Customer Segmentation, Pricing, and Lead Time Decisions: A Stochastic-User-Equilibrium Perspective

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ABSTRACT

Multiple price/lead time options are often used as a competitive tool in supply chain operations when customers have heterogeneous price and lead time preferences. We study a two-echelon supply chain network consisting of manufacturers and retailers facing customers that differ in their price- and time-sensitivity. We examine how many price/lead time options should be provided by manufacturers and retailers under decentralized and centralized supply chain management with time-cost tradeoffs. We adopt a stochastic-user-equilibrium (SUE) approach in a supply chain network by incorporating discrete choice theory and using a multinomial logit-based variational inequality to express equilibrium conditions. This is one of the first papers to study time-cost tradeoffs in a supply chain network by introducing an SUE approach. One critical part of our analysis is the establishment of concavity of profit functions, which allows for analytical derivation of the equilibrium strategies. Another critical part of our analysis is the development of SUE conditions in decentralized and centralized supply chain networks. We demonstrate that the variance of heterogeneous customers' time-sensitivity distribution plays a crucial role in customer segmentations in a time-cost tradeoff supply chain. We find that under SUE conditions, there exists a unique equilibrium in the decentralized and centralized supply chain networks, respectively. We conduct comparative statics analyse to demonstrate the effect of SUE and UE conditions and the influence of decentralized versus centralized supply chain management paradigms. We show how SUE approach can be applied to supply chain network management with time-cost tradeoffs. Our approach can be extended to other tradeoffs decision problems in supply chain network management, especially those in which customers are heterogeneous.

1. Introduction

Responsive supply chains rely on coordination between upstream and downstream firms where each face tradeoffs between lead time and costs. That is time-cost tradeoffs. When customers have heterogeneous price and lead time preferences, the competitiveness of each supply chain in such a network is shaped by pricing and lead time decisions made by individual firms. Matching time-cost tradeoffs in supply with heterogeneous demand has always been a challenge for supply chain management (Hu and Zhou, 2022; Liu et al., 2007). Although significant progress has been made (Namakshenas et al., 2022; Li et al., 2018; Nagurney et al., 2018; Yu and Nagurney, 2013; Masoumi et al., 2012), many firms admit that increasing their investment in human and financial capital to improve time-based competitiveness has not resulted in higher profits or competitive advantage. The major difficulties that cause many firms to be trapped within time-based competition emanate from ignoring customers' heterogeneity (Yu and Nagurney, 2013), firms' actual time-cost tradeoffs (Kim et al., 2012), and the capabilities of competitors (Stalk and Webber, 1993). However, it is possible to develop a network framework model in both decentralized and centralized supply chains to optimize and coordinate price/lead time (P/T) decisions for particular customer characteristics. Many online firms, such as Amazon, Dell, and Walmart, and offline firms, such as German supermarket Globus (Lütke Entrup et al., 2005), California supermarket Lucky (So, 2000), theme parks (Sainathan, 2020), UPS, and FedEx (Zhao et al., 2012) have offered competitive P/T menus for customers.

Golrezaei et al. (2020) noted that customers have become increasingly time-sensitive for almost all products or services: Some customers prefer to pay more for timely service, and others choose the opposite. When customers differ in both price- and time-sensitivity, it is common that firms in time-cost tradeoff supply chains offer a P/T menu

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for customers to choose from. For example, when customers send packages by FedEx, there are five price-delivery time options to choose from. Dell provides several lead times (an identical laptop with different delivery time services) for customers (Dobson and Stavrulaki, 2007). This is defined as a “differentiated quotation mode" by Zhao et al. (2012). However, others provide only a single P/T option to all customers (e.g., Ameristock. http://www.ameristock.com). Both modes are extensively used in practice. Only a few models have addressed optimal P/T menu designs or the optimal market segmentation problem from a monopoly provider (e.g., Afeche and Pavlin, 2016; Braouezec, 2012). The effectiveness of equilibrium P/T menu designs for time-cost tradeoff supply chains and heterogeneous customers is still an open question (Federgruen and Hu, 2016).

In time-cost tradeoff supply chains, each firm has its own relationship between lead time and cost due to factors such as production schedules, personnel assignments, and geographic locations. Therefore, how an individual firm decides on how many options to provide their customers is a critical question, especially when the time-cost tradeoffs faced by upstream and downstream firms differ. In this context, we examine how firms in either centralized or decentralized supply chains determine how many P/T options should be provided when customers have heterogeneous preferences. Our purpose is to find how customers' time-sensitivity distribution, time-cost tradeoff, and management paradigms affect customer segmentation, pricing, and profits in an equilibrium where customers are heterogeneous in their price- and time-sensitivity.

Much of the literature recognizes that competition is no longer between stand-alone companies, but rather between supply chains (Ma et al., 2020; Christopher, 2016). Some scholars deem that competition has now shifted from the level of an integrated supply chain to a decentralized supply chain, where individual firms collaborate with each other (Ray et al., 2005). We design a corresponding centralized supply chain as a benchmark. We want to answer the following questions for both decentralized and centralized supply chain competition. Our first set of questions concern customer segmentation: What underlies optimal customer segmentation under a time-cost tradeoff supply chain? What characterizes equilibrium prices, times, and flows? How is the supply chain's management approach (decentralized and centralized) affected by the distribution of customer heterogeneity (i.e., the mean and variance of customer preferences)? Our second set of questions concern time-cost coordination between time-cost tradeoff supply chains: What affects how many P/T options should be provided by individual firms? How much market share will these supply chains gain in competition? How are profits affected by whether the supply chain is centralized or decentralized?

To address the above questions, we adopt *a stochastic-user-equilibrium* approach to study a supply chain network by integrating a discrete choice model into a supply chain economy. (Wardrop, 1952, p. 345) defined an equilibrium condition (*User-equilibrium* (UE)) in a transportation network whereby “*The journey times of all routes actually used are equal, and less than those which would be experienced by a single vehicle on any unused route*”. In other words, “*An equilibrium will be reached when no traveler can be improved by unilaterally changing routes*” (Sheffi, 1985, p. 19). The assumption that underlies Wardrop's network equilibrium condition is that all individuals have full information and have identical (homogeneous) preferences. Sheffi generalizes and relaxes Wardrop's condition calling it *a stochastic-user-equilibrium* (SUE) condition by introducing discrete choice theory into the transportation network: “*An equilibrium will be reached when no traveler believes that his travel time can be improved by unilaterally changing routes*” (Sheffi, 1985, p. 20). The main assumption of Sheffi's equilibrium condition is that travelers are heterogeneous and do not have full route information. Sheffi's equilibrium condition is more general than Wardrop's equilibrium condition and is particularly applicable in an actual transportation network.

The Wardrop's network equilibrium condition is widely used in supply chain network equilibrium models. As mentioned in literature review, not enough attention has been paid to SUE condition in supply chain network problems (Salarpour and Nagurney, 2021; Dasaklis et al., 2012). If participants do not have full information and are heterogeneous, then the difference between the equilibrium solutions reflecting SUE and UE conditions in a supply chain network may be large. Compared with a transportation network where technology such as a global positioning system (GPS) can be used to get more route information, it is less realistic to assume that all participants, especially firms, have full information about customers. Figure 1 shows the evolution of Wardrop's model to our proposed supply chain network model.

We model customer heterogeneity using a discrete choice model, specifically a multinomial logit model, that represents the probability of a customer choosing one of the goods (or services) provided by supply chains. The introduction of a discrete choice model into a supply chain network results in two technical challenges. First, firm profits are not concave in pricing and lead time. Therefore, traditional convex optimization cannot be used to solve for our supply chain network equilibrium. A critical contribution in our analysis is the establishment of concavity of the objective functions, which allows for analytical derivation of the equilibrium strategies. Second, the equilibrium

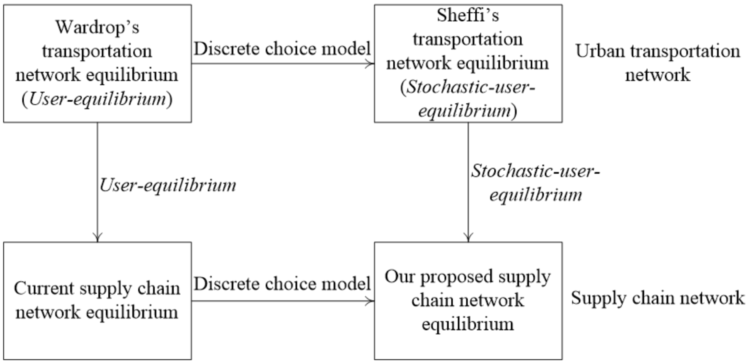


Figure 1: The assumption of the proposed supply chain network equilibrium.

conditions are not easy to use in practice, except that they can be characterized and formulated as equilibrium conditions mathematically. How to integrate customer discrete choice behavior into a supply chain network equilibrium model and then express the equilibrium conditions is the second challenge. The applications of the multinomial logit model to supply chain network models are limited (Federgruen and Hu, 2016). In our analysis, another contribution is that equilibrium conditions are formulated as multinomial logit based-variational inequality problems for both decentralized and centralized supply chains.

Using SUE conditions, we develop a general two-echelon supply chain model in which manufacturers and retailers offer multiple P/T options that we call a P/T menu. Retailers in the second echelon serve heterogeneous customers by combining their time-cost tradeoffs with manufacturers' P/T menus. In our model, a supply chain may provide a P/T menu as a competitive tool by coordinating time-cost tradeoffs of upstream and downstream firms. We consider two management paradigms, decentralized and centralized supply chain management. The novelty in our work is to bridge P/T menu designs with time-cost tradeoff supply chains based on SUE conditions. We then study how the UE and SUE conditions influence the firms' (supply chains') profits, prices, and lead times. We also explore the impact of customers' time-sensitivity distribution on the optimal customer segmentation, equilibrium prices and lead times. In addition, we study how the equilibrium flows and lead times of the chains are affected by the time-cost tradeoffs of the firms already participating in the chains.

Our approach has several advantages. First, our model better matches time-cost tradeoff supply and heterogeneous customers' demand. It accomplishes this by incorporating customers' price- and time-sensitivity using a multinomial logit model in a competitive supply chain network. Specifically, we consider joint pricing in supply chains in which the different echelons coordinate decisions to maximize profit. Customers are segmented by their P/T preferences. Each supply chain competes and optimizes its profit in a time-cost tradeoff setting without full information about customers, that in turn are utility maximizers. Second, our model optimizes supply chain profits without converting a multi-objective problem into a single-objective problem at the firm level. We adopt the discrete choice model and use multinomial logit-based variational inequalities to formulate the equilibrium conditions for a centralized and a decentralized supply chain network. Third, our model allows for heterogeneous customers in demand markets, including price- and time-sensitive customers. This distinctive feature not only uses the mean of customers' time-sensitivity distribution (Liu et al., 2007) but also the variance of customers' time-sensitivity distribution. We further analyze and discuss the impacts of customer heterogeneity on equilibrium solutions and customer segmentation.

The remaining sections proceed as follows. Section 2 reviews relevant literature regarding time-cost tradeoff supply chain management and customer segmentation. Section 3 gives the notation and assumptions used in our model. Section 4 develops a supply chain SUE model for managing time-cost tradeoff supply chains (both decentralized and centralized) and heterogeneous customers. Here, multinomial logit-based variational inequalities are used to express equilibrium conditions. Section 5 provides our analysis and results. Section 6 shows numerical examples and a sensitivity analysis. Section 7 summarizes our results, provides the implications, and indicates areas of interest for further research.

2. Literature Review

The supply chain management literature has studied how a supply chain should deal with heterogeneous customers with price- and time-sensitivity; e.g., Golrezaei et al. (2020), Afeche and Pavlin (2016), Besbes and Lobel (2015), Shang and Liu (2011), and Liu et al. (2007). The Marketing and Operations literature has explored the critical impact of customers' time-sensitivity on their consuming behaviors in practice (Golrezaei et al., 2020; Larson et al., 1991). From the customers' psychological perspective, research shows that different customer types exist in any sort of product or service market (Golrezaei et al., 2020; Suzuki, 2000; Kumar et al., 1997). For example, some customers prefer to pay more for timely service, while others choose the opposite. Accordingly, we model such individual choice behavior by using a discrete choice model and then analyze competitive actions between supply chains.

P/T decisions in the firm level have been discussed extensively, including single firm optimization (Afeche et al., 2019; Mussa and Rosen, 1978) and competition among firms (Sainathan, 2020; So, 2000; Easton and Moodie, 1999). The above decisions are described as profit maximization problems at the service level and cost constraints in a monopoly or oligopoly setting (in which lead time is a critical factor affecting the service level). Considering prioritization of customers by service providers, Afeche et al. (2019) design the pricing/lead time menu for a monopoly service in which customers differ in their demand rates. Sainathan (2020) also considers prioritization of customers and derives the equilibrium conditions for service providers that characterize three different types of equilibrium associated with providing single service and(or) differentiated service. The work above focuses on optimization problems at the firm level, the coordination of supply chain competition is not part of their models.

In supply chain network management much of the network equilibrium literature considers time-cost tradeoff problems where operational time can be converted into operational cost at the firm level. This research includes Kadziński et al. (2017) and Chan and Chung (2004). Many methods in literature have been used to improve the supply efficiency of a time-cost tradeoff supply chain including using the weighting method, quality deterioration, marginal value of time, discarding cost, and exponential scoring. Considering discarding costs affected by the operational time on each link, Masoumi et al. (2012) develop a pharmaceutical product network model and construct a generalized network oligopoly model for perishable products by introducing a concept called arc multiplier. A loss function (Ahumada and Villalobos, 2011) and a product's marginal value of time (Blackburn and Scudder, 2009) are two common tools to convert processing time into cost in time-cost tradeoff supply chains.

Studies in time-cost tradeoff supply chains have illustrated different methods to optimize the profits of firms considering operation time, including linear and nonlinear conversions (Sabri and Beamon, 2000; Masoumi et al., 2017; Kadziński et al., 2017). Additionally, there is much literature that focuses on the Pareto frontier of time-cost tradeoff supply chains (Farahani and Elahipanah, 2008). A decision-maker might select an optimal option from a series of non-differential optimal options according to the decision-maker's preference. These studies focus on internal supply efficiency, whereby customers' preferences or the connection between supply and demand are not considered. However, in a supply chain composed of different firms with time-cost tradeoffs, it is possible to collaborate time-cost arrangements at the supply chain level based on customer heterogeneity.

Equilibrium models for between-supply chain competition have been further studied in last two decades, including a supply chain network equilibrium model (Nagurney, 2021a; Zhang, 2006; Nagurney et al., 2002a), an identical linear assembly chain model (Corbett and Karmarkar, 2001), and model extensions and applications (Rezapour et al., 2017; Yu and Nagurney, 2013; Nagurney and Nagurney, 2010). Most existing supply chain network equilibrium models for time-sensitive products focus on characteristics that are continuous and significant change in the quality from origin to destination of a supply chain (Besik and Nagurney, 2017; Akkerman et al., 2010). Furthermore, the methods to manage the quality of time-sensitive products have been discussed from time control (Yu and Nagurney, 2013) to temperature control (Rong et al., 2011) in supply chain management. We are not limited in the linear or nonlinear conversion at the firm level between the conflicting objectives. Our study focuses on the coordination between the time-cost tradeoff supplies and heterogeneous demands.

Another stream of literature shows how to optimize time-cost tradeoffs from the demand side. Market demands impacted by time-cost tradeoffs can be classified into several categories: (1) price/quality (Nagurney et al., 2018; Jabarzare and Rasti-Barzoki, 2020; Wang et al., 2017), (2) price/lead time (Nagurney et al., 2014; Zhu, 2015; Hua et al., 2010), and (3) utility/surplus (Zhao et al., 2012; Xia and Rajagopalan, 2009). In Nagurney et al. (2018), the dynamics of quality affected by processing time and temperature in a food supply chain network is a single parameter in competitive demand markets. The demand functions are generated from price and quality, and a supply chain network equilibrium condition is given by using variational inequalities. In time-sensitive supply chains, Jabarzare and Rasti-Barzoki (2020) develop non-cooperative and cooperative game models to analyze the effects of the competition on optimal pricing and quality decisions. Considering the influence of time-sensitive product quality on demand functions in decentralized or centralized supply chains, Wang et al. (2017) develop a game model to illustrate the impact of customers and markets on manufacturers' channel choice. Taleizadeh et al. (2018) further investigate pricing strategies and quality decisions in closed-loop supply chains. Similarly, Maiti and Giri (2015) assume price and quality dependent demands and develop a game model to analyze the impact of supply chain structures on pricing strategies and profits. Along this line, Wang and Li (2012) investigate the optimal pricing in a supply chain for a perishable product by using quality deterioration and price-quality functions. Jin and Ryan (2012) bridge supply and demand of time-cost tradeoff supply chains through exponential score functions at the firm level and by a multinomial logit model as the demand side. In our approach, there are two tradeoffs in a firm's time-cost and customers preference and the connection between them: how a time-cost tradeoff supply chain strikes a balance between operational time and cost to match the heterogeneous customers' preferences in a time-sensitive market environment. We introduce discrete choice theory into time-cost tradeoff supply chains and formulate the equilibrium conditions as multinomial logit-based variational inequality problems based on SUE conditions. To adopt SUE conditions makes the model and computational framework more applicable when firms (or supply chains) are not able to obtain full information about customers.

In the supply chain management literature, Liu et al. (2007) adopt a leader-follower game model in a decentralized supply chain in which the supplier determines the promised delivery lead time and wholesale price first and then the retailer determines the retail price. Then, they examine the effect of market factors (price and lead time sensitivity factors) and operational factors on equilibrium solutions. Jin and Ryan (2012) develop an outsourced supply chain equilibrium model in which a buyer has to work with multiple suppliers and the demand function depends on the retail prices and service levels. In dual-channel supply chains, Hua et al. (2010) present a Stackellberg game for decentralized and centralized supply chains to analyze the impacts of lead time and customers' channel choice on firms' pricing strategies. In these articles, a Stackelberg game is used as the competitive framework in a decentralized supply chain. Lead time is treated as a decision variable affecting firm profit. We also consider the first-mover advantage in decentralized supply chains. Our work focuses on between-supply chain competition and collaboration of time-cost tradeoff supply chains.

Three articles on P/T menu design in time-cost tradeoff supply chains are close to our study. Ma et al. (2020) design a time-based supply chain network model to bridge time-cost tradeoff supply chains with heterogeneous price- and time-sensitive customers. Their model considers only fixed P/T options for customers' choice and is used to analyze the impact of existing P/T menus on heterogeneous customers. Lead time as a decision variable plays a critical role in the operational cost of firms in supply chains (Nagurney et al., 2014). Our objective is to design an optimal P/T menu for both time-cost tradeoff supply chains and heterogeneous customers. Afeche and Pavlin (2016) design a fixed price/lead time menu to optimize revenues from heterogeneous time-sensitive customers for a monopoly provider. Like our model, theirs also concerns customer segmentation to optimize revenues. However, we further consider the variance of heterogeneous customer preference distribution, which has a key role in our findings. The discrete choice theory in the model extends supply chain versus supply chain competition to multiple dimensions without the multi-objective conversion at the firm level.

Our study builds upon earlier cost-time tradeoff supply chain models, customer segmentation, and makes several contributions. First, we adopt the SUE conditions approach rather than Wardrop's UE conditions, as SUE conditions are more feasible and applicable in practice than existing models. Second is use of heterogeneous customer types. Not only does the mean of time-sensitivity coefficients have a crucial impact on optimal customer segmentation and equilibrium flows, but also the variance has a critical role in our findings. Third is optimal customer segmentation and P/T design in time-cost tradeoff supply chains. We provide an equilibrium condition for both decentralized and centralized competing supply chain networks with three features: (1) An optimal P/T menu which serves both supply chains and heterogeneous customers; (2) Cooperation and competition in supply chains to optimize revenues and meet heterogeneous customers; and (3) Discrete choice theory is incorporated into the supply chain network to bridge heterogeneous customers with time-cost tradeoff supply chains without assuming firms have full information about their customers. The possible impacts of SUE conditions in supply chain management are listed in Table 1. Table 2 compares the other key related articles with this study.

Table 1
Some inconsistent results that can be explained by SUE conditions.

Scenario	Results under UE conditions	Some inconsistent results that can be explained by SUE conditions
Supply chain inventory management	•Unneeded inventory •Stockout	•Systematic high inventory(Chopra and Sodhi, 2014) •Inventory Routing Problems(Bertazzi et al., 2013)
Supply chain risk management	•Overestimating the chances •Opportunity loss	•Uncertain supply yields(Xie et al., 2021) •Uncertain price(Li and Kouvelis, 1999) •Product substitution(Rajaram and Tang, 2001)
Decision-maker behavior	•Identical perceptions •Perfect information	•No full information(Wakolbinger and Cruz, 2011) •Bounded rationality(Su, 2008)
Demand management	•Supply and demand mismatch	•Overproduction(Swinney, 2011)

Table 2
Price/lead time supply chain literature.

Characteristics	Afeche and Pavlin (2016)	Boyaci and Gallego (2004)	Liu et al. (2007)	Zhao et al. (2012)	Jin and Ryan (2012)	Shang and Liu (2011)	This study
Heterogeneous customers	×		×		×	×	×
Two echelons		×	×	×	×	×	×
Price/lead time menu	×	×		×			×
Inter supply chains competition		×					×
Cost/lead time tradeoff	×				×		×

3. Notation and assumptions

In this section we introduce our notation and assumptions including the supply chain SUE conditions, and three definitions essential to understanding the way we model time-cost tradeoff supply chains. For the sake of simplicity and clarity we do not consider the effect of inventory as we implicitly assume that markets clear in our supply chain network equilibrium. Supply being balanced with demand in equilibrium is a common implicit assumption in the literature (Liu and Wang, 2019; Daultani et al., 2015; Zhang, 2006; Nagurney et al., 2002b).

Definition 1 (Operation link and interface link). An *operation link* denotes substantial business functions in a supply chain, such as production, delivery, and procurement. An *interface link* describes a coordination function between two contiguous operation links in a supply chain.

Let A represent the set of all operation links and a denote a typical one, $a \in A$. Firms provide a quote associated with lead time options on each operation link, where t_a^j is lead time j on operation link a, and $|T_a|$ is the cardinality of the set $T_a = \left\{t_a^1, \cdots, t_a^j, \cdots, t_a^{|T_a|}\right\}$. Let M and N denote the number of manufacturers and retailers, and let m and n denote a typical manufacturer and retailer, respectively. Thus, t_m^j denote lead time j on operation (manufacturing) link m.

Each manufacturer in a supply chain has to coordinate its production schedule and delivery arrangement with downstream firms. Let B represent the set of interface links and b denote a typical interface link $b \in B$. Let t_b^k denote lead time k on interface link b, and $|T_b|$ is the cardinality of the set $T_b = \left\{t_b^1, \cdots, t_b^k, \cdots, t_b^{|T_b|}\right\}$.

Hence, let S denote the set of chains. A typical chain s, made up of operation and interface links, provides a quote associated with delivery of products (or service) to the end markets for their customers. Let t_s^l be lead time l on chain

s. And $|T_s|$ is the cardinality of the set $T_s = \left\{t_s^1, \cdots, t_s^l, \cdots, t_s^{|T_s|}\right\}$. Therefore, lead time l on chain s is the aggregate lead times on link a and b that participate in the lead time l on chain s,

$$t_s^l = \sum_{a\in s}\sum_{j=1}^{|T_a|} t_a^j \cdot \varphi_{jl} + \sum_{b\in s}\sum_{k=1}^{|T_b|} t_b^k \cdot \varphi_{kl}, \tag{1}$$

where φ_{jl} and φ_{kl} denote the binary link-chain coefficient. If the lead time j of link a is contained in the lead time l of chain s, then $\varphi_{jl} = 1$, otherwise $\varphi_{jl} = 0$. For example, we assume there is a supply chain that consists of a producer and a 3PL, in which the producer provides two P/T options for its downstream firm and the 3PL provides only one P/T option for customers. Hence, this supply chain evolves into two chains from the perspective of graph theory. Figure 3 shows the lead time in a typical supply chain. The lead time on the top chain is, $t_{s_1}^1 = t_{a_1}^1 + t_{b_1}^1 + t_{a_2}^1$.

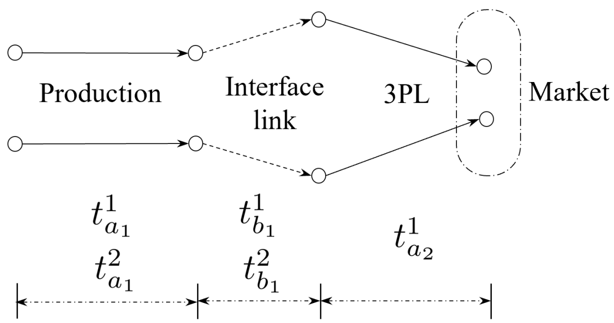


Figure 2: The lead time in a typical supply chain.

The firm that provides more than one P/T option, $|T_s| \geq 2$, in a supply chain has higher operational costs to coordinate production, storage, and transportation in its supply chain. The operational cost and lead time on an interface link embody the coordination effectiveness between echelons. Thus, the cost of chain s is the aggregate cost incurred in all the operation and interface links of this chain in transporting the final product to customers. We assume that each operation link does not charge its participating chains at a uniform rate because the lead time requirement varies across different chains.[1] Here let x_s denote the quantity supplied on chain s, x_a denote the quantity supplied on link a, $\bar{C}_s(t_s, x_s)$ denote the operational cost on chain s, $C_s(t_s)$ denote the cost per unit on chain s, $C_s(t_s) = d\bar{C}_s(t_s, x_s)/dx_s$, $\bar{c}_a(t_a, x_a)$ and $\bar{c}_b(t_b, x_b)$ denote the operational cost on link a and link b, $c_a(t_a)$ denote the cost per unit on operation link a, $c_a(t_a) = d\bar{c}_a(t_a, x_a)/dx_a$, and $c_b(t_b)$ denote the cost per unit on interface link b. Let $\bar{c}_{as}(t_a, x_a)$ denote the operational cost that the operation link a charges chain s. The chain cost function on chain s is:

$$\bar{C}_s(\cdot) = \sum_{a\in S}\sum_{s=1}^{|T_a|} \bar{c}_{as}(\cdot)\cdot\delta_{as} + \sum_{b\in B_s} \bar{c}_b(\cdot), \tag{2}$$

where δ_{as} denotes the binary link-chain coefficient and B_s denotes the set of interface links participating in chain s. If link a is contained in chain s, then $\delta_{as} = 1$, otherwise $\delta_{as} = 0$. In our model, the link cost functions are monotone. The link cost includes such costs as production cost, delivery cost, insurance, labor, energy, etc. Each firm provides $|T_a|$ P/T options over a selling season through a supply chain.

Definition 2 (Degree of Time-Cost Tradeoff). We define the degree of a time-cost tradeoff as the first-order derivative of the time-cost function with respect to the lead time, $dc(\cdot)/dt = c'(\cdot) < 0$.

Definition 2 reflects the cost that firms (supply chains) incur in reducing a unit lead time. In our model, the time-cost function can be treated as a time-cost relationship on operation and interface links. It is common to assume the cost

[1] Under the conditions of time-cost tradeoff supply chains, this assumption is common. For example, UPS charges its downstream firms based on delivery time even if from the same origin to destination.

function is a decreasing function of lead time to reflect the time-cost tradeoff relationship in a single firm or a supply chain operation (Adak and Mahapatra, 2020; Nagurney et al., 2014; Feng et al., 2000; Demeulemeester et al., 1996; De et al., 1995). It is also supported by empirical evidence in the software (Badiru, 1991) and airline industries (Ramdas and Williams, 2006).

Definition 3 (Stochastic-user-equilibrium (SUE)). We generalize the supply chain network equilibrium conditions as: An equilibrium is reached when no supply chain (or firm) believes that their profit can be improved by unilaterally changing its P/T menu.

We adopt the SUE conditions and integrate discrete choice into our supply chain network equilibrium models. A probabilistic function is used to describe customer choice behavior as all factors affecting customer choice behavior cannot be completely observed by firms (Baltas and Doyle, 2001).

Supply chain s offers a P/T option i, with selling price p_{ni} and lead time t_s^i to serve customers that are partitioned into segments. We denote a typical option by i. Selling price p_{ni} denotes the retail price of retailer n for customers in segment i. Processing time t_s^i is the aggregate processing time in all links participating in P/T option i. For illustration, we use a two-dimensional structure where a customer faces a choice set of I P/T options offered by different supply chains (Berbeglia et al., 2022; Gilbride and Allenby, 2004). The customer obtains utility $U_i(p_{ni}, t_s^i, \tilde{\alpha}, \tilde{\beta})$ from choosing option i at selling price p_{ni} and lead time t_s^i:

$$U_i(p_{ni}, t_s^i, \tilde{\alpha}, \tilde{\beta}) = V_i + \varepsilon = w_i - \tilde{\alpha} \cdot p_{ni} - \tilde{\beta} \cdot t_s^i + \varepsilon, \ \forall i \in I, \tag{3}$$

where V_i denotes the observed deterministic component (Kim and Park, 2017; Louviere et al., 2000), $\tilde{\alpha}$ and $\tilde{\beta}$ represent the customer price- and time-sensitivity coefficients, respectively, which are random variables that can follow any distribution, including the normal distribution, uniform distribution, and double exponential distribution. The ratio $\tilde{\alpha}/\tilde{\beta}$ represents the typical consumer time value per unit time (Anderson et al., 1992). Let w_i denote a customer's intrinsic value of option i, and ε is a random variable which reflects other factors that affect a consumer's utility that firms do not know (Chikaraishi and Nakayama, 2016; Li, 2011). All customers choose the chain that maximizes their utility in equilibrium.

We simplify our analysis by treating price-sensitivity as a constant, $\bar{\alpha}$, leaving the time-sensitivity coefficient as a random variable. This incurs little loss of generality as one can create a P/T menu with a list of prices matched with chosen lead times. In what follows we economize on our notation so that $U_i(p_{ni}, t_s^i, \tilde{\alpha}, \tilde{\beta}) = U_i(\cdot)$. The gradient of customer's utility decreases in the decision variables, selling price and lead time. If the utility of choosing any option i is negative, $U_i(\cdot) < 0, \forall i \in I$, then the customer will reject all options (individual rationality, IR). If the utility of choosing option i is positive, $U_i(\cdot) > 0$ and $U_i(\cdot) = \max_{i \in I} \{U_i(\cdot)\}$, then the customer will choose option i (incentive compatibility, IC).

Assumption 1 (Customer Sensitivity Distributions). *The customer time-sensitivity coefficient, $\tilde{\beta}$, follows a normal distribution $N(\bar{\beta}, \sigma^2)$, where $\bar{\beta}$ and σ denote the mean and standard deviation, respectively, where $\tilde{\beta} > 0$.*

Such an assumption is common in literature (Zhang et al., 2015; Masiero and Nicolau, 2012). Statistical analysis methods are often used to estimate the customer sensitivity distributions from an observed data set collected by market survey (Ma et al., 2020; Raab et al., 2009; Lawson and Montgomery, 2006). We focus on the effect of customers' time-sensitivity distribution on the equilibria. Using discrete choice theory, we obtain the probability of a customer choosing option i as[2]

$$P_i(\cdot) = \frac{e^{U_i(\cdot)}}{1 + \sum_{i=1}^{I} e^{U_i(\cdot)}}, i = 1, \cdots, I. \tag{4}$$

Definition 4 (P/T menu). A *P/T menu* is a price quotation strategy offered to downstream firms or end market customers by firms in a supply chain.

[2] In some practical supply chains, especially in fashion and pharmaceuticals (Valletti, 2006; Szymanski and Valletti, 2005), parallel trade is prevented by firms. Therefore, the market segments are sealed. Then, if the market segments are perfectly sealed, the probability of choosing option i can be rewritten as, $P_i(\cdot) = \frac{e^{U_i(\cdot)}}{1+e^{U_i(\cdot)}}, i = 1, \cdots, I$.

The firm (or supply chain) offers multiple predetermined lead times and associated prices for downstream firms (or customers) to choose from. A P/T menu reflects the time-cost tradeoffs in firms whereby different lead times correspond to different costs in such things as production, storage, transportation, and communication. The P/T menu is also often referred to as a price menu or third-degree price discrimination (cf. price menu in Zhang et al. (2015) and price discrimination in Besbes and Lobel (2015)).

Each customer has a particular sensitivity to lead time, represented by coefficients $\tilde{\beta}$ in (3), and maximizes their utility by choosing a P/T menu option. Each supply chain provides $\left|T_s\right|$ options to customers. Taking all options as distinct, there exists I, $I = \sum_{s=1}^{S} \left|T_s\right|$, options in the end market.

4. Model Formulation

In this section we develop a supply chain network equilibrium model for decentralized supply chains and then for centralized supply chains, each facing different P/T options in a competitive environment. We derive several lemmas, propositions, and theorems. Then, we illustrate the equilibrium conditions that characterize decentralized and centralized supply chain networks, respectively.

From the customers' time-sensitivity distribution the number of possible customer segments of chain s is $\left|T_s\right| \geq 1$. Chain s provides different P/T options to different customer segments associated with price p_i as well as lead-times t_s^i, $i = \left(1, \cdots, \left|T_s\right|\right)$. Let h denote the scale of customers' time-sensitivity coefficients being selected by firms (or supply chains) to segment customers, $h \in [0, +\infty)$. If $h = 0$, then it represents customers that have no time-sensitivity. Otherwise, if $h = +\infty$, it represents customers that have extreme time-sensitivity. Customers are segmented consecutively by $h_i, i \in (1, \cdots, \left|T_s\right|)$ into $|T_s|$ groups. All customers belong to the interval covered by $h_i, i \in (1, \cdots, \left|T_s\right|)$. For example, if a supply chain decides to provide two P/T options for their customers, it means the customers are segmented consecutively by h_1 into two groups based on customers' time-sensitivity. For a given distribution of customer time-sensitivity $\tilde{\beta}$, let $f(\tilde{\beta})$ represent its probability density and $\bar{\beta}_i$ denote the expectation (mean) of customers' time-sensitivity in the customer segment that supply chain s targets with option $i, i \in (1, \cdots, \left|T_s\right|)$ such that

$$\bar{\beta}_i = E\left[h_{i-1}, h_i\right] = \int_{h_{i-1}}^{h_i} \tilde{\beta} \cdot f\left(\tilde{\beta}\right) d\tilde{\beta}, \text{ for } i = 1, \cdots, \left|T_s\right|. \tag{5}$$

If we substitute $\bar{\beta}_i$, $i \in \left\{1, \cdots, \left|T_s\right|\right\}$ into the utility function in (3), then using (4) we can obtain the probability of each customer segment choosing option i as the price-sensitivity coefficient is independent of the time-sensitivity coefficient (the mean price-sensitivity coefficient in each segment is its overall mean, $\bar{\alpha}$). Let Q represent the quantity of a continuum of customers in the end market. Therefore, we obtain the demand function of P/T option i, $d(\cdot)_i = Q \cdot P_i(p_{ni}, t_s^i, \bar{\alpha}, \bar{\beta}_i)$, where $P_i(p_{ni}, t_s^i, \bar{\alpha}, \bar{\beta}_i)$ represents the probability of customers choosing option i from (4) which in turn depends on (3). To economize on space, let $P_i(\cdot)$ represent $P_i(p_{ni}, t_s^i, \bar{\alpha}, \bar{\beta}_i)$.

From the above probability of customers choosing option i, the following lemma can be obtained.

Lemma 1. $\frac{\partial P_i(\cdot)}{\partial \bar{\alpha}} < 0$ *and* $\frac{\partial P_i(\cdot)}{\partial \bar{\beta}_i} < 0$, $\forall i$.

The proofs of this lemma and other lemmas, propositions, and theorems are provided in the appendix.

From Lemma 1, we know that the probability of choosing option i, $P_i(\cdot)$, is decreasing in the mean customer price-sensitivity and the segment-specific mean time-sensitivity. Lemma 1 provides two simple rules to understand the effects on the probability of choosing option i of customer sensitivity.

Recognizing that $\bar{\beta}_i$ depends on $f(\tilde{\beta})$ which in turn is a function of σ, from the above probability of customers choosing option i, we get the following lemma.

Lemma 2. *A threshold is the variance of customers' time-sensitivity distribution, σ. For a given customer segment $\left(h_{i-1}, h_i\right)$,*

(a) if $(h_{i-1}, h_i) \in [\bar{\beta} - \sigma, \bar{\beta} + \sigma]$, then the probability of choosing option i, $P_i(\cdot)$, is decreasing with the variance of customers' time-sensitivity distribution σ;

(b) if $(h_{i-1}, h_i) \notin [\bar{\beta} - \sigma, \bar{\beta} + \sigma]$, then the probability of choosing option i, $P_i(\cdot)$, is increasing with the variance of customers' time-sensitivity distribution σ, i.e.,

$$\frac{\partial P_i(\cdot)}{\partial \sigma} \begin{cases} \leq 0 & \text{if } h_{i-1}, h_i \in [\bar{\beta} - \sigma, \bar{\beta} + \sigma] \\ > 0 & \text{if } h_{i-1}, h_i \notin [\bar{\beta} - \sigma, \bar{\beta} + \sigma] \end{cases}, \forall i.$$

Lemma 2 shows that the effect on firms' market shares in different market segments depends on the variance of customers' time-sensitivity distribution. The impact of the variance of customers' time-sensitivity distribution on the choice probability of different customer segments is different and depends on σ. From Lemma 2, if a customer segment is located in $[\bar{\beta} - \sigma, \bar{\beta} + \sigma]$, then the smaller the variance is of customers' time-sensitivity coefficient, the higher the probability is of choosing the option. Lemma 2 provides a guideline to understand the impact of σ on the probability of choosing option i in different segments under normally distributed customer time-sensitivity.

In the next sections, we illustrate equilibrium solutions in different supply chain networks. For the convenience, we adopt superscript D and superscript C to represent decentralized and centralized supply chains, respectively. For instance, π^D and π^C denote the profits for a decentralized and a centralized supply chain, respectively.

4.1. Decentralized supply chain network

In this section, we design a two-echelon decentralized supply chain network consisting of manufacturers and retailers. Next, we model the optimal behavior for firms in decentralized supply chains. We then derive the equilibrium conditions expressed by multinomial logit-based variational inequalities.

We consider competition as a leader-follower Stackelberg game. In our decentralized supply chain, the manufacturer (leader) pursues its own profit maximization by choosing wholesale prices and its own lead times. The retailer (follower) maximizes its own profit by deciding on the selling prices and its own lead times. Let $\pi^D_{mi}\left(X_{mi}\right)$ and $\pi^D_{ni}\left(X_{ni}\right)$ denote the profit of option i of manufacturer m and retailer n in a supply chain, respectively, where X_{mi} is the vector of equilibrium prices and lead times for option i of manufacturer m, and X_{ni} is the vector of equilibrium prices and lead times for option i of retailer n. Figure 3 shows the decision structure of a typical decentralized supply chain where $X_{mi} := \arg\max \sum_i \pi^D_{mi}(X_{mi})$ and $X_{ni} := \arg\max \sum_i \pi^D_{ni}(X_{ni})$. In Figure 3, the interaction between manufacturers and retailers is divided into two stages: In Stage 1, manufacturers decide on their wholesale prices and the corresponding lead times considering retailers' orders. In Stage 2, retailers decide on their retail prices and the corresponding lead-times and then give the orders to manufacturers based on customer demand.

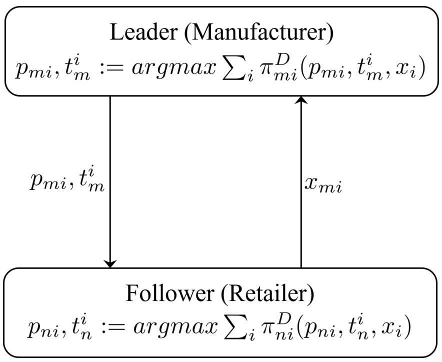


Figure 3: A decentralized supply chain decision structure.

Manufacturer m's optimization problem in this network can be expressed as:

$$\max_{p_{mi}, t^i_m} \pi^D_m(X_m) = \max_{p_{mi}, t^i_m} \sum_{i=1}^{|T_m|} [p_{mi} - c_{mi}(\cdot)] \cdot x_i, \tag{6}$$

where p_{mi} and $c_{mi}(\cdot)$ are manufacturer m's wholesale price and cost per unit of option i, respectively. Let x_i denote the quantity supplied by the supply chain for option i. The basis for the analysis of decentralized supply chains is the first-mover advantage framework with the manufacturer as a leader in the supply chain. Hence, we give the retailer's best response to the manufacturer's decisions in a given customer segment. The retailers can decide on the optimal retail prices based on their transaction cost and the manufacturers' wholesale prices. Accordingly, retailer n's optimization problem in this network can be expressed as:

$$\max_{p_{ni},t_n^i} \pi_n^D(X_n) = \max_{p_{ni},t_n^i} \sum_{i=1}^{|T_n|} [p_{ni} - c_{ni}(\cdot) - p_{mi}] \cdot x_i, \tag{7}$$

where p_{ni} represents the retail price and $c_{ni}(\cdot)$ denotes transaction cost of option i incurred by the retailer in the second echelon, including such things as production cost, transportation cost, and insurance cost.

In a supply chain network equilibrium, all customers choose the chain that maximizes their utility (IC). Therefore, a feasible chain x_i^* constitutes a supply chain network equilibrium if and only if the following equality holds.

$$Q \cdot P_i(\cdot) = x_i^*, \text{ if } p_i > 0, t_s^i > 0, \forall i, s, \tag{8}$$

for all P/T options i, $i = 1, \cdots, I$.

We adopt a Lagrangian function and an augmented Lagrangian function (AL) to solve the constrained optimization problems (6) and (7) restricted by the flow conservation conditions (8). A Lagrangian function is commonly used to convert a constrained optimization problem into an unconstrained optimization problem. However, the necessary condition of variational inequality problem (VIP) for iterative convergence is that the objective functions are convex. (Nagurney and Dong, 2002). When the unconstrained optimization problems are non-convex converted by Lagrangian functions, AL functions might be feasible. The objective functions converted into the AL functions are listed as follows:

$$AL\left(X, \lambda_i, \phi\right) = \pi(X) + \sum_{i=1}^{I} \lambda_i g_i(X) + \frac{\phi}{2}[g_i(X)]^2;$$

$$det(\bar{H}) \begin{cases} < 0 & \text{if } 1 + \sum_{j=1}^{I} e^{U_j(\cdot)} > e^{U_i(\cdot)} \text{ and } -c_i'(\cdot) < \bar{\beta}_i/\bar{\alpha}, \text{ introducing Lagrangian functions} \\ < 0 & \text{if } \phi \text{ is sufficiently large, introducing AL} \end{cases}, \ i \neq j,$$

where $\bar{H}$ denotes the bordered Hessian matrix and ϕ represents the penalty parameter in the AL function. We then get the following lemma.

Lemma 3. *The following holds for any given P/T menu option $i = 1, \cdots, I$:*

(a) If the decision variables are in a specific range, $1 + \sum_{j=1}^{I} e^{U_j(\cdot)} > e^{U_i(\cdot)}$ and $-c_i'(\cdot) < \bar{\beta}_i/\bar{\alpha}$, then the bordered Hessian matrix of objective functions using Lagrangian function in decentralized supply chains is negative definite, $det(\bar{H}) < 0$;

(b) If the penalty parameter ϕ from the AL is sufficiently large, then the bordered Hessian matrix of objective functions using the AL function in decentralized supply chains is negative definite, $det(\bar{H}) < 0$. The AL functions are concave in the entire domain of decision variables.

Lemma 3(a) shows that the new objective functions-unconstrained problems of the firms in decentralized supply chains are not concave across the entire domain of decision variables (see the appendix for details). This can be overcome using the AL function (Huang and Yang, 2003, 2005), because the quadratic penalty makes the new objective strongly convex if the penalty parameter is sufficiently large, and X^* and Lagrange multiplier λ^* meet the second-order sufficiency conditions for the original problem. Lemma 3(b) shows that when an AL function is introduced the objective functions of manufacturers and retailers are completely concave in the entire domain of decision variables (see the appendix for details). In other words, when an AL function is introduced and if the penalty parameter ϕ is sufficiently large, then traditional convex optimization methods work.

Based on the above leader-follower framework with the manufacturer as a leader in the supply chain, for a given manufacturer lead time, t_m^i, and a retailer lead time, t_n^i under a given customer segment, the retailer's best pricing strategy p_{ni}^* is the solution of the following equation.

$$p_{mi} = p_{ni}^* + P_i(\cdot) / \frac{\partial P_i(\cdot)}{\partial p_{ni}^*} - c_{ni}(\cdot), \tag{9}$$

where $\partial P_i(\cdot)/\partial p^*_{ni}$ is the first-order derivative of the probability of choosing option i (see the appendix for details). Combining the above retailer's best response pricing strategy with Lemma 1, we have Lemma 4.

Lemma 4. $\frac{\partial p_{ni}}{\partial \bar{\beta}_i} > \frac{\partial p_{mi}}{\partial \bar{\beta}_i}$, $\forall i$,

where $\partial p_{ni}/\partial \bar{\beta}_i > 0$. Lemma 4 illustrates the impact on manufacturers' wholesale prices and retailers' prices of customer time-sensitivity coefficients. Based on Lemma 4, we see that retailer's price p_{ni} are increasing in the customer time-sensitivity coefficient $\bar{\beta}_i$ in a leader-follower framework. Compared with the impact on manufacturer's price p_{ni}, the customer time-sensitivity coefficient $\bar{\beta}_i$ has a relatively larger positive impact on retailer's price p_{mi} under the leader-follower advantage framework. In other words, if customers become increasingly time-sensitive, then the retailer's selling prices increase more than manufacturers' wholesale prices under the leader-follower advantage framework. Combined with Lemma 2, the impact of the variance of customers' time-sensitivity distribution on retailers' prices would be different in various customer segments. Thus, the following lemma can be obtained.

Lemma 5. $\begin{cases} \frac{\partial p_{ni}}{\partial \sigma} \leq 0 & if\, (h_{i-1}, h_i) \in [\bar{\beta} - \sigma, \bar{\beta} + \sigma] \\ \frac{\partial p_{ni}}{\partial \sigma} > 0 & if\, (h_{i-1}, h_i) \notin [\bar{\beta} - \sigma, \bar{\beta} + \sigma] \end{cases}$, $\forall i$.

For a given customer segment (h_{i-1}, h_i), if the segment is within the threshold, h_{i-1},$h_i \in [\bar{\beta} - \sigma, \bar{\beta} + \sigma]$, then the retailer's price p_{ni} are decreasing with the variance of customers' time-sensitivity distribution, σ. In contrast, if h_{i-1},$h_i \notin [\bar{\beta} - \sigma, \bar{\beta} + \sigma]$, then the retailer's price p_{ni} are increasing with the variance of the customers' time-sensitivity distribution in a leader-follower framework. Thus, outside the limits of $\bar{\beta} \pm \sigma$ the impact of the variance of customers' time-sensitivity distribution is positive on retailer's price p_{ni}. Lemma 5 provides managerial insight about the impact of customer segments on retailers' prices when customer time-sensitivity coefficients are normally distributed. The threshold σ is a key criterion to understand the impact of customers' time-sensitivity distribution on retailers' prices. It has an opposite effect on firms' prices between inside and outside one standard deviation from the mean of customers' time-sensitivity distribution.

The stochastic equilibrium conditions in the decentralized supply chain network must satisfy (6), (7), flows conservation (8), and the retailer's best response (9). We state the stochastic equilibrium conditions of the decentralized supply chain network with constraints as a variational inequality formulation in the appendix. The manufacturers' profit functions (6), the retailers' profit functions (7), and the retailer's best response (9) in our model can be rewritten to standard variational inequality below based on the classic variational inequality problem (see Nagurney et al., 2013; Zhang, 2006):

$$\left\langle \nabla \pi_s \left(X^*\right), \left(X_s - X_s^*\right) \right\rangle \geq 0, \forall X \in \Omega, \tag{10}$$

where $\nabla \pi_s(X)$ is the gradient of $\pi_s(X)$ with respect to X_s and $\pi_s(X)$ being the function that enters the variational inequality problem. Therefore, the derivative of objective functions with respect to link flows, selling prices, lead-times, and customers' segments are captured in the multinomial logit-based variational inequality (see the appendix for details). The next theorem establishes when the equilibrium is unique.

Theorem 1. *Under SUE conditions, there exists a unique equilibrium in the decentralized supply chain network if the time-cost and profit functions are continuous and the marginal profit functions $\nabla \pi_m(X)$ and $\nabla \pi_n(X)$ are strictly monotone, $\forall m \in M$ and $\forall n \in N$.*

4.2. Centralized supply chain network

In this section, we consider the same product being produced by a two-echelon supply chain network and using the same cost structure but now in a centralized supply chain. Centralization means that each supply chain is controlled by a single decision-maker (vertically integrated), and makes its decision, X_s, consistent with profit maximization. In Figure 4, based on the time-cost functions from manufacturers and retailers, the supply chain decision-maker decides on selling prices and lead times for both manufacturers and retailers according to market demand. Figure 4 also shows the decision structure of a typical centralized supply chain, $X_{si} := \arg\max \pi^C_{si}(X_{si})$, where X_{si} is the vector of equilibrium price and lead time of option i on chain s, and $\pi^C_{si}(X_{si})$ is the profit function of option i on chain s in a centralized supply chain. Consistent with vertical integration, the manufacturers' wholesale prices are taken as their marginal cost.

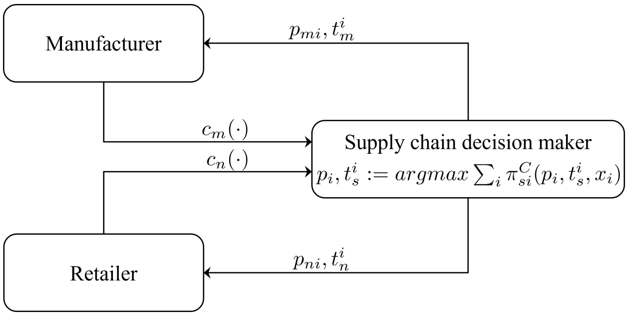


Figure 4: A centralized supply chain decision structure.

Each supply chain provides $|T_s|$ P/T options over a selling season. The supply chain's optimization problem in this network can be expressed as:

$$\max_{p_{ni}, t_s^i} \pi_s^C(X_s) = \max_{p_{ni}, t_s^i} \sum_{i=1}^{|T_s|} [p_{ni} - c_{ni}(\cdot) - c_{mi}(\cdot)] \cdot x_i. \tag{11}$$

In a centralized supply chain network, we can obtain an equivalent lemma as in section 4.1.

Lemma 6. *The following holds for any given option $i = 1, \cdots, I$:*

(a) If the decision variables are in a specific range, $1 + \sum_{j=1}^{I} e^{U_j(\cdot)} > e^{U_i(\cdot)}$ and $-c_i'(\cdot) < \bar{\beta}_i/\bar{\alpha}$, then the bordered Hessian matrix of objective functions using Lagrangian function in centralized supply chains is negative definite, $det(\bar{H}) < 0$;

(b) If the penalty parameter ϕ is sufficiently large, then the bordered Hessian matrix of objective functions using AL function in centralized supply chains is negative definite, $det(\bar{H}) < 0$. The AL functions are concave in the entire domain of decision variables.

The proof of Lemma 6 can be found in the appendix. In a supply chain network equilibrium, all customers choose the chain that maximizes their utility (IC). Therefore, a feasible chain X_i^* constitutes a supply chain network equilibrium if and only if the flow conservation conditions in (8) hold. In other words, the equilibrium conditions in the centralized supply chain network must satisfy (8) and (11). The equilibrium conditions of the centralized supply chain with constraints can be written as a variational inequality which we provide in the appendix. We then use the modified projection method of Salarpour and Nagurney (2021) to find the equilibrium pattern satisfying variational inequalities (G1) and (J1) that we provide in the appendix. Next, we provide the second theorem that establishes when the equilibrium is unique.

Theorem 2. *Under SUE conditions, there exists a unique equilibrium in the centralized supply chain network if the time-cost and profit functions are continuous and the marginal profit functions $\nabla \pi_s(X)$ are strictly monotone, $\forall s \in S$.*

5. Analysis and Results

In this section, we provide some results and an analysis based on the SUE conditions of decentralized and centralized supply chain networks. The goal of this portion of our study is to explore firms' P/T decisions under SUE conditions in supply chain network competition. The nonparametric sensitivity analysis of the variational inequality problem is implemented to analyze our results (Nagurney et al., 2002b). To begin, we focus on the impact of SUE and UE conditions on the supply chain network competition. Next, we discuss whether the choice behaviours of heterogeneous customers and firms' time-cost tradeoffs can affect the competition.

5.1. Impact of SUE and UE

In this subsection, we focus our analysis on the effect of SUE and UE conditions on competition. Let $AL(X)$ denote the function that enters the variational inequality problem. When an AL function is introduced into the objective functions in our model, $\nabla AL(X)$ is strongly monotonic in the entire domain of decision variables (see the appendix for details). In order to conduct comparative statics analysis, let $\hat{AL}(X)$ denote the perturbed function with solution $\hat{X}$ that has a small change in $AL(X)$. In turn, we obtain the following lemma.

Lemma 7. $[\nabla \hat{AL}(X) - \nabla AL(X)]^T \cdot [\hat{X}^* - X^*] \leq 0$.

The SUE conditions in decentralized and centralized supply chain networks are given in the appendix by (G1) and (J1), respectively. We adopt superscript "$*$" to represent the equilibrium solutions under SUE conditions and superscript "$\prime *$" to represent the equilibrium solutions under UE conditions, respectively. If the probability of customers choosing option i (4) is substituted by the following Wardrop's UE conditions, then the solutions correspond to UE conditions:

$$P_i'(\cdot) = \begin{cases} 1 & \text{if } V_i = \max\limits_{i \in I} \{V_i\} \\ 0 & otherwise \end{cases} . \tag{12}$$

The above Wardrop's UE conditions imply that firms believe they have full information about customers, because the random variable, ε, is ignored in firms' (supply chains') decisions. Based on the SUE conditions of the decentralized and centralized supply chain networks, (4), (12), and Lemma 7, we obtain the following proposition.

Proposition 1. *Under SUE conditions,*
(a) the proportion of customers that reject all options (IR constraint) is higher relative to UE conditions in both decentralized and centralized supply chain networks; while the proportion of customers that reject all options (IR constraint) in decentralized supply chains is higher relative to centralized supply chains, i.e.,

$$1 - \sum_{i=1}^{I} P_i^{*\xi} > 1 - \sum_{i=1}^{I} P_i'^{*\xi} \text{ and } 1 - \sum_{i=1}^{I} P_i^{*D} > 1 - \sum_{i=1}^{I} P_i^{*C}, \forall \xi \in \{D, C\};$$

(b) firms in decentralized supply chains provide higher retail prices and longer lead times in equilibrium relative to centralized supply chains, i.e.,

$$p_{ni}^{*D} > p_{ni}^{*C} \text{ and } t_{\kappa i}^{*D} > t_{\kappa i}^{*C}, \forall \kappa \in \{m, n\};$$

(c) firms in supply chain networks always have higher profits than they would under UE conditions; while firms in decentralized supply chains always have lower profits than they would in centralized supply chains, i.e.,

$$\pi_{\kappa i}^{*\xi} > \pi_{\kappa i}'^{*\xi} \text{ and } \pi_{\kappa i}^{*D} < \pi_{\kappa i}^{*C}, \forall \xi \in \{D, C\}, \forall \kappa \in \{m, n\}.$$

If a firm joins the supply chain in a supply chain network and UE conditions are adopted to support its decisions, then the expected demand under UE is higher than the expected demand under SUE which might cause some inventory backlog that is not due to strategic considerations. These systematic expected demand biases when adopting UE conditions lead to overproduction. Another problem is that the decision-makers supported by UE conditions possibly believe the proportion of customers that reject all options is lower as they believe customers have a lower IR constraint relative to SUE conditions. In turn, it will result in firms always having lower profits under UE conditions relative to SUE conditions. The difference in the equilibrium solutions between those obtained under SUE and UE conditions is caused by the decision-makers supported by UE conditions that believe they have full information about customers. These findings further indicate that it is still important if a firm or a supply chain has more information about their customers, even though we assume that all factors affecting customer choice behavior cannot be completely observed by firms. Of course, if a firm or a supply chain has more information about their customers, then the equilibrium solutions under SUE and UE conditions are closer. Proposition 1 further shows that firms in decentralized supply chains under SUE conditions always provide higher retail prices and longer lead times for their customers. In turn, firms in centralized supply chains always have higher profits than they would in decentralized supply chains under SUE conditions because of double marginalization (Liu et al., 2007).

5.2. Impact of time-cost tradeoffs

In this subsection, we focus our analysis on the effect of firms' time-cost tradeoffs on competition. Recall that we assume the cost function is a decreasing function of lead time. Hence, the gradient of the cost function reflects the degree of time-cost tradeoffs. If the gradient of the cost function is high, then the marginal cost of reducing a unit of time is higher. Based on the equilibrium conditions of decentralized and centralized supply chain networks, (G1) and (J1) in the appendix, and Lemma 7, we obtain the following lemma.

Proposition 2. *Under SUE conditions,*

(a) the equilibrium flows of the chains decrease in the degree of the time-cost tradeoffs of the links already participating in the chains;

(b) the equilibrium lead times of the chains increase in the degree of the time-cost tradeoffs of the links already participating in the chains;

(c) the impact of the degree of the time-cost tradeoffs of the links participating in the chains on the equilibrium flows and lead times in decentralized supply chains is lower than its impact in centralized supply chains.

The flows and lead times in a supply chain are impacted by time-cost tradeoffs of firms participating in the supply chain. In other words, a firm's flows and lead times in a supply chain are affected by not only its own time-cost tradeoffs but also by the time-cost tradeoffs of other firms participating in the supply chain. This proposition has at least two implications: (i) The firms with a low degree of the time-cost tradeoffs could contribute their supply chains in supply chain versus supply chain time-based competition. For instance, besides the cost advantage, firms with flexible production schedules can be adopted as supply chain partners in time-based competition which can contribute to the entire supply chain; (ii) The prices and lead times in decentralized supply chains are not sensitive to the degree of the time-cost tradeoffs in comparison of centralized supply chains.

5.3. Impact of customers' time-sensitivity distribution

In this subsection, we start by examining the effect of customer's time-sensitivity distribution on customer segmentation. Let $\sigma_h^{2\xi}$, $\xi \in \{D, C\}$, denote a threshold of the variance of customers' time-sensitivity distribution in decentralized and centralized supply chains. Then combining with Lemma 2 and Lemma 7, we obtain the following proposition.

Proposition 3. *If $\sigma^2 > \sigma_h^{2\xi}$, $\xi \in \{D, C\}$, then supply chain profits (both decentralized and centralized) increase in the number of customer segments where the corresponding threshold of the variance of customers' time-sensitivity distribution in a decentralized supply chain is higher than the corresponding threshold in a centralized supply chain, $\sigma_h^D > \sigma_h^C$.*

From Proposition 3, the variance of customers' time-sensitivity coefficients has a critical impact on customer segmentation. In other words, the more spread the distribution of customers' time-sensitivity coefficients, the more P/T options the supply chain can provide profitably. For example, cheese producers provide different options and operate with a downstream 3PL to deliver to several supermarkets in an area with different time-sensitive customer groups. The results also indicate that under SUE conditions centralized supply chains can generate more revenue by offering more P/T options for the same variance of customers' time-sensitivity distribution as compared to decentralized supply chains. In other words, in centralized supply chains it is easier to reach the threshold in contrast to decentralized supply chains.

We extend Proposition 3 to determine the effect of customers' time-sensitivity coefficient on the equilibrium prices and lead times. Using Lemma 2, Lemma 4, Lemma 7 and the SUE conditions of the decentralized and centralized supply chain networks, we obtain the following propositions.

Proposition 4. *Under SUE conditions,*

(a) in decentralized and centralized supply chain networks, for a given customer segment, if $(h_{i-1}, h_i) \in [\bar{\beta} - \sigma, \bar{\beta} + \sigma]$, then the equilibrium retail prices decrease in the standard deviation of customers' time-sensitivity distribution, otherwise vice versa;

(b) in decentralized and centralized supply chain networks, for a given customer segment, if $(h_{i-1}, h_i) \in [\bar{\beta} - \sigma, \bar{\beta} + \sigma]$, then the equilibrium lead times increase in the standard deviation of customers' time-sensitivity distribution, otherwise vice versa;

(c) the standard deviation of customers' time-sensitivity distribution has a relatively greater impact on the retail prices in decentralized supply chains relative to the retail prices in centralized supply chains if $(h_{i-1}, h_i) \notin [\bar{\beta}-\sigma, \bar{\beta}+\sigma]$*;*

(d) in decentralized supply chain networks, the equilibrium prices of manufacturers and retailers increase in the expectation of customers' time-sensitivity coefficient, $\bar{\beta}$*. However, it has a relatively greater impact on retail prices relative to manufacturers' wholesale prices.*

From Proposition 4(a) and (b), when considering different customer segmentations, the variance of customers' time-sensitivity distribution has a different effect on the prices and lead times of firms. Note that in our setting, the manufacturers in decentralized supply chains are assumed to be the leader in a leader-follower framework. Proposition 4(c) shows the different effects of the standard deviation of customers' time-sensitivity distribution on the retail prices between decentralized and centralized supply chains. The findings indicate that, in contrast to centralized supply chains, the equilibrium retail prices in decentralized supply chains are increasing more in the spread of customers' time-sensitivity distribution. Proposition 4(d) shows the impact of the expectation of customers' time-sensitivity coefficient on retail prices. It has an interesting interpretation: Retailers' prices are more sensitive to the expectation of customers' time-sensitivity coefficient than manufacturers' prices in decentralized supply chains. As a result, if customers become increasingly time-sensitive, then the retailers' selling prices increase more than the manufacturers' wholesale prices in a decentralized supply chain.

6. Numerical Examples.

In this section, we design three numerical examples, two cheese supply chain networks and a general food supply chain network, to illustrate the impact of SUE conditions, customers' time-sensitivity distribution and time-cost tradeoff on the equilibria. A modified projection method is adopted to solve the multinomial logit-based variational inequality for the numerical examples.

Example 1. We provide a simple illustrative example showing the difference in the equilibrium solutions between SUE and UE conditions. There are two centralized food supply chains consisting of two cheese producers and two retailers, respectively (Figure 5). Both of them sell their substitutable cheese in a demand market.

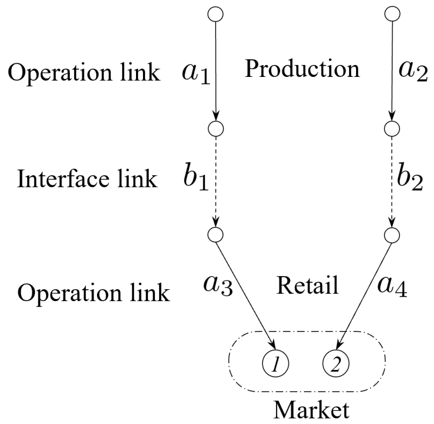


Figure 5: The cheese supply chain network topology.

The customers can discern their processing time from the "best before" tag. The number of customers is $N = 100$. We assume the customer price- and time-sensitivity coefficients are uncorrelated and follow a normal distribution, $\bar{\alpha} = 0.5$ and $\tilde{\beta} = N\left(0.5, 0.1^2\right)$. The customer's intrinsic value of option i is $w_i = 10$. The operation costs on chain 1 and chain 2 are $c_{s_1} = 10/t_{s_1}$ and $c_{s_2} = 12/t_{s_2}$, respectively. The interface link costs are $c_{b_1} = 1$ and $c_{b_2} = 1$, respectively. The SUE conditions in a centralized supply chain network are given in section 4.2 (G1). If the probability of customers choosing option i is substituted by the Wardrop's UE conditions (12), then the solutions correspond to UE conditions.

Table 3
The equilibrium solutions under SUE and UE conditions.

Solution		SUE	UE
Selling prices	p_{s_1}	11.1	12.7
	p_{s_2}	10.9	10.3
Lead times	t_{s_1}	7.7	6.6
	t_{s_2}	9.4	10.5
Supplies	x_{s_1}	50.0	93.1
	x_{s_2}	22.7	0.5
Demands	$Q \cdot P_{s_1}$	50.0	45.9
	$Q \cdot P_{s_2}$	22.7	21.7

The equilibrium solutions under SUE and UE conditions are shown in Table 3. If the customers' utility is negative, then they will reject the option under both SUE and UE conditions. It is optimal for both chains to provide one P/T option for their customers. Now, we make some observations by using Table 3. If UE conditions are adopted to support their decisions, then Chain 1's (left chain in Figure 5) expected supply under UE is higher than the demand which will cause inventory backlogs. Another problem is that the decision-makers supported by UE conditions may believe the number of customers that reject all options is low. In other words, the number of customers that satisfy the IR constraint under UE conditions is higher than those under SUE conditions. The ratio of customers that reject all options under SUE conditions is approximately 27.3%, while the decision-makers supported by UE conditions believe that the ratio of customers that reject all options is around 6.4%. If the decision-makers adopt the P/T options under UE conditions, then the ratio of customers that reject all options is around 32.4%. The solutions in Example 1, which show the IR constraint gap between SUE and UE conditions specifically, is consistent with the results in Proposition 1. The above analysis stems from the assumption that a firm (a supply chain) does not have full information about their customers.

Example 2. We provide a simple example to illustrate the optimal P/T options. To compare the results, we design scenarios with two different customers' sensitivity distributions. Here we set customers' time-sensitivity coefficients to follow truncated normal distributions, $\tilde{\beta} \sim N(1, 0.2^2)$ and $\tilde{\beta} \sim N(1, 0.5^2)$. The means of both scenarios are the same, but the variances are 0.2^2 and 0.5^2, respectively. We set customers' price-sensitivity coefficient, $\bar{\alpha} = 0.5$. There is a centralized food supply chain consisting of a food producer and a retailer selling their fresh food as a vertically integrated monopoly. The operation cost on chain 1 is $c_{s_1}(\cdot) = 3/t_{s_1}$. The interface link cost is $\bar{c}_{b_1}(\cdot) = 50$. The number of customers is $Q = 100$.

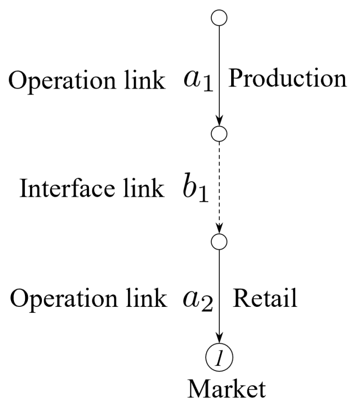


Figure 6: The food supply chain topology.

We show prices, lead times, flows, and profits in Table 4. The food supply chain cannot improve its profit by providing two P/T options for customers in Scenario 1. However, it is feasible in Scenario 2 to offer two P/T options for customers. This numerical example further validates that the variance of customers' time-sensitivity coefficients

Table 4
Equilibrium solutions on the food supply chain.

Solution	Scenario 1 $\alpha = \beta = N(1, 0.2^2)$		Scenario 2 $\alpha = \beta = N(1, 0.5^2)$
$\lvert T_s \rvert$	1	2	2
Selling prices p_{s_1}	7.1	(7.1, 6.3)	(7.2, 5.2)
Lead times t_{s_1}	1.8	(2.1, 4.0)	(2.5, 5.0)
Flows x_{s_1}	75.9	(70.7, 8.1)	(76.4, 2.5)
Profits π_{s_1}	362.4	345.9	370.0

Table 5
Cost functions in the cheese supply chain network.

Link	Cost function	Link	Cost function
a_1	$c_{a_1} = 4/t_{a_1} + 1$	a_2	$c_{a_2} = 5/t_{a_2} + 0.5$
b_1	$c_{b_1} = 2, t_{b_1} = 1$	b_2	$c_{b_2} = 1, t_{b_2} = 0.5$
a_3	$c_{a_3} = 6/t_{a_3} + 1$	a_4	$c_{a_4} = 7/t_{a_4} + 0.5$

has a critical impact on customer segmentations when the mean of customers' time-sensitivity distribution remains the same. When $\sigma^2 > \sigma_h^2$, the equilibrium profit of the supply chain increases with the augmented number of customer segments. When $\sigma^2 > \sigma_h^2$, the number of customers that reject all options is lower and the average margin is higher if the supply chain provides more options.

Example 3. We consider the same network structure displayed in Figure 5 for both decentralized and centralized supply chains. We assume that the producers and retailers pay for the coordinating cost and time of interface links together in the decentralized supply chain. Each firm has its own tradeoff time-cost function (Table 5). Supply chains compete to sell their substitutable products to heterogeneous time-sensitive customers in the demand market. The customers can discern the processing time from the "best before" tag. In this example, the number of customers is $Q = 1000$. For the sake of comparison facing competition, supply chain 2 (SC_2) is assumed to serve customers with a single P/T option. The multinomial logit-based variational inequality satisfying the utility function condition is the variational inequalities (G1) and (J1). Henceforth the customers' time-sensitivity coefficient, $\tilde{\beta}$, characterize a particular demand market in a competitive supply chain environment with a complicated partial derivative, even with a specific linear $U_i(\cdot)$. We also assume a general marginal time-cost function. How should supply chain 1 (SC_1) design a P/T menu and scheduling policy of link a_1 and link a_3 to maximize their revenues for heterogeneous time-sensitive customers? Given these conditions, we show our solutions with different variances and time-cost tradeoffs of competing firms.

Scenario 1: In this scenario, the decision-makers in SC_1 and SC_2 face typical heterogeneous time-sensitive customers that follow a normal distribution $\tilde{\beta} \sim N(0.5, 0.08^2)$ and $\bar{\alpha} = 0.5$. The decision-maker in SC_1 is planning to design an optimal P/T menu to compete with SC_2 and to satisfy the heterogeneous customers. The equilibrium solutions, optimal customer segmentations, and P/T menu with the above time-cost functions are reported in Table 6.

Scenario 2: In this scenario we increase the variance to $\sigma^2 = 0.2^2$. The decision-maker wants to know whether the P/T menu in Scenario 1 still works for Scenario 2. From the solutions in Table 6, the optimal number of customer segmentations is two and each firm provides two options. In other words, it is more advantageous for supply chains to provide more P/T options if the variance of customer time-sensitivity coefficients is higher, which is line with the findings obtained in Proposition 3. The equilibrium prices, lead times, and profits in Table 7 further reflect the difference between decentralized and centralized supply chains under SUE conditions, which have been illustrated in Proposition 1. In this scenario, it is interesting that each firm provides two options to serve customers. Hence, we design Scenario 3 in next subsection.

Scenario 3: We continue the example from the prior subsection. For the sake of the comparison with Scenario 2, the cost function of Manufacturer 1 has a higher degree of time-cost tradeoff in Scenario 3. The decision-maker considers redesigning supply chain SC_1. The time-cost function for Manufacturer 1 is:

Table 6
Computed equilibrium solutions of Scenario 1-3 ($\rho = 10^{-3},\ \varepsilon = 10^{-4}$).

Solutions	Scenario 1 ($\sigma^2 = 0.08^2$) CS[1]	DS[2]	Scenario 2 ($\sigma^2 = 0.15^2$) CS	DS	Scenario 3 ($\sigma^2 = 0.15^2$) CS	DS
$\lvert T_s \rvert$	1	1	2	2	2	2
x_1	95.80	39.50	112.01	48.37	80.85	36.19
x_2	129.97	52.54	114.66	46.61	138.01	49.01
P/T menu of chain 1	(8.66,5.45)	(10.81,5.46)	(8.23,6.20) (9.02,4.94)	(10.32,6.22) (11.07,5.05)	(9.24,5.83) (10.16,4.63)	(11.16,5.93) (12.03,4.72)
P/T menu of chain 2	(8.18,5.38)	(10.32,5.38)	(8.14,5.44)	(10.43,5.39)	(7.85,5.41)	(7.79,5.44)
(c_{a_1}, t_{a_1})	(2.00,2.00)	(2.00,2.00)	(1.71,2.34) (2.20,1.82)	(1.70,2.35) (2.20,1.82)	(3.03,1.75)	(2.80,2.05)
p_{a_1}	-[3]	4.29	-	4.06, 4.68	-	5.24
(c_{a_3}, t_{a_3})	(2.45,2.45)	(2.45,2.45)	(2.10,2.86) (2.70,2.22)	(2.09,2.87) (2.69,2.23)	(1.90,3.16) (2.90,2.07)	(2.08,2.88) (3.59,1.67)
(c_{a_2}, t_{a_2})	(2.13,2.35)	(2.24,2.24)	(2.24,2.23)	(2.23,2.24)	(2.22,2.25)	(2.23,2.24)
p_{a_2}	-	4.56	-	4.69	-	4.63
(c_{a_4}, t_{a_4})	(2.51,2.79)	(2.65,2.65)	(2.58,2.71)	(2.64,2.65)	(2.63,2.66)	(2.64,2.65)
U_1	218.44	172.65	261.43	213.38	180.19	155.47
U_2	298.72	233.05	259.02	212.17	276.02	220.62

1 The equilibrium solutions in the centralized supply chain network.

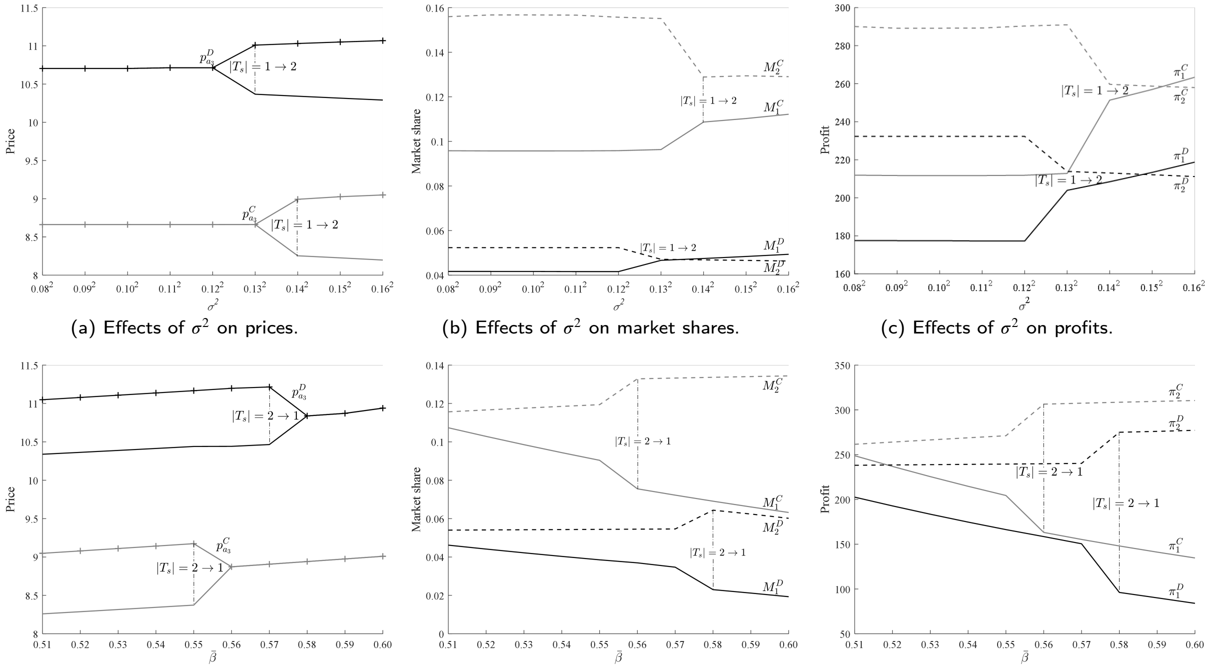
2 The equilibrium solutions in the decentralized supply chain network.

3 The wholesale prices are endogenous variables in the centralized supply chain, are not considered.

$$c_{a_1}(\cdot) = 4/\sqrt{t_{a_1}}.$$

From the solutions in Table 6, the optimal number of customer segments is still two. However, Manufacturer 1 provides one option, and Retailer 1 provides two options to coordinate and serve the heterogeneous customers. There exist two interface links to coordinate two P/T options. Besides cost and lead time advantages, a firm with a low degree of time-cost tradeoff can also contribute to the competitiveness of its supply chain. Additionally, the option that serves higher time-sensitive customers would have higher margins compared to the option that serves the lower time-sensitive customers despite the increase of operational cost.

Sensitivity Analysis. In this subsection, we conduct a sensitivity analysis to examine effects of the variance and mean of customers' time-sensitivity distribution on equilibrium prices, market shares, and profits. We use the same cost structure and parameters in Scenario 1 of Example 2, but the variance of customers' time-sensitivity distribution increases from 0.08^2 to 0.16^2 in Figures 7(a), (b), and (c); and the mean of customers' time-sensitivity distribution increases from 0.51 to 0.60 in Figures 7(d), (e), and (f). Note that in Figures 7(a), (b), and (c), when the variance of customer time-sensitivity distribution increases, the values of optimal customer segmentations of SC_1 in centralized and decentralized supply chains also increase from 1 to 2, respectively. The thresholds in decentralized and centralized supply chains further reflect the differences in the effect of the variance of customer time-sensitivity distribution on those two supply chain paradigms. When σ^2 is varied from 0.12^2 to 0.13^2, we observe that the optimal number of customer segments in centralized supply chains vary from 1 to 2, while the optimal number of customer segments in decentralized supply chains stays the same, which shows the advantage of centralized supply chains over decentralized supply chains in terms of customer segmentation. As illustrated in Figure 7(a), when $\sigma^2 > \sigma_h^2$, the equilibrium prices tend to become polarized as σ increases. The equilibrium price that serves time-sensitive customers is high in both centralized and decentralized supply chains and the price to time-insensitive customers is low. Figure 7(b) shows the effects of variance of customers' time-sensitivity distribution on market shares. For clarity, let M denote the market share. For instance, M_1^D denotes the market share of SC_1 under a decentralized supply chain. Furthermore, any increase in customer segmentation also increases market share and profit performance of the chain which the P/T menu fits for customers' sensitivity coefficient (see Figures 7(b) and (c)). Figures 7(d), (e), and (f) show that the optimal number of customer segments of SC_1 in centralized and decentralized supply chains decrease from 2 to 1 when the mean of customers' time-sensitivity distribution increases. Whereas the intuition is that firms would decrease P/T options to save costs when they face high time-sensitive customers.



(a) Effects of σ^2 on prices. (b) Effects of σ^2 on market shares. (c) Effects of σ^2 on profits.

(d) Effects of $\bar{\beta}$ on prices. (e) Effects of $\bar{\beta}$ on market shares. (f) Effects of $\bar{\beta}$ on profits.

Notes. The parameters used for the figure (a)-(c) are $\bar{\beta} = 0.5$ and $\bar{\alpha} = 0.5$. The parameters used for the figure (d)-(f) are $\sigma^2 = 0.15^2$ and $\bar{\alpha} = 0.5$.

Figure 7: Sensitivity analysis with respect to the variance and mean of customers' time-sensitivity distribution.

7. Conclusions

We provide a network stochastic equilibrium model to better match supply and heterogeneous customer demand in a competitive time-cost tradeoff supply chain network. First, we generalize the equilibrium conditions assumption of a traditional supply chain network equilibrium by introducing discrete choice theory. On the firms' side, we implicitly assume that firms (supply chains) do not have full information about their customers. On the customer side, we assume heterogeneous P/T preferences. Under SUE conditions, we found that there exists a unique equilibrium in the centralized and decentralized supply chain networks. Compared with UE conditions, we showed that the IR constraint under SUE conditions is higher. Accordingly, the expected demand under UE is higher relative to that under SUE conditions. The reason for this result is that the decision-makers supported by UE conditions believe that they have full information about their customers, but they do not. Moreover, the decision-makers may believe that the number of customers that reject all options is lower under UE conditions relative to under SUE conditions. Our results suggest that the decision-makers in supply chains should take caution in the decisions of production, lead time, and pricing to avoid unnecessary losses when they do not have full information about their customers. We further highlight the distinction between decentralized and centralized supply chain networks from the impacts of SUE conditions, time-cost tradeoffs, and customers' time-sensitivity distribution. Our results also demonstrate that the decisions in both decentralized and centralized supply chains under SUE conditions is superior to those under UE conditions when firms do not have full information about their customers.

Second, if customers' time-sensitivity coefficients follow a specific distribution such as a normal distribution, we demonstrate that the variance of heterogeneous customers' time-sensitivity distribution plays a crucial role in customer segmentations in a time-cost tradeoff supply chain, although there is a difference in segmentation between decentralized and centralized supply chains. We suggest that if the variance of customers' time-sensitivity distribution is high enough, then firms (supply chains) should provide more P/T options for customers to choose from. The reasoning is that the number of customers that reject the options is low from the differentiated prices and lead times if the variance of customers' time-sensitivity distribution is high enough and if firms provide more P/T options for customers to choose from. In turn, the firms (supply chains) can obtain more benefits to cover costs associated with more P/T options.

Third, the degree of time-cost tradeoff for firms has a critical impact on customer segmentations for both decentralized and centralized supply chains (Lemma 2). Individual firms with a low degree of time-cost tradeoff can be selected as a multiple option provider to optimize the revenue of the entire supply chain. Furthermore, a firm's P/T options in a supply chain are affected by not only its own time-cost tradeoffs but also by the time-cost tradeoffs of other firms participating in the supply chain.

Fourth, we capture customers' choice behavior using a discrete choice model that considers a firm or a supply chain that does not have full information about their customers. Incorporating a discrete choice model into a supply chain equilibrium model leads to a technical challenge involving nonconcave profit objective functions. A critical part of our analysis is the establishment of concavity of the objective functions when incorporating a discrete choice model into a supply chain equilibrium model. The equilibrium conditions of a time-cost tradeoff supply chain network, both decentralized and centralized, are given by formulating multinomial logit-based variational inequalities. From a theoretical perspective, this work can provide a valuable perspective for the management and analysis of a supply chain network with SUE conditions.

Inspired by the valuable comments from two reviewers, we provide the following implications for practitioners.

First, compared with SUE conditions, we find that if UE conditions are adopted to support decisions in practice, then the decision-makers face higher losses when the chain they participate in is the dominant chain. In contrast, the decision-makers lose opportunities when the chain they participate in is subservient. This is because we make an implicit assumption that a firm or a supply chain does not have full information about customers in the model with SUE conditions. Furthermore, a probabilistic function is adopted to describe customer choice behavior. Although methods can be used to obtain more information from customers, it is less realistic to assume firms have full information about customers. Of course, if a firm or a supply chain has more information about customers, then the equilibrium solutions under SUE and UE conditions are closer. Our work broadens the extant literature about supply chain network management considering SUE conditions, where a multinomial logit model is designed to illustrate customers choice behavior.

Second, our model optimizes supply chain (firm) profits without converting a multi-objective problem into a single-objective problem at the firm level. These characteristics differentiate our work from other supply chain network equilibrium models in that they rely on multi-objective transformation at the firm level, which implies that customers have the same time value per unit. Further, our model can be extended from time-cost tradeoffs optimization to other conflicting objectives optimization in a supply chain network. This captures current practice and has real-world implications for conflicting objectives of optimization in supply chain networks from fresh food to fashion and medicine, etc.

Third, our model considers the coordination between different time-cost tradeoff firms to maximize profit in supply chain P/T menu design. Firms with a high degree of time-cost tradeoff might get more benefit from selecting a partner with a low degree of time-cost tradeoff in a supply chain. A firm with a low degree of time-cost tradeoff has an advantage to collaborate and coordinate easily with other firms in the same supply chain. In other words, besides cost and lead time advantages, a firm with a low degree of time-cost tradeoff can also contribute to the supply chain that it joins in supply chain versus supply chain competition.

Besides the observations of inferior performance of decentralized supply chains due to the double marginalization effect from Liu et al. (2007), we find that the double marginalization effect can be reduced under some typical customers' time-sensitivity coefficients and an individual firm's time-cost relationship in two scenarios: (1) The optimal number of customer segmentations provided by a decentralized supply chain are more than those in a centralized supply chain facing a specific customers' time-sensitivity distribution; (2) The degree of time-cost tradeoff is high in both supply chain paradigms. Additionally, we demonstrate that to achieve successful coordination with firms in a decentralized supply chain, it is necessary for a manufacturer to improve internal operational efficiency. We also illustrate the impacts of internal operational efficiency from a time-cost relationship. Considering customers' characteristics and individual firm's time-cost relationship, we conclude that a supply chain (centralized or decentralized) has more opportunities to coordinate and optimize its benefits under SUE conditions than under UE conditions.

Our proposed models have a widespread application in decentralized and centralized supply chain management. For example, a manufacturer can use the model to analyze which downstream firms are profitable, providing multiple options or a single option. The model can be used as an analysis tool to determine whether to join a supply chain with

Table 7
The limitations of our work.

Limitation	Description
Customers' behaviors are not correlated	The mixed logit model could be adopted to expand our model.
Customers' sensitivity	Only two variables for customers' sensitivity and not other preferences.
Customers' time-sensitivity	Further research could expand the model by considering other distributions of customers' sensitivity.

a dominant firm or a decentralized supply chain. In addition, there are a number of opportunities for future research. One is to study supply chain competition while considering customers' behaviors by integrating discrete choice and spatial price theory. Further study on customers' behaviors would provide valuable opportunities for practitioners in industries and benefits for customers. Furthermore, if the customers' behaviors are correlated, then the equilibrium conditions formulated by multinomial logit-based variation inequality needs to be extended. The detailed limitations of our work are listed in Table 7.

Acknowledgments

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Appendix A Table of Notation

Appendix B Proofs of Lemmas, Propositions, and Theorems

Proof of Lemma 1.

Taking the partial derivative of the probability function of a customer choosing option i with respect to $\bar{\alpha}$ we have

$$\frac{\partial P_i(\cdot)}{\partial \bar{\alpha}} = \frac{-p_i e^{U_i(\cdot)}[1+\sum_{i=1}^{I} e^{U_i(\cdot)} - e^{U_i(\cdot)}]}{[1+\sum_{i=1}^{I} e^{U_i(\cdot)}]^2} = -p_i P_i(\cdot)[1-P_i(\cdot)] < 0, \tag{B1}$$

where p_i is the retail price of option i, $p_i > 0$.

Taking the partial derivative of the probability function of a customer choosing option i with respect to $\bar{\beta}_i$ we have

$$\frac{\partial P_i(\cdot)}{\partial \bar{\beta}_i} = \frac{-t_s^i e^{U_i(\cdot)}[1+\sum_{i=1}^{I} e^{U_i(\cdot)} - e^{U_i(\cdot)}]}{[1+\sum_{i=1}^{I} e^{U_i(\cdot)}]^2} = -t_s^i P_i(\cdot)[1-P_i(\cdot)] < 0, \tag{B2}$$

where t_s^i is the lead time of option i on chain s, $t_s^i > 0$.

Proof of Lemma 2.

Partially differentiating the expected function of customers' time-sensitivity with respect to σ yields

$$\frac{\partial \bar{\beta}_i}{\partial \sigma} = \frac{\partial \int_{h_{i-1}}^{h_i} \tilde{\beta} \cdot f(\tilde{\beta}) d\tilde{\beta}}{\partial \sigma} = \frac{\partial \int_{h_{i-1}}^{h_i} \tilde{\beta} \cdot \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(\tilde{\beta}-\bar{\beta})^2}{2\sigma^2}} d\tilde{\beta}}{\partial \sigma}.$$

Let $\tilde{\beta} = \sigma y - \bar{\beta}$. Therefore,

$$\frac{\partial \bar{\beta}_i}{\partial \sigma} = \frac{1}{\sqrt{2\pi}} \left[(1+y_{i-1}^2) e^{-\frac{y_{i-1}^2}{2}} - (1+y_i^2) e^{-\frac{y_i^2}{2}} \right] + \frac{\bar{\beta}}{\sqrt{2\pi}\sigma} \left[y_{i-1} e^{-\frac{y_{i-1}^2}{2}} - y_i e^{-\frac{y_i^2}{2}} \right]$$

Table A1
Notation for time-cost tradeoff supply chain equilibrium model.

Notation	Description
Sets:	
$H = [G, L, T]$	The graph consisting of nodes G and lead time T on directed links L representing the activities associated with each firm in a supply chain network.
L	The set of all links divided into operation link and interface link representing a business function and a coordination function, respectively.
Parameters:	
δ_{as}	Binary link-chain coefficient, $\delta_{as} \in [0, 1]$.
φ	Binary link-chain coefficient, $\varphi \in [0, 1]$.
w	Intrinsic value of the product (or service) for customers, $w > 0$.
$\tilde{\alpha}$	Customers' price-sensitivity coefficient, $\tilde{\alpha} \geq 0$.
$\bar{\alpha}$	Mathematical expectation of customers' price-sensitivity coefficient.
$\tilde{\beta}$	Customers' time-sensitivity coefficient, $\tilde{\beta} \geq 0$.
$\bar{\beta}$	Mathematical expectation of customers' time-sensitivity coefficient.
Q	Number of customers in a demand market, $Q = constant$.
x_a	Flow on operation link a, $x_a \geq 0$.
x_b	Flow on interface link b, $x_b \geq 0$.
x_s	Flow on chain s, $x_s \geq 0$.
σ_h^2	Thresholds of the variance of customers' time-sensitivity distribution.
$N(\bar{\beta}, \sigma^2)$	Normal distribution.
Decision variables:	
t_a	Lead time on operation link a, $t_a \geq 0$.
t_b	Lead time on interface link b, $t_b \geq 0$.
t_s	Aggregate lead time on chain s, $t_s \geq 0$.
p	Price of products; let p_{mi} denote the wholesale price of Manufacturer m and p_{ni} denote the retail price of option i in a demand market, $p \geq 0$.
Functions:	
$P_i(\cdot)$	Probability of a customer choosing option i, $0 \leq P_i(\cdot) \leq 1$.
$\bar{c}_a(\cdot)$	Operational cost on operation link a.
$c_a(\cdot)$	Cost per unit on operation link a.
$\bar{c}_b(\cdot)$	Operational cost on interface link b.
$c_b(\cdot)$	Cost per unit on interface link b.
$\bar{C}_s(\cdot)$	Operational cost on chain s.
$C_s(\cdot)$	Cost per unit on chain s.
$U_i(\cdot)$	A customer's utility function from choosing option i.
$\pi_m(\cdot)$	Profit of Manufacturer m.
$\pi_n(\cdot)$	Profit of Retailer n.
$\pi_s(\cdot)$	Profit of chain s.

$$= \frac{1}{\sqrt{2\pi}} \left[(1 + \frac{\bar{\beta}}{\sigma} y_{i-1} + y_{i-1}^2) e^{-\frac{y_{i-1}^2}{2}} - (1 + \frac{\bar{\beta}}{\sigma} y_i + y_i^2) e^{-\frac{y_i^2}{2}} \right],$$

where $y_{i-1} = \frac{h_{i-1} - \bar{\beta}}{\sigma}$ and $y_i = \frac{h_i - \bar{\beta}}{\sigma}$. If $(h_{i-1}, h_i) \in [\bar{\beta} - \sigma, \bar{\beta} + \sigma]$, then $\partial \bar{\beta}_i / \partial \sigma < 0$. If $(h_{i-1}, h_i) \in (0, \bar{\beta} - \sigma)$ or $(\bar{\beta} + \sigma, +\infty)$, $\partial \bar{\beta}_i / \partial \sigma > 0$. According to (B2), $\partial P_i(\cdot) / \partial \bar{\beta}_i = -[t_n^i + t_m^i] P_i(\cdot)[1 - P_i(\cdot)] < 0$. Hence, if $h_{i-1}, h_i \in [\bar{\beta} - \sigma, \bar{\beta} + \sigma]$, then $P_i(\cdot)$ is increasing in σ. If $(h_{i-1}, h_i) \in (0, \bar{\beta} - \sigma)$ or $(\bar{\beta} + \sigma, +\infty)$, then $P_i(\cdot)$ is decreasing in σ.

Proof of Lemma 3.

(Lagrangian function). To solve (6) and (7) with respect to constraints (8), we associate a Lagrange multiplier λ with the ith constraint to construct a Lagrangian function as (D1). The constraint functions $g_i(X)$ must satisfy certain regularity conditions (Mokhtar et al., 1979).

$$L\left(X,\lambda_i\right)=\pi(X)+\sum_{i=1}^{I}\lambda_i g_i(X). \tag{D1}$$

The Lagrangian function (D1) in our model can be rewritten as a standard variational inequality (D2), based on the classic variational inequality problem (Zhang, 2006; Nagurney et al., 2013).

$$\left\langle \nabla L\left(X_s^*\right),\left(X_s-X_s^*\right)\right\rangle \geq 0,\forall X_s\in\Omega, \tag{D2}$$

where $\nabla\mathrm{L}(X_s)$ is the gradient of $L(X_s)$ with respect to X_s and $L(X_s)$ being the function that enters the variational inequality problem.
In this proof, we give the range over which the objective functions using a Lagrangian in a decentralized supply chain are concave from a negative definite of bordered Hessian matrix perspective. Our first step is to construct the Lagrangian for profit function π with respect to flow conservation constraints (8) and best response (9). Then taking the second-order partial derivatives of π and the first-order partial derivatives of flow conservation constraints (8) with respect to x_i, p_i, and t_s^i, we get the bordered Hessian matrix:

$$det(\bar{H})=\begin{bmatrix} 0 & 1 & -Q\frac{\partial P_i(\cdot)}{\partial p_i} & -Q\frac{\partial P_i(\cdot)}{\partial t_s^i} \\ 1 & 0 & \frac{1}{1-P_i(\cdot)} & -c_i'(\cdot)+\frac{\bar{\beta}_i P_i(\cdot)}{\bar{\alpha}[1-P_i(\cdot)]} \\ -Q\frac{\partial P_i(\cdot)}{\partial p_i} & \frac{1}{1-P_i(\cdot)} & \frac{-\bar{\alpha}P_i(\cdot)x_i}{1-P_i(\cdot)}-\lambda_i\cdot Q\frac{\partial^2 P_i(\cdot)}{\partial^2 p_i} & \frac{-\bar{\beta}_i P_i(\cdot)x_i}{1-P_i(\cdot)}-\lambda_i\cdot Q\frac{\partial^2 P_i(\cdot)}{\partial p_i\partial t_s^i} \\ -Q\frac{\partial P_i(\cdot)}{\partial t_s^i} & -c_i'(\cdot)+\frac{\bar{\beta}_i P_i(\cdot)}{\bar{\alpha}[1-P_i(\cdot)]} & \frac{-\bar{\beta}_i P_i(\cdot)x_i}{1-P_i(\cdot)}-\lambda_i\cdot Q\frac{\partial^2 P_i(\cdot)}{\partial t_s^i\partial p_i} & [-c_i''(\cdot)-\frac{\bar{\beta}_i^2 P_i(\cdot)}{\bar{\alpha}[1-P_i(\cdot)]}]x_i-\lambda_i\cdot Q\frac{\partial^2 P_i(\cdot)}{\partial^2 t_s^i} \end{bmatrix}$$

$$\begin{aligned} &=\underbrace{-\frac{2\bar{\beta}_i^2\cdot Q\cdot x_i P_i(\cdot)}{1-P_i(\cdot)}}_{I}\underbrace{-2\bar{\alpha}\cdot\bar{\beta}_i^2\cdot\lambda_i\cdot Q^2\cdot P_i^2[1-P_i][1-2P_i]}_{II} \\ &\underbrace{+\bar{\alpha}^2[1-P_i(\cdot)]^2Q^2P_i(\cdot)^2\left[[\frac{\bar{\beta}_i P_i(\cdot)}{\bar{\alpha}[1-P_i(\cdot)]}-c_i'(\cdot)]^2-\frac{\bar{\beta}_i^2}{\bar{\alpha}^2[1-P_i(\cdot)]^2}\right]}_{III} \\ &\underbrace{-\left[\frac{-\bar{\beta}_i P_i(\cdot)x_i}{1-P_i(\cdot)}-\lambda_i\cdot Q\cdot\bar{\alpha}\cdot\bar{\beta}_i P_i(\cdot)[1-P_i(\cdot)][1-2P_i(\cdot)]\right]^2}_{IV}, \end{aligned} \tag{D3}$$

where $c_i'(\cdot)$ are firms' or supply chains' marginal cost in decentralized or centralized supply chains, respectively. If the utility of choosing option i is negative, $U_i(\cdot)<0$, then customers will reject option i (IR). The second term in formula D3 is less than or equal to zero, if $1+\sum_{j=1}^{I}e^{U_j(\cdot)}>e^{U_i(\cdot)}$, $i\neq j$. The third term is less than or equal to zero, if $-c_i'(\cdot)<\bar{\beta}_i/\bar{\alpha}$, and $\bar{\alpha}/\bar{\beta}_i$ represents the typical consumer time value per unit time (Anderson et al., 1992). The first and fourth term are less than zero. Therefore, if $1+\sum_{j=1}^{I}e^{U_j(\cdot)}>e^{U_i(\cdot)}$, $i\neq j$, and $-c_i'(\cdot)<\bar{\beta}_i/\bar{\alpha}$, then the bordered Hessian matrix has $det(\bar{H})<0$.

(Augmented Lagrangian function). To solve (6) and (7) with respect to constraints (8), we associate an augmented Lagrange multiplier λ with the ith constraint to construct an augmented Lagrangian as (D4). The constraint functions $g_i(X)$ must also satisfy certain regularity conditions (Mokhtar et al., 1979).

$$AL\left(X,\lambda_i,\phi\right)=\pi(X)+\sum_{i=1}^{I}\lambda_i g_i(X)+\frac{\phi}{2}[g_i(X)]^2. \tag{D4}$$

The augmented Lagrangian function (D4) in our model can be rewritten as a standard variational inequality (D5), based on the classic variational inequality problem (Zhang, 2006; Nagurney et al., 2013).

$$\left\langle \nabla AL\left(X_s^*\right), \left(X_s - X_s^*\right)\right\rangle \geq 0, \forall X \in \Omega, \tag{D5}$$

where $\nabla AL(X_s)$ is the gradient of $AL(X_s)$ with respect to X_s and $AL(X_s)$ being the function that enters the variational inequality problem.

In this proof, we also analyze whether the objective functions using an augmented Lagrangian function in a decentralized supply chain are concave from a negative definite of bordered Hessian matrix perspective. Our first step is to construct the augmented Lagrangian for profit function π with respect to flow conservation constraints (8) and best response (9). Then taking the second-order partial derivatives of π and the first-order partial derivatives of flow conservation constraints (8) with respect to x_i, p_i, and t_s^i, we get the bordered Hessian matrix. For simplicity, let $Y_1 = Q\cdot[-\lambda_i - \phi[x_i - Q\cdot P_i(\cdot)]]P_i(\cdot)[1-P_i(\cdot)][1-2P_i(\cdot)]$, $Y_2 = Q^2\cdot\phi\cdot P_i(\cdot)^2[1-P_i(\cdot)]^2$, and $Y_3 = Q\cdot P_i(\cdot)[1-P_i(\cdot)]$.

$$det(\bar{H}) = \begin{bmatrix} 0 & \bar{\alpha}\cdot Y_3 & 1 & \bar{\beta}_i\cdot Y_3 \\ \bar{\alpha}\cdot Y_3 & \bar{\alpha}^2[Y_1+Y_2] & \bar{\alpha}\cdot\phi\cdot Y_3 - 1 & \bar{\alpha}\cdot\bar{\beta}_i[Y_1+Y_2] \\ 1 & \bar{\alpha}\cdot\phi\cdot Y_3 - 1 & \phi & c_i'(\cdot)+\bar{\beta}_i\cdot\phi\cdot Y_3 \\ \bar{\beta}_i\cdot Y_3 & \bar{\alpha}\cdot\bar{\beta}_i[Y_1+Y_2] & c_i'(\cdot)+\bar{\beta}_i\cdot\phi\cdot Y_3 & c_i''(\cdot)x_i + \bar{\beta}_i^2[Y_1+Y_2] \end{bmatrix}$$

$$= \underbrace{-Y_3^2[\bar{\alpha}[c_i'(\cdot)+\bar{\beta}_i\cdot\phi\cdot Y_3]}_{I}\underbrace{-\bar{\beta}_i[\bar{\alpha}\cdot\phi\cdot Y_3 - 1]]^2}_{II}\underbrace{-\bar{\alpha}^2c_i''(\cdot)x_i[Y_1+Y_2]}_{III}. \tag{D6}$$

If ϕ is sufficiently large, then the second term in (D6) is less than or equal to zero. Therefore, if ϕ is sufficiently large, then the bordered Hessian matrix $det(\bar{H}) < 0$ and X^*, λ^* meet the second-order sufficiency conditions of the proposed problem in the decentralized supply chain network.

Proof of Lemma 4.

Partially differentiating the retailer's best response pricing function with respect to $\bar{\beta}_i$ yields

$$\frac{\partial p_{mi}^*}{\partial\bar{\beta}_i} = \frac{\partial p_{ni}^*}{\partial\bar{\beta}_i} + \frac{1}{\bar{\alpha}[1-P_i(\cdot)]^2}\frac{\partial P_i(\cdot)}{\partial\bar{\beta}_i}.$$

Combining with (B2) in the proof of Lemma 1, $\partial P_i(\cdot)/\partial\bar{\beta}_i = -t_s^i P_i(\cdot)[1-P_i(\cdot)]$, then

$$\frac{\partial p_{mi}^*}{\partial\bar{\beta}_i} = \frac{\partial p_{ni}^*}{\partial\bar{\beta}_i} - \frac{t_s^i\cdot P_i(\cdot)}{\bar{\alpha}[1-P_i(\cdot)]},$$

where $0 < P_i(\cdot) < 1$. Thus, we get $\partial p_{mi}^*/\partial\bar{\beta}_i < \partial p_{ni}^*/\partial\bar{\beta}_i$. For convenience of calculations, the objective functions are converted into the form of minimization problems. Converting the objective functions into minimization problems does not alter the essence of the problem. Let $\psi(p_{ni}^*)$ denote the partial derivative of the objective functions in decentralized supply chain network which are the necessary first-order conditions (FOC) for firms by choice of retail prices. Employing the implicit function rule,

$$\frac{\partial p_{ni}^*}{\partial\bar{\beta}_i} = -\frac{\partial\psi(p_{ni}^*)/\partial\bar{\beta}_i}{\partial\psi(p_{ni}^*)/\partial p_{ni}} > 0,$$

where $\partial\psi(p_{ni}^*)/\partial\bar{\beta}_i = [\partial\psi(p_{ni}^*)/\partial x_i][\partial x_i/\partial\bar{\beta}_i] < 0$ and $\partial\psi(p_{ni}^*)/\partial p_{ni} > 0$ since the objective functions are converted into minimization problems. In a supply chain network equilibrium, $Q\cdot P_i(\cdot) = x_i^*$. Then we get Lemma 4.

Proof of Lemma 5.

According to Lemma 4, we know that $\partial p_{ni}/\partial\bar{\beta}_i > 0$. Combining with Lemma 2, we can obtain if $(h_{i-1}, h_i) \in [\bar{\beta}-\sigma, \bar{\beta}+\sigma]$ where $\partial\bar{\beta}_i/\partial\sigma \leq 0$, then $\partial p_{ni}/\partial\sigma \leq 0$. If $(h_{i-1}, h_i) \notin [\bar{\beta}-\sigma, \bar{\beta}+\sigma]$ where $\partial\bar{\beta}_i/\partial\sigma > 0$, then $\partial p_{ni}/\partial\sigma > 0$. Then we get Lemma 5.

The decentralized supply chain network SUE conditions.

The manufacturers' prices can be obtained from the retailers' best response functions. For simplicity, let $Z_i = Q \cdot P_i(\cdot)[1 - P_i(\cdot)][\lambda_i + \phi[x_i - Q \cdot P_i(\cdot)]] - \sum_{j=1, j\neq i}^{|T_s|} Q \cdot P_i(\cdot)P_j(\cdot)[\lambda_j + \phi[x_j - Q \cdot P_j(\cdot)]]$ and $M_i = \lambda_i + \phi[x_i - Q \cdot P_i(\cdot)]$. The decentralized supply chain network SUE conditions combined with the augmented Lagrangian functions leading to $(p_{ni}^*, t_m^{i*}, t_n^{i*}, h_i^*)$ are the solutions of the following multinomial logit-based variational inequality formulation:

$$\underbrace{\sum_{s=1}^{S}\sum_{i=1}^{|T_s|}\left[c_{ni}(\cdot) + c_{mi}(\cdot) - p_{ni} - \frac{1}{\bar{\alpha}[1 - P_i(\cdot)]} + M_i\right] \times [x_i - x_i^*]}_{I} + \underbrace{\sum_{s=1}^{S}\sum_{i=1}^{|T_s|}\left[-[\frac{1}{1 - P_i(\cdot)}]x_i + \bar{\alpha} \cdot Z_i\right] \times [p_{ni} - p_{ni}^*]}_{II}$$
$$+\underbrace{\sum_{s=1}^{S}\sum_{i=1}^{|T_s|}\left[-[\frac{x_{i+1}}{\bar{\alpha}[1 - P_{i+1}(\cdot)]^2}\frac{\partial P_{i+1}(\cdot)}{\partial h_i} + \frac{x_i}{\bar{\alpha}[1 - P_i(\cdot)]^2}\frac{\partial P_i(\cdot)}{\partial h_i}] - Q[\frac{\partial P_{i+1}(\cdot)}{\partial h_i}M_{i+1} + \frac{\partial P_i(\cdot)}{\partial h_i}M_i]\right] \times [h_i - h_i^*]}_{III}$$
$$+\underbrace{\sum_{s=1}^{S}\sum_{i=1}^{|T_s|}\left[[\frac{\partial c_{mi}(\cdot)}{\partial t_m^{i*}} - \frac{\bar{\beta}_i P_i(\cdot)}{\bar{\alpha}[1 - P_i(\cdot)]}]x_i + \bar{\beta}_i \cdot Z_i\right] \times [t_m^i - t_m^{i*}]}_{IV} + \underbrace{\sum_{s=1}^{S}\sum_{i=1}^{|T_s|}\left[[\frac{\partial c_{ni}(\cdot)}{\partial t_n^{i*}} - \frac{\bar{\beta}_i P_i(\cdot)}{\bar{\alpha}[1 - P_i(\cdot)]}]x_i + \bar{\beta}_i \cdot Z_i\right] \times [t_n^i - t_n^{i*}]}_{V}$$
$$+\underbrace{\sum_{s=1}^{S}\sum_{i=1}^{|T_s|}[x_i - Q \cdot P_i(\cdot)] \times [\lambda_i - \lambda_i^*]}_{VI} \geq 0, \forall p, x, t \in R_+^{MN}, \tag{G1}$$

where $\Omega = (x_i \geq 0, t_a^i \geq 0, p_{ni} \geq 0, h_i \leq h_{i+1})$ is the feasible set of variables for the supply chain network, the term λ_i is the Lagrange multiplier associated with constraint (8) for option i, and $\partial P_i(\cdot)/\partial h_i$ are partial derivative of the probability of customers choosing option i (see (H5)). The first term is the necessary first-order conditions of the augmented Lagrangian function (D4) with respect to the quantity supplied of the supply chain for option i. The second term is the necessary first-order conditions with respect to retail price. The third term is the necessary first-order conditions with respect to scale of customers' time-sensitivity coefficient being selected by firms to segment customers. The fourth and fifth terms are the necessary first-order conditions with respect to lead times provided by manufacturers and retailers, respectively. The sixth term is the necessary first-order conditions with respect to Lagrange multiplier.

Statement in the equilibrium conditions (G1).

In the proof of the equilibrium conditions of the decentralized supply chain network, the equilibrium conditions with link flow are formulated with multinomial logit-based variational inequality (G1). Let $\partial P_i(\cdot)/\partial p_i$, $\partial P_i(\cdot)/\partial t_s^i$, and $\partial P_i(\cdot)/\partial h_i$ be the partial derivatives of the probability of customers choosing option i with respect to p_i, t_s^i, and h_i, respectively,

$$\frac{\partial P_i(\cdot)}{\partial p_i} = \frac{\partial(\frac{e^{U_i(\cdot)}}{1+\sum_{i=1}^{I} e^{U_i(\cdot)}})}{\partial p_i} = \frac{-e^{U_i(\cdot)}\bar{\alpha}[1 + \sum_{i=1}^{I} e^{U_i(\cdot)} - e^{U_i(\cdot)}]}{[1 + \sum_{i=1}^{I} e^{U_i(\cdot)}]^2} = -\bar{\alpha}P_i(\cdot)[1 - P_i(\cdot)]$$

and

$$\frac{\partial P_i(\cdot)}{\partial t_s^i} = \frac{-e^{U_i(\cdot)}\bar{\beta}_i[1 + \sum_{i=1}^{I} e^{U_i(\cdot)} - e^{U_i(\cdot)}]}{[1 + \sum_{i=1}^{I} e^{U_i(\cdot)}]^2} = -\bar{\beta}_i P_i(\cdot)[1 - P_i(\cdot)], \tag{H1}$$

where

$$\bar{\beta}_i = \int_{h_i}^{h_{i+1}} \tilde{\beta} f(\tilde{\beta}) d\tilde{\beta}. \tag{H2}$$

By replacing $\bar{\beta}_i$ using (H2) and reorganizing terms, (H1) can be written as:

$$\frac{\partial P_i(\cdot)}{\partial t_s^i} = \frac{-e^{U_i(\cdot)} \int_{h_i}^{h_{i+1}} \tilde{\beta} f(\tilde{\beta}) d\tilde{\beta} [1 + \sum_{i=1}^{I} e^{U_i(\cdot)} - e^{U_i(\cdot)}]}{[1 + \sum_{i=1}^{I} e^{U_i(\cdot)}]^2} = -\int_{h_i}^{h_{i+1}} \tilde{\beta} f(\tilde{\beta}) d\tilde{\beta} P_i(\cdot)[1 - P_i(\cdot)].$$

$$\frac{\partial P_i(\cdot)}{\partial h_i} = \frac{\partial P_i(\cdot)}{\partial U_i(\cdot)} \frac{\partial U_i(\cdot)}{\partial h_i}, \tag{H3}$$

and

$$\frac{\partial P_{i+1}(\cdot)}{\partial h_i} = \frac{\partial P_{i+1}(\cdot)}{\partial U_{i+1}(\cdot)} \frac{\partial U_{i+1}(\cdot)}{\partial h_i}, \tag{H4}$$

where $\frac{\partial U_i(\cdot)}{\partial h_i} = -t_s^i \frac{\partial \int_{h_{i-1}}^{h_i} \tilde{\beta} f(\tilde{\beta}) d\tilde{\beta}}{\partial h_i} = -h_i t_s^i \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{[h_i - \bar{\beta}]^2}{2\sigma^2}} = -h_i t_s^i f(h_i),$
and $\frac{\partial U_{i+1}(\cdot)}{\partial h_i} = -t_s^i \frac{\partial \int_{h_i}^{h_{i+1}} \tilde{\beta} f(\tilde{\beta}) d\tilde{\beta}}{\partial h_i} = h_i t_s^i \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{[h_i - \bar{\beta}]^2}{2\sigma^2}} = h_i t_s^i f(h_i).$
By substituting $\partial U_i(\cdot)/\partial h_i$ and $\partial U_{i+1}(\cdot)/\partial h_i$ into (H3) and (H4), we can get the following equations:

$$\frac{\partial P_i(\cdot)}{\partial h_i} = -\frac{e^{U_i(\cdot)} h_i t_s^i \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{[h_i - \bar{\beta}]^2}{2\sigma^2}} [1 + \sum_{i=1}^{I} e^{U_i(\cdot)} - e^{U_i(\cdot)}]}{\left[1 + \sum_{i=1}^{I} e^{U_i(\cdot)}\right]^2} = -h_i \cdot t_s^i f(h_i) P_i(\cdot)[1 - P_i(\cdot)] \text{ and} \tag{H5}$$

$$\frac{\partial P_{i+1}(\cdot)}{\partial h_i} = -\frac{e^{U_i(\cdot)} h_i t_s^i \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{[h_i - \bar{\beta}]^2}{2\sigma^2}} [1 + \sum_{i=1}^{I} e^{U_i(\cdot)} - e^{U_i(\cdot)}]}{[1 + \sum_{i=1}^{I} e^{U_i(\cdot)}]^2} = -h_i \cdot t_s^i f(h_i) P_{i+1}(\cdot)[1 - P_{i+1}(\cdot)]. \tag{H6}$$

Proof of Lemma 6.

(Lagrangian function). In this proof, we give the range of the objective functions in a centralized supply chain that are concave from a negative definite of bordered Hessian matrix perspective. The first step is to construct the Lagrange function for profit function π with respect to flow conservation constraints (8). Then taking the second-order partial derivatives of π and the first-order partial derivatives of flow conservation constraints (8) with respect to x_i, p_i, and t_s^i, we get the bordered Hessian matrix:

$$det(\bar{H}) = \begin{bmatrix} 0 & 1 & -Q\frac{\partial P_i(\cdot)}{\partial p_i} & -Q\frac{\partial P_i(\cdot)}{\partial t_s^i} \\ 1 & 0 & 1 & -c_i'(\cdot) \\ -Q\frac{\partial P_i(\cdot)}{\partial p_i} & 1 & -\lambda_i Q\frac{\partial^2 P_i(\cdot)}{\partial^2 p_i} & -\lambda_i Q\frac{\partial^2 P_i(\cdot)}{\partial p_i \partial t_s^i} \\ -Q\frac{\partial P_i(\cdot)}{\partial t_s^i} & -c_i'(\cdot) & -\lambda_i Q\frac{\partial^2 P_i(\cdot)}{\partial t_s^i \partial p_i} & c_i''(\cdot)x_i - \lambda_i Q\frac{\partial^2 P_i(\cdot)}{\partial^2 t_s^i} \end{bmatrix}$$

$$= \underbrace{\lambda_i \cdot Q^2 \cdot \bar{\alpha} \cdot \bar{\beta}_i^2 P_i(\cdot)^2 [1 - P_i(\cdot)]^2 [1 - 2P_i(\cdot)][-2 - \lambda_i \cdot \bar{\alpha}(1 - 2P_i(\cdot))]}_{I} + \underbrace{[1 - P_i(\cdot)]^2 Q^2 P_i(\cdot)^2 [[\bar{\alpha} \cdot c_i'(\cdot)]^2 - \bar{\beta}_i^2]}_{II}. \tag{I1}$$

If $1 + \sum_{j=1}^{I} e^{U_j(\cdot)} > e^{U_i(\cdot)}, i \neq j$, then the first term in (I1) is less than zero. If $-c_i'(\cdot) < \bar{\beta}_i/\bar{\alpha}$, then the second term is less than zero. Therefore, if $1 + \sum_{j=1}^{I} e^{U_j(\cdot)} > e^{U_i(\cdot)}, i \neq j$, and $-c_i'(\cdot) < \bar{\beta}_i/\bar{\alpha}$, then the bordered Hessian matrix $det(\bar{H}) < 0$.

The centralized supply chain network SUE conditions.

The centralized supply chain network SUE conditions combined with the augmented Lagrangian functions leading to $(p_{ni}^*, t_{ni}^*, t_{mi}^*, h_i^*)$ are solutions of the following variational inequality formulation:

$$\underbrace{\sum_{s=1}^{S}\sum_{i=1}^{|T_s|}\left[c_{ni}(\cdot)+c_{mi}(\cdot)-p_{ni}+M_i\right]\times[x_i-x_i^*]}_{I}+\underbrace{\sum_{s=1}^{S}\sum_{i=1}^{|T_s|}\left[-x_i+\bar{\alpha}\cdot Z_i\right]\times[p_{ni}-p_{ni}^*]}_{II}$$
$$+\underbrace{\sum_{s=1}^{S}\sum_{i=1}^{|T_s|}\left[-Q(\frac{\partial P_{i+1}(\cdot)}{\partial h_i}M_{i+1}+\frac{\partial P_i(\cdot)}{\partial h_i}M_i)\right]\times[h_i-h_i^*]}_{III}+\underbrace{\sum_{s=1}^{S}\sum_{i=1}^{|T_s|}\left[\frac{\partial c_{mi}(\cdot)}{\partial t_m^{i*}}x_i+\bar{\beta}_i\cdot Z_i\right]\times[t_m^i-t_m^{i*}]}_{IV}+ \quad \text{(J1)}$$
$$\underbrace{\sum_{s=1}^{S}\sum_{i=1}^{|T_s|}\left[\frac{\partial c_{ni}(\cdot)}{\partial t_n^{i*}}x_i+\bar{\beta}_i\cdot Z_i\right]\times[t_n^i-t_n^{i*}]}_{V}+\underbrace{\sum_{s=1}^{S}\sum_{i=1}^{|T_s|}\left[x_i-Q\cdot P_i(\cdot)\right]\times[\lambda_i-\lambda_i^*]}_{VI}\geq 0,\forall p,x,t\in R_+^{MN},$$

where $\Omega=(x_s\geq 0, t_a^i\geq 0, p_{ni}\geq 0, h_i\leq h_{i+1})$ is the feasible set of chain variables for the supply chain network, the term λ_i is the Lagrange multiplier associated with constraint (8) for option i and $\partial P_i(\cdot)/\partial h_i$ is the partial derivative of the probability of customers choosing option i (see (H5)). The first term is the necessary first-order conditions of the augmented Lagrangian function with respect to the quantity supplied of the supply chain for option i. The second term is the necessary first-order conditions with respect to retail price. The third term is the necessary first-order conditions with respect to scale of customers' time-sensitivity coefficient being selected by firms to segment customers. The fourth and fifth terms are the necessary first-order conditions with respect to the lead times provided by manufacturers and retailers, respectively. The sixth term is the necessary first-order conditions with respect to Lagrange multiplier.

Existence proof of Theorems 1 and 2.

Suppose that the time-cost and profit functions are continuous. There is at least one equilibrium solution for variational inequalities (G1) and (J1).

The profit functions of a chain and the link cost functions are continuous. This implies that the marginal profit of chain and marginal link cost are continuous. The constraint set Ω is bounded, closed, and convex. According to the variational inequality theory (Zhang, 2006; Nagurney and Yu, 2012), variational inequalities (G1) and (J1) have at least one equilibrium solution.

Uniqueness proof of Theorems 1 and 2.

If the marginal profit functions $\nabla\pi(X^*)$ of variational inequality (G1) and (J1) are strictly monotone, then there is a unique equilibrium solution for the decentralized and centralized supply chain network.

In this proof, we prove that there exists a unique equilibrium solution (G1) and (J1) from a negative definite of bordered Hessian matrix perspective (see (D6)). If the penalty parameter is sufficiently large, then the bordered Hessian matrix $|\bar{H}|<0$. Hence X^*, λ^* meet the second-order sufficiency conditions of the proposed problems. According to the basic theory of variational inequalities (Zhang, 2006; Nagurney and Yu, 2012), variational inequalities (G1) and (J1) have a unique equilibrium solution, respectively. Theorems 1 and 2 are immediate from the negative definite of the bordered Hessian matrix perspective.

Proof of Lemma 7.

Let $AL(X)$ denote the function that enters the variational inequality problem and is strongly monotonic in the entire domain of decision variables. Let $\hat{AL}(X)$ denote the perturbed function with solution $\hat{X}$ that has a small change in $AL(X)$. Equilibrium solutions, vectors X^* and $\hat{X}^*$, can be obtained by the following variational inequalities, respectively.

$$\nabla AL(X)^T\cdot(X-X^*)\geq 0,\forall x\in\Omega, \quad \text{(M1)}$$

$$\nabla \hat{AL}(X)^T \cdot (X - \hat{X}^*) \geq 0, \forall X \in \Omega. \tag{M2}$$

Let $X = \hat{X}^*$ in (M1) and $X = X^*$ in (M2). Then combining the above variational inequalities, we have,

$$[\nabla \hat{AL}(X) - \nabla AL(X)]^T \cdot [\hat{X}^* - X^*] \leq 0.$$

Proof of Proposition 1.

(a) Assuming that the random variable follows a Gumbel distribution (Li, 2011; Chikaraishi and Nakayama, 2016). Recall that the random variable, ε, is ignored in firms' (supply chains') decisions under UE conditions. Considering the IC constraint that customers choose with option i, the probability of customers choosing option i under UE is,

$$P_i'^{*\xi}(\cdot) = P(V_i > V_j) = P(w_i - \tilde{\alpha} \cdot p_{ni} - \tilde{\beta} \cdot t_s^i - w_j + \tilde{\alpha} \cdot p_{nj} + \tilde{\beta} \cdot t_s^j > 0), \forall j \in \{j \neq i\} \text{ and } \forall \xi \in \{D, C\};$$

We assume that all factors affecting customer choice behavior cannot be completely observed by firms. The random variable reflects other factors that affect a consumer's utility that firms do not know. Then, considering the IC constraint that customers choose option i, the probability of customers choosing option i under SUE is,

$$P_i^{*\xi}(\cdot) = P(U_i > U_j) = P(w_i - \tilde{\alpha} \cdot p_{ni} - \tilde{\beta} \cdot t_s^i - w_j + \tilde{\alpha} \cdot p_{nj} + \tilde{\beta} \cdot t_s^j + \varepsilon_i - \varepsilon_j > 0), \forall j \in \{j \neq i\} \text{ and } \forall \xi \in \{D, C\},$$

where random variables ε_i and ε_j are assumed to not be related, which follow a Gumbel distribution with a mean of zero. In terms of the algebra of probability distributions, we have $1 - \sum_{i=1}^{I} P_i^{*\xi} > 1 - \sum_{i=1}^{I} P_i'^{*\xi}, \forall \xi \in \{D, C\}$.

The First term of the centralized supply chain network SUE conditions (J1) is the necessary first-order condition (FOC) for firms by choice of own output for option i, $Q \cdot P_i(\cdot)$, which is increasing in the output (see the proof of Lemma 3). Let $\nabla \hat{AL}(P_i(\cdot))$ and $\nabla AL(P_i(\cdot))$ denote the first terms of the decentralized and centralized supply chain network SUE conditions (G1 and J1), respectively. Then, $\nabla \hat{AL}(P_i(\cdot)) - \nabla AL(P_i(\cdot)) = -1/[\bar{\alpha}[1 - P_i(\cdot)]] < 0$. In terms of Lemma 7, we have $1 - \sum_{i=1}^{I} P_i^{*D} > 1 - \sum_{i=1}^{I} P_i^{*C}$.

(b) The second terms of the decentralized and centralized supply chain network SUE conditions (G1 and J1) are the necessary first-order conditions (FOC) for firms by choice of retail prices. Let $\nabla \hat{AL}(p_{ni})$ and $\nabla AL(p_{ni})$ denote the second terms of the decentralized and centralized supply chain network SUE conditions (G1 and J1), respectively. Then, $\nabla \hat{AL}(p_{ni}) - \nabla AL(p_{ni}) = -[P_i(\cdot)x_i]/[1 - P_i(\cdot)] < 0$. Using a similar approach, we have $p_{ni}^{*D} > p_{ni}^{*C}$.

The fourth and fifth terms of the decentralized and centralized supply chain network SUE conditions (G1 and J1) are the necessary first-order conditions (FOC) for firms by choice of lead times. Let $\nabla \hat{AL}(t_{\kappa i}^D)$ and $\nabla AL(t_{\kappa i}^C)$, $\kappa \in \{m, n\}$, denote the the fourth and fifth terms of the decentralized and centralized supply chain network SUE conditions (G1 and J1), $\nabla \hat{AL}(t_{\kappa i}^D) - \nabla AL(t_{\kappa i}^C) = -[\bar{\beta}_i x_i P_i(\cdot)]/[\bar{\alpha}_i[1 - P_i(\cdot)]] < 0$. In terms of Lemma 7, we have $t_{\kappa i}^{*D} > t_{\kappa i}^{*C}, \forall \kappa \in \{m, n\}$.

(c) In terms of the uniqueness proof of Theorems 1 and 2, there exists a unique equilibrium solution for decentralized and centralized supply chain networks under SUE. In other words, in equilibrium no supply chain (or firm) can improve the profit by unilaterally changing its P/T menu. It is straightforward that $\pi_{\kappa i}^{*\xi} > \pi_{\kappa i}'^{*\xi}, \forall \xi \in \{D, C\}, \forall \kappa \in \{m, n\}$ under SUE conditions.

The objective functions in a centralized supply chain network are (11). Considering the best response pricing function, the objective functions in a decentralized supply chain network can be rewritten as,

$$\max_{p_{ni}, t_s^i} \pi_s^D(X_s) = \max_{p_{ni}, t_s^i} \sum_{i=1}^{|T_s|} [p_{ni} - \frac{1}{\bar{\alpha}[1 - P_i(\cdot)]} - c_{ni}(\cdot) - c_{mi}(\cdot)] \cdot x_i,$$

where $-1/[\bar{\alpha}[1 - P_i(\cdot)]] < 0$. Comparing the objective functions in decentralized and centralized supply chains, we have $\pi_{\kappa i}^{*D} < \pi_{\kappa i}^{*C}, \forall \kappa \in \{m, n\}$.

Proof of Proposition 2.

(a) Let $\nabla \hat{AL}(\hat{x}_i)$ and $\nabla AL(x_i)$ denote the first term of the decentralized supply chain equilibrium conditions considering a slightly higher degree of the time-cost tradeoffs of the links participating option i and not, respectively. Then, in terms of the form of the first term of the decentralized supply chain equilibrium conditions, we have

$\nabla \hat{AL}(\hat{x}_i) - \nabla AL(x_i) > 0$. Substituting $\nabla \hat{AL}(\hat{x}_i) - \nabla AL(x_i) > 0$ into Lemma 7 yields $\hat{x}_i^{*D} - x_i^{*D} < 0$. Based on the form of the first term of the decentralized supply chain equilibrium conditions, the equilibrium flows of a firm will be impacted by the degree of the time-cost tradeoffs of all links participating option i. Similarly, we can obtain $\hat{x}_i^{*C} - x_i^{*C} < 0$ as well.

(b) Let $\nabla \hat{AL}(\hat{t}_{\kappa i})$ and $\nabla AL(t_{\kappa i})$, $\kappa \in \{m, n\}$, denote the fourth and fifth terms of the decentralized supply chain equilibrium conditions considering a slightly higher degree of the time-cost tradeoffs of the links participating option i and not, respectively. Then, from the form of the fourth and fifth terms of the decentralized supply chain equilibrium conditions, we have $\nabla \hat{AL}(\hat{t}_{\kappa i}) - \nabla AL(t_{\kappa i}) > 0$. Substituting $\nabla \hat{AL}(\hat{t}_{\kappa i}) - \nabla AL(t_{\kappa i}) > 0$ into Lemma 7 yields $\hat{t}_{\kappa i}^{*D} - t_{\kappa i}^{*D} < 0, \forall \kappa \in \{m, n\}$. Using a similar approach, we can also obtain $\hat{t}_{\kappa i}^{*C} - t_{\kappa i}^{*C} < 0, \forall \kappa \in \{m, n\}$.

(c) Recall that the first term of the decentralized and centralized supply chain equilibrium conditions (G1) and (J1) are the necessary first-order conditions (FOC) for firms by choice of flows which are monotonic increasing functions, considering augmented Lagrangian functions. Let $\psi^D(c_i', x_i^D(\cdot))$ and $\psi^C(c_i', x_i^C(\cdot))$ denote the first terms of the decentralized and centralized supply chain network SUE conditions (G1 and J1), respectively. Then, $\psi^D(c_i', x_i^D(\cdot)) - \psi^C(c_i', x_i^C(\cdot)) = -1/[\bar{\alpha}[1 - P_i(\cdot)]] < 0$. The difference between the first terms of the decentralized and centralized supply chain network SUE conditions is decreasing in $P_i(\cdot)$,

$$\frac{\partial[\psi^D(c_i', x_i^D(\cdot)) - \psi^C(c_i', x_i^C(\cdot))]}{\partial P_i} = -\frac{1}{\bar{\alpha}[1 - P_i(\cdot)]^2} < 0, \tag{M1}$$

where $Q \cdot P_i(\cdot) = x_i^*$ in equilibrium. The partial derivative of the cost functions with respect to lead times reflects the degree of the time-cost tradeoff. Employing the implicit function rule,

$$\frac{\partial x_i^D}{\partial c_i'} = -\frac{\partial \psi^D(c_i', x_i^D(\cdot))/\partial c_i'}{\partial \psi^D(c_i', x_i^D(\cdot))/\partial x_i^D} \text{ and } \frac{\partial x_i^C}{\partial c_i'} = -\frac{\partial \psi^C(c_i', x_i^C(\cdot))/\partial c_i'}{\partial \psi^C(c_i', x_i^C(\cdot))/\partial x_i^C}.$$

From the forms of the first term of the decentralized and centralized supply chain equilibrium conditions (G1) and (J1),

$$\begin{aligned}\frac{\partial x_i^D}{\partial c_i'} - \frac{\partial x_i^C}{\partial c_i'} &= \frac{\partial \psi^C(c_i', x_i^C(\cdot))/\partial c_i'}{\partial \psi^C(c_i', x_i^C(\cdot))/\partial x_i^C} - \frac{\partial \psi^D(c_i', x_i^D(\cdot))/\partial c_i'}{\partial \psi^D(c_i', x_i^D(\cdot))/\partial x_i^D} \\ &= \frac{\partial \psi^C(c_i', x_i^C(\cdot))/\partial c_i'}{\partial \psi^C(c_i', x_i^C(\cdot))/\partial x_i^C} - \frac{\partial \psi^C(c_i', x_i^C(\cdot))/\partial c_i' + \partial[\psi^D(c_i', x_i^D(\cdot)) - \psi^C(c_i', x_i^C(\cdot))]/\partial c_i'}{\partial \psi^C(c_i', x_i^C(\cdot))/\partial x_i^C + \partial[\psi^D(c_i', x_i^D(\cdot)) - \psi^C(c_i', x_i^C(\cdot))]/\partial x_i^D},\end{aligned}$$

where $\partial \psi^D(c_i', x_i^D(\cdot))/\partial x_i^D > 0$ and $\partial \psi^C(c_i', x_i^C(\cdot))/\partial x_i^C > 0$ since the objective functions are transformed into minimization problems in (G1 and J1), and $\partial[\psi^D(c_i', x_i^D(\cdot)) - \psi^C(c_i', x_i^C(\cdot))]/\partial c_i' = 0$. Combining with M1, we obtain that the effect of a change in the degree of the time-cost tradeoffs of the links participating option i on the equilibrium flows in decentralized supply chains is lower than the effect of a change on the equilibrium flows in centralized supply chains, $\partial x_i^D/\partial c_i' < \partial x_i^C/\partial c_i'$.

Recall that the fourth and fifth terms of the decentralized and centralized supply chain equilibrium conditions (G1) and (J1) are the necessary first-order conditions (FOC) for firms by choice of lead times. Let $\psi^D(c_i', t_{\kappa i}^D(\cdot))$ and $\psi^C(c_i', t_{\kappa i}^C(\cdot))$, $\kappa \in \{m, n\}$, denote the fourth and fifth terms of the decentralized and centralized supply chain network SUE conditions (G1 and J1), respectively. Then, $\psi^D(c_i', t_{\kappa i}^D(\cdot)) - \psi^C(c_i', t_{\kappa i}^C(\cdot)) = -[\bar{\beta}_i x_i P_i(\cdot)]/[\bar{\alpha}_i[1 - P_i(\cdot)]] < 0$, $\kappa \in \{m, n\}$. The difference between the fourth and fifth terms of the decentralized and centralized supply chain network under SUE conditions is increasing in $t_{\kappa i}^D$, $\kappa \in \{m, n\}$,

$$\frac{\partial[\psi^D(c_i', t_{\kappa i}^D(\cdot)) - \psi^C(c_i', t_{\kappa i}^C(\cdot))]}{\partial t_{\kappa i}^D} = \frac{\bar{\beta}^2 x_i P_i(\cdot)}{\bar{\alpha}[1 - P_i(\cdot)]} > 0, \forall \kappa \in \{m, n\}.$$

Using a similar approach, we obtain that the effect of a change in the degree of the time-cost tradeoffs of the links participating option i on the equilibrium lead times in decentralized supply chains is greater than the effect of a change on the equilibrium lead times in centralized supply chains, $\partial t_{\kappa i}^{*D}/\partial c_i' > \partial t_{\kappa i}^{*C}/\partial c_i'$, $\forall \kappa \in \{m, n\}$.

Proof of Proposition 3.

We adopt mathematical induction to prove Proposition 3. Base case: Suppose a centralized supply chain provides only a single P/T option to its customers under SUE conditions. The probability of customers choosing this option remains unchanged when the variance of customers' time-sensitivity distribution increases or decreases within a certain range. Suppose a supply chain provides two P/T options to its customers. Based on Lemma 2, the probability of customers choosing an option increases with the variance of customers' time-sensitivity distribution if $(h_{i-1}, h_i) \notin [\bar{\beta} - \sigma, \bar{\beta} + \sigma]$. Hence, when the costs of providing two options are feasible, we can find solutions that can meet the following inequality if the variance of customers' time-sensitivity distribution is above the threshold. Thus,

$$\sum_{i=1}^{2} \left[[p_{ni} - C_i(\cdot)] \cdot Q \cdot P_i(\cdot)\right] > [p_{n1} - C_1(\cdot)] \cdot Q \cdot P_1(\cdot) \text{ if } \sigma^2 > \sigma_h^2.$$

Induction step: Given that $\sum_{i=1}^{I}[[p_{ni} - C_i(\cdot)] \cdot Q \cdot P_i(\cdot)] > \sum_{i=1}^{I-1}[[p_{ni} - C_i(\cdot)] \cdot Q \cdot P_i(\cdot)]$ holds for some value of $1 < i < N$. Assuming the cost is zero, it is the optimal solution to provide a unique option for each customer, which means $i = N$. Hence, we know that $I < N$. When $i = I + 1 < N$ and the costs are feasible, we can find solutions that can meet $\sum_{i=1}^{I}[[p_{ni} - C_i(\cdot)] \cdot Q \cdot P_i(\cdot)] > \sum_{i=1}^{I-1}[[p_{ni} - C_i(\cdot)] \cdot Q \cdot P_i(\cdot)]$ based on Lemma 2.

Similarly, when the costs of providing two options are feasible, we can also find solutions that can meet the following inequality if the variance of customers' time-sensitivity distribution is above the threshold in a decentralized supply chain. Thus,

$$\sum_{i=1}^{2} \left[[p_{ni} - \frac{1}{\bar{\alpha}[1 - P_i(\cdot)]} - c_{ni}(\cdot) - c_{mi}(\cdot)] \cdot Q \cdot P_i(\cdot)\right] > [p_{n1} - \frac{1}{\bar{\alpha}[1 - P_1(\cdot)]} - c_{n1}(\cdot) - c_{m1}(\cdot)] \cdot Q \cdot P_1(\cdot) \text{ if } \sigma^2 > \sigma_h^2.$$

Let $\psi(\sigma)$ denote the difference between the objective functions of the decentralized and centralized supply chains. Combining with Lemma 2 and (M1), we have $\partial\psi(\sigma)/\partial\sigma < 0$ if $(h_{i-1}, h_i) \notin [\bar{\beta} - \sigma, \bar{\beta} + \sigma]$. The induction step in decentralized settings is similar to the step in centralized settings. Then we have $\sigma_h^D > \sigma_h^C$ which means that decentralized supply chains require a greater variance relative to centralized supply chains for the same profits. Therefore, we further conclude that the variance of customers' time-sensitivity distribution is a key factor in customer segmentations.

Proof of Proposition 4.

(a) Recall that Lemma 2 shows the effect of the variance of customers' time-sensitivity distribution on firms' market shares under a given customer segment. Combining Lemma 2 with Lemma 5, we have the effect of the standard deviation of customers' time-sensitivity distribution on firms' prices.

(b) Let $\psi^D(\sigma, t_{ni}^D)$ and $\psi^C(\sigma, t_{ni}^C)$ denote the fifth terms of the decentralized and centralized supply chain network SUE conditions (G1 and J1), respectively. Employing the implicit function rule,

$$\frac{\partial t_{ni}^D}{\partial \sigma} = -\frac{\partial \psi^D(\sigma, t_{ni}^D)/\partial\sigma}{\partial \psi^D(\sigma, t_{ni}^D)/\partial t_{ni}^D} \text{ and } \frac{\partial t_{ni}^C}{\partial \sigma} = -\frac{\partial \psi^C(\sigma, t_{ni}^C)/\partial\sigma}{\partial \psi^C(\sigma, t_{ni}^C)/\partial t_{ni}^C},$$

where $\partial\psi^D(\sigma, t_{ni}^D)/\partial\sigma > 0$ and $\partial\psi^C(\sigma, t_{ni}^C)/\partial\sigma > 0$ if $(h_{i-1}, h_i) \notin [\bar{\beta} - \sigma, \bar{\beta} + \sigma]$. Then, we have $\partial t_{ni}^D/\partial\sigma < 0$ and $\partial t_{ni}^C/\partial\sigma < 0$ if $(h_{i-1}, h_i) \notin [\bar{\beta} - \sigma, \bar{\beta} + \sigma]$; otherwise, vice versa. Using a similar approach, we have $\partial t_{mi}^D/\partial\sigma < 0$ and $\partial t_{mi}^C/\partial\sigma < 0$ if $(h_{i-1}, h_i) \notin [\bar{\beta} - \sigma, \bar{\beta} + \sigma]$ as well; otherwise, vice versa.

(c) The second terms of the decentralized and centralized supply chain network SUE conditions (G1 and J1) are the necessary first-order conditions (FOC) for firms by choice of retail prices. Let $\psi^D(\sigma, p_{ni}^D)$ and $\psi^C(\sigma, p_{ni}^D)$ denote the second terms of the decentralized and centralized supply chain network SUE conditions (G1 and J1), respectively. Then $\psi^D(\sigma, p_{ni}^D) - \psi^C(\sigma, p_{ni}^C) = -[P_i(\cdot)x_i]/[1 - P_i(\cdot)] < 0$. Combining with the proof of Lemma 2, the difference between the first terms of the decentralized and centralized supply chain network SUE conditions is decreasing in the standard deviation, σ, if $(h_{i-1}, h_i) \notin [\bar{\beta} - \sigma, \bar{\beta} + \sigma]$; otherwise, vice versa.

$$\frac{\partial[\psi^D(\sigma, p_{ni}^D) - \psi^C(\sigma, p_{ni}^C)]}{\partial\sigma} = \frac{-x_i \partial P_i(\cdot)/\partial\sigma}{[1 - P_i(\cdot)]^2} \begin{cases} \geq 0 & \text{if } (h_{i-1}, h_i) \in [\bar{\beta} - \sigma, \bar{\beta} + \sigma] \\ < 0 & \text{if } (h_{i-1}, h_i) \notin [\bar{\beta} - \sigma, \bar{\beta} + \sigma] \end{cases}, \forall i.$$

Employing the implicit function rule,

$$\frac{\partial p_{ni}^D}{\partial \sigma} = -\frac{\partial \psi^D(\sigma, p_{ni}^D)/\partial \sigma}{\partial \psi^D(\sigma, p_{ni}^D)/\partial p_{ni}^D} \text{ and } \frac{\partial p_{ni}^C}{\partial \sigma} = -\frac{\partial \psi^C(\sigma, p_{ni}^C)/\partial \sigma}{\partial \psi^C(\sigma, p_{ni}^C)/\partial p_{ni}^C};$$

Next, Proposition 4(a) shows that $\partial p_{ni}^D/\partial \sigma > 0$ and $\partial p_{ni}^C/\partial \sigma > 0$ if $(h_{i-1}, h_i) \notin [\bar{\beta} - \sigma, \bar{\beta} + \sigma]$. In terms of the forms of the second terms of the decentralized and centralized supply chain network SUE conditions (G1 and J1), we have

$$\begin{aligned}\frac{\partial p_{ni}^D}{\partial \sigma} - \frac{\partial p_{ni}^C}{\partial \sigma} &= \frac{\partial \psi^C(\sigma, p_{ni}^C)/\partial \sigma}{\partial \psi^C(\sigma, p_{ni}^C)/\partial p_{ni}^C} - \frac{\partial \psi^D(\sigma, p_{ni}^D)/\partial \sigma}{\partial \psi^D(\sigma, p_{ni}^D)/\partial p_{ni}^D} \\ &= \frac{\partial \psi^C(\sigma, p_{ni}^C)/\partial \sigma}{\partial \psi^C(\sigma, p_{ni}^C)/\partial p_{ni}^C} - \frac{\partial \psi^C(\sigma, p_{ni}^C)/\partial \sigma + \partial[\psi^D(\sigma, p_{ni}^D) - \psi^C(\sigma, p_{ni}^C)]/\partial \sigma}{\partial \psi^C(\sigma, p_{ni}^C)/\partial p_{ni}^C + \partial[\psi^D(\sigma, p_{ni}^D) - \psi^C(\sigma, p_{ni}^C)]/\partial p_{ni}^D},\end{aligned}$$

where $\partial[\psi^D(\sigma, p_{ni}^D) - \psi^C(\sigma, p_{ni}^C)]/\partial p_{ni}^D = x_i \bar{\alpha} P_i(\cdot)/[1 - P_i(\cdot)] > 0$ and $\partial[\psi^D(\sigma, p_{ni}^D) - \psi^C(\sigma, p_{ni}^C)]/\partial \sigma < 0$ if $(h_{i-1}, h_i) \notin [\bar{\beta} - \sigma, \bar{\beta} + \sigma]$. Then, we have $\partial p_{ni}^{*D}/\partial \sigma > \partial p_{ni}^{*C}/\partial \sigma$ if $(h_{i-1}, h_i) \notin [\bar{\beta} - \sigma, \bar{\beta} + \sigma]$.

(d) Recall that Lemma 4 shows the effect of the customers' time-sensitivity coefficients on firms' prices in a leader-follower framework. Combining Lemma 4 with the decentralized and centralized supply chain network SUE conditions (G1 and J1), we get Proposition 4(d).

Algorithm.

Using the modified projection method (Khanh and Phan, 2014; Nagurney et al., 2013), the necessary condition for iterative convergence is that $\nabla AL(X)$ is monotone, where $AL(X)$ is the function that enters the variational inequality problem. If such a condition is not met like our proposed model, as is very common in supply chain management, then the algorithm will not converge iteratively (He and Yuan, 2012). We use AL functions to convert constrained problems into unconstrained problems to ensure that $\nabla AL(X)$ is monotone and this method still works for the proposed problems. Based on variational inequality theory, the functions that enter the variational inequality problem must be the minimization functions. The resulting variational inequality subproblem is then transformed into a quadratic programming problem in the modified projection method (Salarpour and Nagurney, 2021; Nagurney et al., 2013). Glowinski and Oden (1985) present a Lagrangian multiplier update method which is effective to improve the computation speed. The algorithm combined with the Lagrangian multiplier update method is stated below. Some applications of the modified projection method and the projection method to the solutions of supply chain network models can be found in Nagurney (2021b), Saberi et al. (2018), and Zhang (2006).

Set any initial feasible point $X^0 \in \Omega$ and a convergence tolerance $\varepsilon, \varepsilon > 0$. Let $k := 1$, where k is the iteration counter. Then set ρ, such that $0 < \rho \leq \frac{1}{l}$, where l is the Lipschitz constant for function $L(X)$ in the variational inequality problem (G1) and (J1).

Step 1: Set the Lagrange multiplier based on the Lagrangian multiplier update method

$$\lambda^k = \bar{\lambda}^k - \phi g_m(\bar{X}^k),$$

where the term ϕ is the penalty parameter of augmented Lagrangian function.

Step 2: Construction and computation

Compute $\bar{X}^k = (\bar{x}_i^k, \bar{p}^k, \bar{t}^k, \bar{\lambda}^k, \bar{h}^k)$ by solving the following variational inequality subproblem:

$$\left[\bar{x}^k + (\rho F\left(x^{k-1}\right) - x^{k-1})\right]^T \cdot \left[x' - \bar{x}^k\right] \geq 0, \forall x' \in \Omega.$$

The above variational inequality subproblem can be solved by using the following equivalent quadratic programming problem (Nagurney et al., 2013):

$$\min_{x \in \Omega} \frac{1}{2} \bar{x}^k H \bar{x}^k + (\rho F\left(x^{k-1}\right) - x^{k-1})^T \cdot \bar{x}^k,$$

where H is a fixed symmetric positive definite matrix.

Step 3: Adaptation

Compute $X^k = (x_i^k, p^k, t^k, \lambda^k, h^k)$ by solving the variational inequality subproblem:

$$\left[x^k + (\rho F\left(\bar{x}^k\right) - \bar{x}^k)\right]^T \cdot \left[x' - x^k\right] \geq 0, \forall x' \in \Omega.$$

The above variational inequality subproblem can be solved by using the following quadratic programming problem:

$$\min_{x\in\Omega} \frac{1}{2} x^k H x^k + (\rho F\left(\bar{x}^k\right) - \bar{x}^k)^T \cdot x^k.$$

Step 4: Convergence

If $\left|X^k - X^{k-1}\right| \leq \epsilon$ for all $i \in I$, then stop. $X^k = (x_i^k, p^k, t^k, \lambda^k, h^k)$ is an acceptable approximate solution; otherwise, set $k := k + 1$, and go to Step 1.

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