# Organization of Public Safety Networks: Spillovers, Interoperability, and Participation

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September 9, 2016

## Online Appendix. Proof of Lemmas and Propositions

### Proof of Proposition 1 – Decentralized Provision

Under decentralized provision, districts make their network asset and interoperability effort decisions simultaneously to maximize the total surplus within their own districts. The resulting network assets ($g^*_{1dd}$, $g^*_{2dd}$) and interoperability efforts ($e^*_{1dd}$, $e^*_{2dd}$) chosen by the two districts are:

$$g^*_{1dd} = \frac{m_1\left[1-\kappa\right]}{2p},\ g^*_{2dd} = \frac{m_2\left[1-\kappa\right]}{2p},\ e^*_{1dd} = e^*_{2dd} = \frac{m_1 m_2 \kappa \left[1-\kappa\right]}{4\bar{e}p\delta}.$$

Based on the above results $g^*_{1dd}$ is independent of $m_2$ and $g^*_{2dd}$ is independent of $m_1$. In addition, $e^*_{1dd} = e^*_{2dd}$. To show that the interoperability efforts of the two districts increase in the degree of spillover $\kappa$, we take the partial derivative of the interoperability effort of each district with respect to $\kappa$:

$$\frac{\partial e^*_{1dd}}{\partial \kappa} = \frac{\partial e^*_{2dd}}{\partial \kappa} = \frac{m_1 m_2 [1 - 2\kappa]}{4\bar{e}p\delta}.$$

As $m_1 > 0$, $m_2 > 0$, $\bar{e} > 0$, $p > 0$, $\delta > 0$, and $\kappa \in [0, 0.5]$, we have $\frac{\partial e^*_{1dd}}{\partial \kappa} = \frac{\partial e^*_{2dd}}{\partial \kappa} > 0$. Therefore, the combined interoperability effort, $e^*_{1dd} + e^*_{2dd}$, and hence the interoperability $\left[\frac{e^*_{1dd}+e^*_{2dd}}{\bar{e}}\right]$ increase in $\kappa$ because both $e^*_{1dd}$ and $e^*_{2dd}$ increase in $\kappa$.

Q.E.D.

**Proof of Proposition 2 – Centralized Provision**

Under centralized provision, the federal government chooses network assets, $g_{1cc}$ and $g_{2cc}$, and a single interoperability effort $e_{cc}$ that it implements to maximize the social welfare across both districts. The resulting network assets ($g^*_{1cc}$, $g^*_{2cc}$) and interoperability effort ($e^*_{cc}$) are:

$$g^*_{1cc} = \frac{m_1 [1-\kappa] [4\bar{e}^2 p\delta - [m_1^2 - m_2^2] \alpha^2 \kappa^2]}{2p [4\bar{e}^2 p\delta - [m_1^2 + m_2^2] \alpha^2 \kappa^2]},\ g^*_{2cc} = \frac{m_2 [1-\kappa] [4\bar{e}^2 p\delta + [m_1^2 - m_2^2] \alpha^2 \kappa^2]}{2p [4\bar{e}^2 p\delta - [m_1^2 + m_2^2] \alpha^2 \kappa^2]},$$

$$e^*_{cc} = \frac{2\bar{e} m_1 m_2 \alpha\kappa [1-\kappa]}{4\bar{e}^2 p\delta - [m_1^2 + m_2^2] \alpha^2 \kappa^2}.$$

To show that the network assets for one district increase in the network asset preference of the other district, we take the partial derivatives of the network assets of each district with respect to the network asset preference of the other district:

$$\frac{\partial g^*_{1cc}}{\partial m_2} = \frac{2 m_1 m_2 [1-\kappa] \alpha^2 \kappa^2 [4\bar{e}^2 p\delta - m_1^2 \alpha^2 \kappa^2]}{p [4\bar{e}^2 p\delta - [m_1^2 + m_2^2] \alpha^2 \kappa^2]^2}$$

$$\frac{\partial g^*_{2cc}}{\partial m_1} = \frac{2 m_1 m_2 [1-\kappa] \alpha^2 \kappa^2 [4\bar{e}^2 p\delta - m_2^2 \alpha^2 \kappa^2]}{p [4\bar{e}^2 p\delta - [m_1^2 + m_2^2] \alpha^2 \kappa^2]^2}.$$

As $m_1 > m_2 > 0$ and $4\bar{e}^2 p\delta - m_2^2 \alpha^2 \kappa^2 > 4\bar{e}^2 p\delta - m_1^2 \alpha^2 \kappa^2 > 4\bar{e}^2 p\delta - [m_1^2 + m_2^2] \alpha^2 \kappa^2 > 0$, we have $\frac{\partial g^*_{1cc}}{\partial m_2} > 0$ and $\frac{\partial g^*_{2cc}}{\partial m_1} > 0$.

To show that the interoperability effort $e^*_{cc}$ and hence interoperability $\frac{e^*_{cc}}{\bar{e}}$ increase in degree of spillover $\kappa$, we take the partial derivative of the interoperability effort $e^*_{cc}$ with respect to $\kappa$:

$$\frac{\partial e^*_{cc}}{\partial \kappa} = \frac{2\bar{e}m_1m_2\alpha\left[4\bar{e}^2p\delta\left[1-2\kappa\right]+\left[m_1^2+m_2^2\right]\alpha^2\kappa^2\right]}{\left[4\bar{e}^2p\delta-\left[m_1^2+m_2^2\right]\alpha^2\kappa^2\right]^2}.$$

Because $\kappa \in [0, 0.5]$, we have $\frac{\partial e^*_{cc}}{\partial \kappa} > 0$.

Q.E.D.

**Proof of Proposition 3 – Mixed Provision**

Under mixed provision, there are two cases: the *cd* case with District 1 opt-in and District 2 opt-out; and the *dc* case with District 1 opt-out and District 2 opt-in.

In the *cd* case, the federal government chooses $g_{1cd}$ as well as chooses and implements $e_{1cd}$ for District 1, and District 2 chooses its own $g_{2cd}$ and $e_{2cd}$. The resulting network assets ($g^*_{1cd}$, $g^*_{2cd}$) and interoperability efforts ($e^*_{1cd}$, $e^*_{2cd}$) are:

$$g^*_{1cd} = \frac{m_1\left[1-\kappa\right]\left[4\bar{e}^2p\delta+m_2^2\gamma^2\kappa^2\right]}{4p\left[2\bar{e}^2p\delta-m_2^2\gamma^2\kappa^2\right]},\ g^*_{2cd} = \frac{m_2\left[1-\kappa\right]}{2p}$$

$$e^*_{1cd} = \frac{m_1m_2\gamma\kappa\left[1-\kappa\right]\left[8\bar{e}^2p\delta-m_2^2\gamma^2\kappa^2\right]}{8\bar{e}p\delta\left[2\bar{e}^2p\delta-m_2^2\gamma^2\kappa^2\right]},\ e^*_{2cd} = \frac{m_1m_2\gamma\kappa\left[1-\kappa\right]\left[4\bar{e}^2p\delta+m_2^2\gamma^2\kappa^2\right]}{8\bar{e}p\delta\left[2\bar{e}^2p\delta-m_2^2\gamma^2\kappa^2\right]}.$$

In the *dc* case, the federal government chooses $g_{2dc}$ as well as chooses and implements $e_{2dc}$ for District 2, and District 1 chooses its own $g_{1dc}$ and $e_{1dc}$. The resulting network assets ($g^*_{1dc}$, $g^*_{2dc}$) and interoperability efforts ($e^*_{1dc}$, $e^*_{2dc}$) are:

$$g^*_{1dc} = \frac{m_1\left[1-\kappa\right]}{2p},\ g^*_{2dc} = \frac{m_2\left[1-\kappa\right]\left[4\bar{e}^2p\delta+m_1^2\gamma^2\kappa^2\right]}{4p\left[2\bar{e}^2p\delta-m_1^2\gamma^2\kappa^2\right]},$$

$$e^*_{1dc} = \frac{m_1m_2\gamma\kappa\left[1-\kappa\right]\left[4\bar{e}^2p\delta+m_1^2\gamma^2\kappa^2\right]}{8\bar{e}p\delta\left[2\bar{e}^2p\delta-m_1^2\gamma^2\kappa^2\right]},\ e^*_{2dc} = \frac{m_1m_2\gamma\kappa\left[1-\kappa\right]\left[8\bar{e}^2p\delta-m_1^2\gamma^2\kappa^2\right]}{8\bar{e}p\delta\left[2\bar{e}^2p\delta-m_1^2\gamma^2\kappa^2\right]}.$$

To show that the network assets for the opt-in district chosen by the federal government increase in the network asset preference of the opt-out district, we take the partial derivatives of the network assets for the opt-in district with respect to the network asset preference of the opt-out district:

$$\frac{\partial g^*_{1cd}}{\partial m_2} = \frac{3\bar{e}^2\delta m_1 m_2 \gamma^2 \kappa^2 [1-\kappa]}{[2\bar{e}^2 p\delta - m_2^2\gamma^2\kappa^2]^2}, \ \frac{\partial g^*_{2dc}}{\partial m_1} = \frac{3\bar{e}^2\delta m_1 m_2 \gamma^2 \kappa^2 [1-\kappa]}{[2\bar{e}^2 p\delta - m_1^2\gamma^2\kappa^2]^2}.$$

Because $m_1 > 0$, $m_2 > 0$, $\delta > 0$, and $\kappa \in [0, 0.5]$, we have $\frac{\partial g^*_{1cd}}{\partial m_2} > 0$ and $\frac{\partial g^*_{2dc}}{\partial m_1} > 0$.

To show that the interoperability efforts increase in the degree of spillover $\kappa$ for both the opt-in and the opt-out districts, we take the partial derivatives of the interoperability efforts with respect to $\kappa$:

$$\frac{\partial e^*_{1cd}}{\partial \kappa} = \frac{m_1 m_2 \gamma [[16\bar{e}^4 p^2\delta^2 + m_2^4\gamma^4\kappa^4][1-2\kappa] + 2\bar{e}^2 p\delta m_2^2\gamma^2\kappa^2 [1+4\kappa]]}{8\bar{e}p\delta [2\bar{e}^2 p\delta - m_2^2\gamma^2\kappa^2]^2}$$

$$\frac{\partial e^*_{2cd}}{\partial \kappa} = \frac{m_1 m_2 \gamma [[8\bar{e}^4 p^2\delta^2 - m_2^4\gamma^4\kappa^4][1-2\kappa] + 2\bar{e}^2 p\delta m_2^2\gamma^2\kappa^2 [5-4\kappa]]}{8\bar{e}p\delta [2\bar{e}^2 p\delta - m_2^2\gamma^2\kappa^2]^2}.$$

$$\frac{\partial e^*_{1dc}}{\partial \kappa} = \frac{m_1 m_2 \gamma [[8\bar{e}^4 p^2\delta^2 - m_1^4\gamma^4\kappa^4][1-2\kappa] + 2\bar{e}^2 p\delta m_1^2\gamma^2\kappa^2 [5-4\kappa]]}{8\bar{e}p\delta [2\bar{e}^2 p\delta - m_1^2\gamma^2\kappa^2]^2}$$

$$\frac{\partial e^*_{2dc}}{\partial \kappa} = \frac{m_1 m_2 \gamma [[16\bar{e}^4 p^2\delta^2 + m_1^4\gamma^4\kappa^4][1-2\kappa] + 2\bar{e}^2 p\delta m_1^2\gamma^2\kappa^2 [1+4\kappa]]}{8\bar{e}p\delta [2\bar{e}^2 p\delta - m_1^2\gamma^2\kappa^2]^2}.$$

As $m_1 > m_2 > 0$, $p\delta > \frac{m_1^2\gamma^2\kappa^2}{2\bar{e}^2}$, and $\kappa \in [0, 0.5]$, we have $\frac{\partial e^*_{1cd}}{\partial \kappa} > 0$, $\frac{\partial e^*_{2cd}}{\partial \kappa} > 0$, $\frac{\partial e^*_{1dc}}{\partial \kappa} > 0$, and $\frac{\partial e^*_{2dc}}{\partial \kappa} > 0$.

In order to compare the interoperability efforts in the *dc* case and those in the *cd* case, we first compare the interoperability efforts for the opt-in district and then compare the interoperability efforts for the opt-out district:

$$e^*_{2dc} - e^*_{1cd} = e^*_{1dc} - e^*_{2cd} = \frac{3\bar{e} m_1 m_2 [m_1 - m_2][m_1 + m_2][1-\kappa]\gamma^3\kappa^3}{4[2\bar{e}^2 p\delta - m_1^2\gamma^2\kappa^2][2\bar{e}^2 p\delta - m_2^2\gamma^2\kappa^2]} > 0$$

as $m_1 > m_2 > 0$, $p\delta > \frac{m_1^2\gamma^2\kappa^2}{2\bar{e}^2}$, and $\kappa \in [0, 0.5]$. Therefore, the resulting interoperability level is greater in the *dc* case than the *cd* case.

Q.E.D.

**Proof of Lemma 1**

Based on the results from Proposition 1 and Proposition 3, we substitute the network assets and interoperability levels back to derive the social welfare for decentralized provision $dd$ and mixed provision $cd$ and $dc$:

$$SW_{dd} = \frac{[1-\kappa]^2 \left[2\bar{e}^2 p\delta \left[m_1^2 + m_2^2\right] + 3m_1^2 m_2^2 \kappa^2\right]}{8\bar{e}^2 p^2 \delta}$$

$$SW_{cd} = \frac{[1-\kappa]^2 \left[16\bar{e}^4 p^2 \delta^2 \left[m_1^2 + m_2^2\right] + 4\bar{e}^2 p\delta m_2^2 \left[5m_1^2 - 2m_2^2\right] \gamma^2 \kappa^2 + m_1^2 m_2^4 \gamma^4 \kappa^4\right]}{32\bar{e}^2 p^2 \delta \left[2\bar{e}^2 p\delta - m_2^2 \gamma^2 \kappa^2\right]}$$

$$SW_{dc} = \frac{[1-\kappa]^2 \left[16\bar{e}^4 p^2 \delta^2 \left[m_1^2 + m_2^2\right] + 4\bar{e}^2 p\delta m_1^2 \left[5m_2^2 - 2m_1^2\right] \gamma^2 \kappa^2 + m_1^4 m_2^2 \gamma^4 \kappa^4\right]}{32\bar{e}^2 p^2 \delta \left[2\bar{e}^2 p\delta - m_1^2 \gamma^2 \kappa^2\right]}$$

Comparing these social welfare levels yields:

$$SW_{dc} - SW_{cd} = \frac{15 m_1^2 m_2^2 \left[m_1 - m_2\right]\left[m_1 + m_2\right] \gamma^4 \kappa^4 [1-\kappa]^2}{16p \left[2\bar{e}^2 p\delta - m_1^2 \gamma^2 \kappa^2\right]\left[2\bar{e}^2 p\delta - m_2^2 \gamma^2 \kappa^2\right]} > 0$$

$$SW_{cd} - SW_{dd} = \frac{m_1^2 m_2^2 \kappa^2 [1-\kappa]^2 \left[4\bar{e}^2 p\delta \left[7\gamma^2 - 6\right] + m_2^2 \gamma^2 \kappa^2 \left[12 + \gamma^2\right]\right]}{32\bar{e}^2 p^2 \delta \left[2\bar{e}^2 p\delta - m_2^2 \gamma^2 \kappa^2\right]} > 0$$

as $m_1 > m_2 > 0$, $\gamma > 1$ and $p\delta > \frac{m_1^2 \gamma^2 \kappa^2}{2\bar{e}^2}$. Therefore, $SW_{dc} > SW_{cd} > SW_{dd}$.

Q.E.D.

**Proof of Proposition 4 – Social Optimum**

From Lemma 1, we know that mixed provision $cd$ and decentralized provision $dd$ cannot be socially optimal. Thus, centralized provision $cc$ and mixed provision $dc$ are the only possible social optimum.

Based on the results from Proposition 2, we substitute the network assets and interoperability levels back to derive the social welfare for centralized provision $cc$:

$$SW_{cc} = \frac{[1-\kappa]^2 \left[4\bar{e}^2 p\delta \left[m_1^2 + m_2^2\right] - \left[m_1^2 - m_2^2\right]^2 \alpha^2 \kappa^2\right]}{4p \left[4\bar{e}^2 p\delta - \left[m_1^2 + m_2^2\right] \alpha^2 \kappa^2\right]}$$

Comparing the social welfare levels of $cc$ and $dc$ yields:

$$SW_{cc} - SW_{dc} = \frac{m_1^2 m_2^2 \kappa^2 \left[1-\kappa\right]^2}{32\bar{e}^2 p^2 \delta \left[2\bar{e}^2 p\delta - m_1^2\gamma^2\kappa^2\right]\left[4\bar{e}^2 p\delta - \left[m_1^2 + m_2^2\right]\alpha^2\kappa^2\right]}$$

$$\left[16\bar{e}^4 p^2\delta^2 \left[4\alpha^2 - 7\gamma^2\right] - 4\bar{e}^2 p\delta \left[m_1^2\left[\alpha^2+\gamma^2\right] - 7m_2^2\alpha^2\right]\gamma^2\kappa^2 + m_1^2\left[m_1^2+m_2^2\right]\alpha^2\gamma^4\kappa^4\right].$$

The sign of $SW_{cc} - SW_{dc}$ depends on the interoperability efficiency of centralized provision parameter $\alpha$. If

$$\alpha \geq \hat{\alpha} = 2\left[\frac{\bar{e}^2 p\delta\gamma^2\left[28\bar{e}^2 p\delta + m_1^2\gamma^2\kappa^2\right]}{64\bar{e}^4 p^2\delta^2 - 4\bar{e}^2 p\delta\left[m_1^2 - 7m_2^2\right]\gamma^2\kappa^2 + m_1^2\left[m_1^2+m_2^2\right]\gamma^4\kappa^4}\right]^{\frac{1}{2}},$$

then $SW_{cc} \geq SW_{dc}$; otherwise, $SW_{cc} < SW_{dc}$. The threshold $\hat{\alpha}$ is such that when $\alpha = \hat{\alpha}$, $SW_{cc} = SW_{dc}$.

Q.E.D.

**Proof of Lemma 2**

Based on the results from Proposition 1 and Proposition 3, we substitute the network assets and interoperability levels back to derive the total surplus of each district for decentralized provision $dd$ and mixed provision $cd$ and $dc$. We first compare District 1's surplus under mixed provision $cd$ and that under decentralized provision $dd$:

$$S_{1cd} - S_{1dd} = -\frac{m_1^2 m_2^2\kappa^2\left[1-\kappa\right]^2\left[16\bar{e}^4 p^2\delta^2\left[3-2\gamma^2\right] + 4\bar{e}^2 p\delta m_2^2\gamma^2\kappa^2\left[17\gamma^2 - 12\right] + m_2^4\gamma^4\kappa^4\left[12+\gamma^2\right]\right]}{64\bar{e}^2 p^2\delta\left[2\bar{e}^2 p\delta - m_2^2\gamma^2\kappa^2\right]^2}$$

We next compare District 2's surplus under mixed provision $dc$ and that under decentralized provision $dd$:

$$S_{1cd} - S_{1dd} = -\frac{m_1^2 m_2^2\kappa^2\left[1-\kappa\right]^2\left[16\bar{e}^4 p^2\delta^2\left[3-2\gamma^2\right] + 4\bar{e}^2 p\delta m_1^2\gamma^2\kappa^2\left[17\gamma^2 - 12\right] + m_1^4\gamma^4\kappa^4\left[12+\gamma^2\right]\right]}{64\bar{e}^2 p^2\delta\left[2\bar{e}^2 p\delta - m_1^2\gamma^2\kappa^2\right]^2}$$

From Assumption 2(b) where $\gamma^2 < 1.5$, mixed provision $cd$ is dominated by decentralized provision $dd$ for District 1 and mixed provision $dc$ is dominated by decentralized provision $dd$ for District 2. Therefore, neither mixed provision can be an equilibrium.

Q.E.D.

**Proof of Lemma 3**

Based on the results from Proposition 2, we substitute the network assets and interoperability levels back into the objective functions to derive the total surplus of each district for centralized provision $cc$ subject to the cost sharing percentage $\phi$ for District 1 and $1-\phi$ for District 2:

$$S_{1cc} = \frac{m_1^2\left[1-\kappa\right]^2\left[16\bar{e}^4p^2\delta^2+\left[m_1^4+6m_1^2m_2^2-7m_2^4\right]\alpha^4\kappa^4-8\bar{e}^2p\delta\left[m_1^2+m_2^2\left[2\phi-1\right]\right]\alpha^2\kappa^2\right]}{4p\left[4\bar{e}^2p\delta-\left[m_1^2+m_2^2\right]\alpha^2\kappa^2\right]^2}$$

$$S_{2cc} = \frac{m_2^2\left[1-\kappa\right]^2\left[16\bar{e}^4p^2\delta^2+\left[m_2^4+6m_1^2m_2^2-7m_1^4\right]\alpha^4\kappa^4-8\bar{e}^2p\delta\left[m_2^2+m_1^2\left[1-2\phi\right]\right]\alpha^2\kappa^2\right]}{4p\left[4\bar{e}^2p\delta-\left[m_1^2+m_2^2\right]\alpha^2\kappa^2\right]^2}$$

Thus, $\frac{\partial S_{1cc}}{\partial\phi} > 0$ and $\frac{\partial S_{2cc}}{\partial\phi} < 0$. Furthermore, we know that the cost sharing percentage $\phi$ does not affect the districts surplus under mixed provision. Thus, $S_{1dc}$ and $S_{2cd}$ do not depend on $\phi$.

The threshold $\phi_1$ is such that when $\phi = \phi_1$, $S_{1cc} = S_{1dc}$. The threshold $\phi_2$ is such that when $\phi = \phi_2$, $S_{2cc} = S_{2cd}$. Comparing $S_{1cc}$ and $S_{1dc}$ yields that if $\phi \leq \phi_1$, then $S_{1cc} \geq S_{1dc}$; otherwise, $S_{1cc} < S_{1dc}$. Similarly, comparing $S_{2cc}$ and $S_{2cd}$ yields that if $\phi \geq \phi_2$, then $S_{2cc} \geq S_{2cd}$; otherwise, $S_{2cc} < S_{2cd}$.

Q.E.D.

**Proof of Lemma 4**

To show that there exists a threshold $\alpha^\phi$ such that if $\alpha \geq \alpha^\phi$, $\phi_1 \geq \phi_2$, we take the partial derivative of $\phi_1 - \phi_2$ with respect to $\alpha$, where $\phi_1$ and $\phi_2$ are defined in Lemma 3:

$$\frac{\partial\left[\phi_1-\phi_2\right]}{\partial\alpha} = \frac{80\bar{e}^8p^4\delta^4\gamma^2-8\bar{e}^6p^3\delta^3\left[m_1^2+m_2^2\right]\left[\alpha^4+4\gamma^4\right]\kappa^2}{\alpha^3\left[2\bar{e}^2p\delta-m_1^2\gamma^2\kappa^2\right]^2\left[2\bar{e}^2p\delta-m_2^2\gamma^2\kappa^2\right]^2}$$

$$+\frac{\bar{e}^4p^2\delta^2\gamma^2\left[\left[19m_1^4-32m_1^2m_2^2+19m_2^4\right]\gamma^4+6\left[m_1^2+m_2^2\right]^2\alpha^4\right]\kappa^4}{2\alpha^3\left[2\bar{e}^2p\delta-m_1^2\gamma^2\kappa^2\right]^2\left[2\bar{e}^2p\delta-m_2^2\gamma^2\kappa^2\right]^2}-\frac{\bar{e}^2p\delta m_1^2m_2^2\left[m_1^2+m_2^2\right]\left[8\alpha^4-5\gamma^4\right]\gamma^4\kappa^6}{2\alpha^3\left[2\bar{e}^2p\delta-m_1^2\gamma^2\kappa^2\right]^2\left[2\bar{e}^2p\delta-m_2^2\gamma^2\kappa^2\right]^2}$$

$$-\frac{\left[\left[m_1^2+m_2^2\right]^2\left[19m_1^4-96m_1^2m_2^2+19m_2^4\right]\alpha^4+8m_1^4m_2^4\gamma^4\right]\gamma^6\kappa^8}{32\alpha^3\left[2\bar{e}^2p\delta-m_1^2\gamma^2\kappa^2\right]^2\left[2\bar{e}^2p\delta-m_2^2\gamma^2\kappa^2\right]^2}$$

$$-\frac{m_1^2m_2^2\left[m_1^2+m_2^2\right]\left[5m_1^2+m_2^2\right]\left[m_1^2+5m_2^2\right]\alpha\gamma^8\kappa^{10}}{32\bar{e}^2p\delta\left[2\bar{e}^2p\delta-m_1^2\gamma^2\kappa^2\right]^2\left[2\bar{e}^2p\delta-m_2^2\gamma^2\kappa^2\right]^2}+\frac{m_1^4m_2^4\left[m_1^2+m_2^2\right]^2\alpha\gamma^{10}\kappa^{12}}{64\bar{e}^4p^2\delta^2\left[2\bar{e}^2p\delta-m_1^2\gamma^2\kappa^2\right]^2\left[2\bar{e}^2p\delta-m_2^2\gamma^2\kappa^2\right]^2}>0$$

as $p\delta > \frac{m_1^2\gamma^2\kappa^2}{2\bar{e}^2} > \frac{m_2^2\gamma^2\kappa^2}{2\bar{e}^2}$ and $\gamma^2 < 1.5$.

Define the threshold $\alpha^\phi$ as when $\alpha = \alpha^\phi$, $\phi_1 = \phi_2$. Therefore, there exists a threshold $\alpha^\phi$ such that if $\alpha \geq \alpha^\phi$, then $\phi_1 \geq \phi_2$; otherwise $\phi_1 < \phi_2$. The threshold $\alpha^\phi$ is such that when $\alpha = \alpha^\phi$, $\phi_1 = \phi_2$.

Q.E.D.

**Proof of Proposition 5 – Utilizing Cost Sharing Rule to Induce Social Optimum**

From Proposition 4, we know that if $\alpha \geq \hat{\alpha}$, then the social optimum is centralized provision *cc*; otherwise, the social optimum is mixed provision *dc*. From Lemma 3 and Lemma 4, we know that if $\alpha \geq \alpha^\phi$, then $\phi_1 \geq \phi_2$ and the federal government can choose a feasible $\phi$ such that $\phi_2 \leq \phi \leq \phi_1$ to induce centralized provision; otherwise, there is no feasible $\phi$ to induce centralized provision.

Comparing the thresholds $\alpha^\phi$ and $\hat{\alpha}$, we find that the sign of $\alpha^\phi - \hat{\alpha}$ depends on

$$32\bar{e}^8p^4\delta^4-8\bar{e}^6p^3\delta^3\left[3m_1^2-4m_2^2\right]\gamma^2\kappa^2+6\bar{e}^4p^2\delta^2\left[3m_1^4-12m_1^2m_2^2-m_2^4\right]\gamma^4\kappa^4$$
$$-\bar{e}^2p\delta m_1^2\left[7m_1^4-30m_1^2m_2^2-15m_2^4\right]\gamma^6\kappa^6-m_1^4m_2^2\left[m_1^2+6m_2^2\right]\gamma^8\kappa^8.$$

This expression is positive since $p\delta > \frac{m_1^2\gamma^2\kappa^2}{2\bar{e}^2} > \frac{m_2^2\gamma^2\kappa^2}{2\bar{e}^2}$ and $\gamma^2 < 1.5$. Thus, we get that $\alpha^\phi > \hat{\alpha}$.

In addition, cost sharing pertains only to centralized provision and not to mixed provision. Therefore, we derive the three cases given in Proposition 5, i.e., $\alpha \geq \alpha^\phi$, $\hat{\alpha} \leq \alpha < \alpha^\phi$, and

$\alpha < \hat{\alpha}$.

Q.E.D.

**Proof of Lemma 5**

The federal goverment may offer grants to the opt-in districts in order to induce the socially optimum organization form. From Lemma 3, we know that in the absence of grants District 1 does not have the incentive to deviate from centralized provision if $\phi \leq \phi_1$ and District 2 does not have the incentive to deviate from centralized provision if $\phi \geq \phi_2$. In the presence of grants, the thresholds $\phi_1$ and $\phi_2$ become functions of the grants, i.e., $\phi_1(F_{1cc})$ and $\phi_2(F_{2cc})$. Specifically,

$$\phi_1(F_{1cc}) = \phi_1(0) + \frac{\left[4\bar{e}^2 p\delta - \left[m_1^2 + m_2^2\right]\alpha^2\kappa^2\right]^2 F_{1cc}}{4\bar{e}^2\delta m_1^2 m_2^2 \left[1-\kappa\right]^2 \alpha^2\kappa^2}$$

$$\phi_2(F_{2cc}) = \phi_2(0) - \frac{\left[4\bar{e}^2 p\delta - \left[m_1^2 + m_2^2\right]\alpha^2\kappa^2\right]^2 F_{2cc}}{4\bar{e}^2\delta m_1^2 m_2^2 \left[1-\kappa\right]^2 \alpha^2\kappa^2}$$

Note that in the above expressions, $\phi_1(0)$ and $\phi_2(0)$ are equivalent to the thresholds derived in Lemma 3. Solving $\phi_1(F_{1cc}) = \phi_2(F_{2cc})$ yields $\alpha^{\phi}(F_{cc})$, where $F_{cc} = F_{1cc} + F_{2cc}$. Furthermore, $\alpha^{\phi}(F_{cc})$ decreases in the total grant given to the opt-in districts. Thus, there exists $F_{cc}^*$ such that $\alpha = \alpha^{\phi}(F_{cc}^*)$. Therefore, if $F_{cc} \geq F_{cc}^*$, then $\alpha \geq \alpha^{\phi}(F_{cc}^*)$.

Q.E.D.

**Proof of Proposition 6 – Utilizing Cost Sharing and Grants to Induce Social Optimum**

Based on the results from Proposition 5 and Lemma 5, we know that if $\alpha \geq \alpha^{\phi}(0)$, then the federal government prefers centralized provision and can choose a feasible $\phi$ to induce centralized provision. Therefore, no grant is needed, i.e., $F_{cc}^* = 0$. If $\hat{\alpha} \leq \alpha < \alpha^{\phi}(0)$, then the federal government prefers centralized provision and has to provide grant $F_{cc}^*$ to the opt-in districts as stated in Lemma 5 to induce centralized provision.

If $\alpha < \hat{\alpha}$, then the federal government prefers the mixed provision $dc$. In order to induce the mixed provision $dc$, two conditions need to be satisfied: $S_{1dc} \geq S_{1cc}$ and $S_{2dc} \geq S_{2dd}$. From Lemma 3, we know that if $\phi \geq \phi_1$, then $S_{1dc} \geq S_{1cc}$. As for District 2, we know that without grant, $S_{2dc}(F_{2dc} = 0) < S_{2dd}$. Therefore, the federal government has to provide grant $F^*_{2dc} = S_{2dd} - S_{2dc}(F_{2dc} = 0)$ to ensure that District 2 opts in.

Q.E.D.