,U_c,I)}{\partial U}\frac{\partial U}{\partial x} \\ &= \frac{\partial PR(\theta,x)}{\partial x} - \frac{\partial K(\theta,U,U_c,I)}{\partial U}\theta f(\theta) = 0 = \psi(\theta,x,U_c,I),\end{aligned}$$

where $\partial U/\partial x$ is only for a given θ. $\psi(\theta,x,U_c,I)$ implicitly defines the optimal value function $x(\theta,U_c,I)$. From Assumption 4 we have

$$\begin{aligned}\frac{\partial \psi(\theta,x,U_c,I)}{\partial \theta} = &\frac{\partial^2 PR(\theta,x)}{\partial x \partial \theta} - \frac{\partial^2 K(\theta,U,U_c,I)}{\partial U \partial \theta}\theta f(\theta) \\ &- \frac{\partial K(\theta,U,U_c,I)}{\partial U} f(\theta) - \frac{\partial K(\theta,U,U_c,I)}{\partial U}\theta \frac{df(\theta)}{d\theta} < 0.\end{aligned}$$

The inequality is certainly true so long as the distribution is upward sloping or if the last term is small as it would be with any reasonably flat distribution: it is zero with θ distributed uniform $[0,1]$. The sign of the second-order condition for profit maximization follows from Assumption 2 and Assumption 3:

$$\frac{\partial^2 \Pi}{\partial x^2} = \frac{\partial^2 PR(\theta,x)}{\partial x^2} - \frac{\partial^2 K(\theta,U,U_c,I)}{\partial [U]^2}\theta^2 f(\theta)^2 = \frac{\partial \psi(\theta,x,U_c,I)}{\partial x} < 0.$$

From Assumption 4 we have

$$\frac{\partial \psi(\theta,x,U_c,I)}{\partial I} = -\frac{\partial^2 K(\theta,U,U_c,I)}{\partial I \partial U} > 0 \text{ and } \frac{\partial \psi(\theta,x,U_c,I)}{\partial U_c} = -\frac{\partial^2 K(\theta,U,U_c,I)}{\partial U_c \partial U} < 0.$$

Directly from the implicit function rule we have

$$\frac{\partial x(\theta,U_c,I)}{\partial \theta} = -\frac{\partial \psi(\cdot)/\partial \theta}{\partial \psi(\cdot)/\partial x} < 0, \quad \frac{\partial x(\theta,U_c,I)}{\partial I} = -\frac{\partial \psi(\cdot)/\partial I}{\partial \psi(\cdot)/\partial x} > 0 \text{ and } \frac{\partial x(\theta,U_c,I)}{\partial U_c} = -\frac{\partial \psi(\cdot)/\partial U_c}{\partial \psi(\cdot)/\partial x} < 0.$$

Q.E.D.

THEOREM 1. *Case 1: If* $s\, x_c(\tilde{\theta}, s) < \partial K(\tilde{\cdot})/\partial \tilde{\theta}$, *then edge providers with a greater bandwidth requirement per unit of output convert and edge providers with a lesser bandwidth requirement per unit of output do not convert.*

Case 2: If $s\, x_c(\tilde{\theta}, s) > \partial K(\tilde{\cdot})/\partial \tilde{\theta}$, *then edge providers with a lesser bandwidth requirement per unit of output convert and edge providers with a greater bandwidth requirement per unit of output do not convert.*

Proof: Totally differentiating $\phi(S,s,I,\tilde{\theta})$ with respect to $\tilde{\theta}$, cancelling terms using the optimality conditions, and rearranging terms, we have

$$\frac{d\phi(S,s,I,\tilde{\theta})}{d\tilde{\theta}} = \frac{\partial PR_c(\tilde{\theta},x_c(\tilde{\theta},s))}{\partial\tilde{\theta}} - \frac{\partial PR(\tilde{\theta},x(\tilde{\theta},U_c,I))}{\partial\tilde{\theta}} - s\,x_c(\tilde{\theta},s) + \frac{\partial K(\tilde{\cdot})}{\partial\tilde{\theta}}.$$

Treating $x_c(\tilde{\theta},s) - x(\tilde{\theta},U_c,I)$ as a small change in x we can rewrite the first two terms as $\partial^2 PR(\tilde{\theta},x)/\partial x\partial\tilde{\theta}$, which from Assumption 4 is slightly negative, if negative at all. Thus the sign of the equation depends on the relative magnitudes of $s\,x_c(\tilde{\theta},s)$ and $\partial K(\tilde{\cdot})/\partial\tilde{\theta}$. Q.E.D.

LEMMA 3. *Case 1: the proportion of edge providers converting is decreasing in the fixed fee, the usage-based fee, and investments in Internet capacity.*

Case 2: the proportion of edge providers converting is increasing in the fixed fee, the usage-based fee, and investments in Internet capacity.

Proof: By totally differentiating (2) with respect to S, s, and I, cancelling terms using the first-order conditions, and using the implicit function rule, we have:

$$\frac{\partial\tilde{\theta}(\cdot)}{\partial S} = -\frac{\partial\phi(S,s,I,\tilde{\theta})/\partial S}{\partial\phi(S,s,I,\tilde{\theta})/\partial\tilde{\theta}} = -\frac{-1}{\partial\phi(S,s,I,\tilde{\theta})/\partial\tilde{\theta}}$$

$$\frac{\partial\tilde{\theta}(\cdot)}{\partial s} = -\frac{\partial\phi(S,s,I,\tilde{\theta})/\partial s}{\partial\phi(S,s,I,\tilde{\theta})/\partial\tilde{\theta}} = -\frac{-\tilde{\theta}x_c(s,\tilde{\theta})}{\partial\phi(S,s,I,\tilde{\theta})/\partial\tilde{\theta}}$$

$$\frac{\partial\tilde{\theta}(\cdot)}{\partial I} = -\frac{\partial\phi(S,s,I,\tilde{\theta})/\partial I}{\partial\phi(S,s,I,\tilde{\theta})/\partial\tilde{\theta}} = -\frac{\partial K(\tilde{\cdot})/\partial I}{\partial\phi(S,s,I,\tilde{\theta})/\partial\tilde{\theta}}.$$

Using Theorem 1, the three derivatives are positive for Case 1 and negative for Case 2. Q.E.D.

THEOREM 2. *Case 1: the broadband provider does not invest in Internet capacity, charges a positive fixed fee and no usage-based fee.*

Case 2: the broadband provider does not invest in Internet capacity and if the impact of the bandwidth requirement per unit of output on edge provider output is no more than moderate, then Case 2 is infeasible.

Proof: Case 1: Using Lemma 3, (5) is negative. Therefore, the broadband provider has no incentive to invest in Internet capacity and chooses $I_{bp} = 0$. Further, assume for the moment that an interior solution obtains for (6) such that the fixed fee is positive, $S>0$. Then $\partial\Lambda_1(I_{bp},S,s)/\partial S=0$. Next, multiply the terms in (6) by $\tilde{\theta}x_c(\theta,s)$, subtract the result from (7), and collect terms. Then using (3) we have

$$-\tilde{\theta}x_c(\tilde{\theta},s)\int_{\tilde{\theta}(\cdot)}^{1} f(\theta)d\theta+\int_{\tilde{\theta}(\cdot)}^{1}\theta x_c(\theta,s)f(\theta)d\theta+s\int_{\tilde{\theta}(\cdot)}^{1}\theta\frac{\partial x_c(\theta,s)}{\partial s}f(\theta)d\theta<0.$$

To show the sign of the above, from Lemma 1 the sum of the first two terms is negative as $x_c(\tilde{\theta},s)>x_c(\theta,s)$ and $\theta\le 1$. The third term is also negative from Lemma 1. Consequently, starting at $s=0$, the broadband provider cannot increase profits by choosing $s>0$.

Now assume that an interior solution obtains for (7) such that the usage-based fee is positive, $s>0$. Then $\partial\Lambda_1(I_{bp},S,s)/\partial s=0$. Next, subtract (7) from (6) and collect terms. Then using (3) we have

$$\tilde{\theta}x_c(\tilde{\theta},s)\int_{\tilde{\theta}(\cdot)}^{1} f(\theta)d\theta-\int_{\tilde{\theta}(\cdot)}^{1}\theta x_c(\theta,s)f(\theta)d\theta-s\int_{\tilde{\theta}(\cdot)}^{1}\theta\frac{\partial x_c(\theta,s)}{\partial s}f(\theta)d\theta>0,$$

which is the reverse of the inequality above. Consequently, an interior solution for the usage-based fee results in an optimal fixed fee with no limit, which is infeasible. Therefore, $s=0$.

Finally, $s=0$ satisfies the constraint in (4), which defines Case 1. Q.E.D.

Case 2: The proof follows a similar structure to that of Case 1. The three first derivatives are

$$\frac{\partial\Lambda_2(I_{bp},S,s)}{\partial I_{bp}}=S\frac{\partial\tilde{\theta}(\cdot)}{\partial I_{bp}}f(\tilde{\theta})+s\frac{\partial\tilde{\theta}(\cdot)}{\partial I_{bp}}\tilde{\theta}x_c(\tilde{\theta},s)f(\tilde{\theta})-1,$$

$$\frac{\partial\Lambda_2(I_{bp},S,s)}{\partial S}=\int_{0}^{\tilde{\theta}(\cdot)} f(\theta)d\theta+S\frac{\partial\tilde{\theta}(\cdot)}{\partial S}f(\tilde{\theta})+s\frac{\partial\tilde{\theta}(\cdot)}{\partial S}\tilde{\theta}\,x_c(\tilde{\theta},s)f(\tilde{\theta}),$$

$$\frac{\partial\Lambda_2(I_{bp},S,s)}{\partial s}=\int_{0}^{\tilde{\theta}(\cdot)}\theta\,x_c(\theta,s)f(\theta)d\theta+S\frac{\partial\tilde{\theta}(\cdot)}{\partial s}f(\tilde{\theta})+s\frac{\partial\tilde{\theta}(\cdot)}{\partial s}\tilde{\theta}x_c(\tilde{\theta},s)f(\tilde{\theta})+s\int_{0}^{\tilde{\theta}(\cdot)}\theta\frac{\partial x_c(\theta,s)}{\partial s}f(\theta)d\theta.$$

Using Lemma 3, $\partial\Lambda_2(I_{bp},S,s)/\partial I_{bp}$ is negative. Therefore, the broadband provider has no incentive to invest in Internet capacity and chooses $I_{bp}=0$. Further, assume for the moment that an interior solution obtains for $\partial\Lambda_2(I_{bp},S,s)/\partial S$ such that the fixed fee is positive, $S>0$. Then $\partial\Lambda_2(I_{bp},S,s)/\partial S=0$. Next, multiply the terms in $\partial\Lambda_2(I_{bp},S,s)/\partial S$ by $\tilde{\theta}x_c(\theta,s)$ and subtract the result from $\partial\Lambda_2(I_{bp},S,s)/\partial s$ and collect terms. Then using (3) we have

$$-\tilde{\theta}x_c(\tilde{\theta},s)\int_{0}^{\tilde{\theta}(\cdot)} f(\theta)d\theta+\int_{0}^{\tilde{\theta}(\cdot)}\theta x_c(\theta,s)f(\theta)d\theta+s\int_{0}^{\tilde{\theta}(\cdot)}\theta\frac{\partial x_c(\theta,s)}{\partial s}f(\theta)d\theta<0.$$

From the premise of the Theorem $x_c(\tilde{\theta}, s)$ is close to $x_c(\theta, s)$. Examining the first two terms and noting that the second term includes an additional θ under integration where $\theta \leq \tilde{\theta}$, the sum of the first two terms is either negative or slightly positive. The third term is negative from Lemma 1. Consequently, starting at $s = 0$, the broadband provider cannot increase profits by choosing $s > 0$, and the constraint in (8) cannot be satisfied.

Next, assume that an interior solution obtains for $\partial\Lambda_2(I_{bp}, S, s)/\partial s$ such that the usage-based fee is positive, $s > 0$. Then $\partial\Lambda_2(I_{bp}, S, s)/\partial s = 0$. Subtract $\partial\Lambda_2(I_{bp}, S, s)/\partial s$ from $\partial\Lambda_2(I_{bp}, S, s)/\partial S$ and collect terms. Then using (3) we have

$$\tilde{\theta} x_c(\tilde{\theta}, s) \int_0^{\tilde{\theta}(\cdot)} f(\theta) d\theta - \int_0^{\tilde{\theta}(\cdot)} \theta x_c(\theta, s) f(\theta) d\theta - s \int_0^{\tilde{\theta}(\cdot)} \theta \frac{\partial x_c(\theta, s)}{\partial s} f(\theta) d\theta > 0,$$

which is the reverse of the inequality earlier in the proof. Using the same reasoning as earlier in the Theorem, the sum of the first two terms is positive or slightly negative. The third term is positive from Lemma 1. Consequently, an interior solution for the usage-based fee results in an optimal fixed fee with no limit, which is infeasible. Therefore, $s = 0$ and the constraint in (8) cannot be satisfied. Q.E.D.

LEMMA 4. *Aggregate output of converting edge providers is decreasing, while aggregate output of non-converting edge providers is increasing, in investments in Internet capacity.*

Proof: Differentiating aggregate output with respect to the investments in Internet capacity:

$$\frac{\partial X_c(\cdot)}{\partial I} = -x_c(\tilde{\theta}, 0) f(\tilde{\theta}) \frac{\partial \tilde{\theta}(\cdot)}{\partial I} < 0$$

and

$$\frac{\partial X(\cdot)}{\partial I} = x(\tilde{\theta}, U_c(\cdot), I) f(\tilde{\theta}) \frac{\partial \tilde{\theta}(\cdot)}{\partial I} + \int_0^{\tilde{\theta}(\cdot)} \frac{\partial x(\theta, U_c(\cdot), I)}{\partial I} f(\theta) d\theta + \int_0^{\tilde{\theta}(\cdot)} \frac{\partial x(\theta, U_c(\cdot), I)}{\partial U_c} \frac{\partial U_c(\cdot)}{\partial I} f(\theta) d\theta > 0.$$

The sign of the first equation is from Lemma 3, and the sign of the second equation is from Lemmas 2, 3, and the first part of Lemma 4. Q.E.D.

THEOREM 3. *If there are no investments in capacity, then (a) all edge providers in the open Internet and their end users are worse off; (b) a two-tier Internet is socially beneficial only if the increase of end user surplus and edge provider profits from the fast-lane Internet outweigh the fixed costs of providing a fast-lane Internet as well as the additional congestion costs and negative externalities on the open Internet that result from higher bandwidth use from edge providers that convert to the fast-lane Internet.*

Proof: (a) is straightforward from the edge providers' optimality conditions and Assumption 3, where edge providers that convert to the fast-lane produce more output and consequently more congestion on the open Internet.

(b) If only the edge provider with the highest bandwidth requirement per unit of output, $\theta = 1$, converts to the fast-lane and the broadband provider is not required to invest in the open Internet, $I = 0$, then the according change in welfare can be written as:

$$\Delta B(0) =$$
$$\Delta EUS(X_c(\cdot), X(\cdot)) + PR_c(1, x_c(1,0)) - PR(1, x(1,0,0)) - [K(1, U(\cdot), x_c(1,0), 0)$$
$$-K(1, U(\cdot), 0, 0)] - [\omega(q(1, U(\cdot), x_c(1,0), 0)) - \omega(q(1, U(\cdot), 0, 0))] - \tau.$$

Again from the edge providers' optimality conditions, the output of converting edge providers is higher than the output of non-converting edge providers, $x_c(1,0) > x(1,0,0)$. Thus, the conversion of the edge provider with the highest bandwidth requirement per unit of output leads to an increase of end user surplus and edge provider profits through increased output representing a positive effect on social welfare. Due to the associated higher bandwidth use in the fast-lane, the congestion costs and negative externalities are higher if the edge provider with the highest bandwidth requirement per unit of output converts away from the open Internet representing a negative effect on welfare. The fixed costs of providing a fast-lane also represent a negative effect on welfare. Q.E.D.

3. Effects of Investment in Internet Capacity on Welfare Components

Effects on End User Surplus: see Lemma 4 and its Proof.

Effects on Edge Provider Surplus: Differentiating edge provider surplus with respect to investments in Internet capacity:

$$\begin{aligned}
&\frac{\partial EPS(\cdot)}{\partial I} = \\
&-[PR_c(\tilde{\theta}, x(\tilde{\theta}, 0)) - S] f(\tilde{\theta}) \frac{\partial \tilde{\theta}(\cdot)}{\partial I} + [PR(\tilde{\theta}, x(\tilde{\theta}, U_c(\cdot), I)) - K(\tilde{\theta}, U(\cdot), U_c(\cdot), I)] f(\tilde{\theta}) \frac{\partial \tilde{\theta}(\cdot)}{\partial I} \\
&+ \int_0^{\tilde{\theta}(\cdot)} \frac{\partial PR(\theta, x(\theta, U_c(\cdot), I))}{\partial x} \frac{\partial x(\theta, U_c(\cdot), I)}{\partial I} f(\theta) d\theta - \int_0^{\tilde{\theta}(\cdot)} \frac{\partial K(\theta, U(\cdot), U_c(\cdot), I)}{\partial I} f(\theta) d\theta \\
&- \int_0^{\tilde{\theta}(\cdot)} \frac{\partial K(\theta, U(\cdot), U_c(\cdot), I)}{\partial U} \frac{\partial U(\cdot)}{\partial I} f(\theta) d\theta - \int_0^{\tilde{\theta}(\cdot)} \frac{\partial K(\theta, U(\cdot), U_c(\cdot), I)}{\partial U_c} \frac{\partial U_c(\cdot)}{\partial I} f(\theta) d\theta.
\end{aligned}$$

Rearranging terms, using (2) and noting that $s = 0$, and cancelling terms we have

$$\begin{aligned}
\frac{\partial EPS(\cdot)}{\partial I} &= \int_0^{\tilde{\theta}(\cdot)} \frac{\partial PR(\theta, x(\theta, U_c(\cdot), I))}{\partial x} \frac{\partial x(\theta, U_c(\cdot), I)}{\partial I} f(\theta) d\theta - \int_0^{\tilde{\theta}(\cdot)} \frac{\partial K(\theta, U(\cdot), U_c(\cdot), I)}{\partial I} f(\theta) d\theta \\
&\quad - \int_0^{\tilde{\theta}(\cdot)} \frac{\partial K(\theta, U(\cdot), U_c(\cdot), I)}{\partial U} \frac{\partial U(\cdot)}{\partial I} f(\theta) d\theta - \int_0^{\tilde{\theta}(\cdot)} \frac{\partial K(\theta, U(\cdot), U_c(\cdot), I)}{\partial U_c} \frac{\partial U_c(\cdot)}{\partial I} f(\theta) d\theta.
\end{aligned}$$

Using Assumption 2 and Lemma 2, the first term is positive. From Assumption 3 the second term is positive. Using Assumption 3 as well as Lemma 4, the third term is negative and the fourth term is positive.

Effects on Broadband Provider Surplus: Differentiating broadband provider surplus with respect to investments in Internet capacity:

$$\frac{\partial BPS(\cdot)}{\partial I} = -S f(\tilde{\theta}) \frac{\partial \tilde{\theta}(\cdot)}{\partial I} - 1 < 0.$$

Using Lemma 3, the derivative is negative.

Effects on Total Social Value of Negative Externalities: Differentiating the aggregate negative externalities with respect to the investments in Internet capacity:

$$\begin{aligned}
\frac{\partial Q(\cdot)}{\partial I} &= q(\tilde{\theta}, U(\cdot), U_c(\cdot), I) f(\tilde{\theta}) \frac{\partial \tilde{\theta}(\cdot)}{\partial I} + \int_0^{\tilde{\theta}(\cdot)} \frac{\partial q(\theta, U(\cdot), U_c(\cdot), I)}{\partial I} f(\theta) d\theta \\
&\quad + \int_0^{\tilde{\theta}(\cdot)} \frac{\partial q(\theta, U(\cdot), U_c(\cdot), I)}{\partial U} \frac{\partial U(\cdot)}{\partial I} f(\theta) d\theta + \int_0^{\tilde{\theta}(\cdot)} \frac{\partial q(\theta, U(\cdot), U_c(\cdot), I)}{\partial U_c} \frac{\partial U_c(\cdot)}{\partial I} f(\theta) d\theta.
\end{aligned}$$

Using Lemma 3 the first term is positive. Using Assumption 5 the second term is negative. Using Assumption 5 and Lemma 4, the third term is positive and the fourth term is negative.