

A Dynamic Pricing Strategy for a 3PL Provider with Heterogeneous Customers

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Abstract

We study the pricing problem for a third-party-logistics (3PL) provider that provides warehousing and transportation services. When customers arrive at the 3PL provider, they specify the delivery dates for their freight, and before the specified delivery dates, their freight is stocked in the 3PL provider's warehouse. We propose a dynamic pricing strategy (DPS) and develop a stochastic-nonlinear-programming (SNLP) model which computes the optimal freight rates for different delivery dates incorporating the 3PL provider's current holding cost and available transportation capacity for each route. As customers are heterogeneous in their valuations and price sensitivities for delivery dates, and the distributions of the customers' delivery date preferences are unknown to the 3PL provider, we modify the standard multinomial logit (MNL) function to predict customer choices. Through a simulation experiment, we show that the proposed MNL function can be a good replacement for the mixed MNL function when the mixed MNL function is not applicable. Through simulation we also compare the proposed DPS with a static pricing strategy. We show that with our DPS both the 3PL provider and its customers are better off, and the 3PL provider has different investment incentives for increasing transportation capacity. Our results can be also applied in similar settings that feature holding costs, limited production capacity and delivery-date-sensitive customers.

Keywords: dynamic pricing, multinomial logit, third-party logistics, stochastic programming

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1. INTRODUCTION

In recent years, the third-party-logistics (3PL) industry has grown swiftly with E-commerce. According to *US census bureau*, E-commerce based manufacturing shipments have grown from US\$996 billion to US\$2,283 billion during the seven years from 2004 to 2010. The growth of E-commerce increases the intensity of competition in many industries, and increasingly companies are encouraged to embrace the one-stop logistics services provided by 3PLs so that they can increase their supply chains' efficiency and concentrate on their core competencies (Vaidyanathan 2005). For example, many online sellers prefer to stock their products at a 3PL distribution center so that they can reduce processing time for shipment after customers place orders. 3PLs have evolved to provide a full set of integrated logistics activities such as transportation, warehousing, freight consolidation and distribution, rate negotiation and logistics information systems (Rabinovich et al. 1999, Sink et al. 1997). Moreover, E-commerce has induced 3PLs to increase their investment in central warehouses. According to the North American Industry Classification System (NAICS), 3PLs with warehousing and storage services belong to the *Warehousing and Storage* industry (NAICS Code: 493). The data from the *Bureau of Labor Statistics* (BLS) in US shows that, from January 2001 to October 2012, the number of employees in Warehousing and Storage increased by 31% while in its NAICS super sector, Transportation and Warehousing, the increase was only 4.3%. The data from the *Bureau of Economic Analysis* (BEA) in the US also shows that, from 2001 to 2011, the GDP added from Warehousing and Storage industry increased by 80.1% while the Transportation and Warehousing sector GDP only increased by 48.0%. In 2010, about 85% of 3PLs reported providing warehousing services (Capgemini 2010). These changes in the 3PL industry have increased the efficiency of traditional supply chains, and consequently drawn the attention of the academic community to the problems faced by 3PLs that provide warehousing and transportation services.

We study the pricing problem for 3PLs that provide these comprehensive services including warehousing and less-than-truckload (LTL) transportation. This work is motivated by a local 3PL whose customers include local manufacturers, wholesalers and retailers. Customers negotiate with the 3PL on the price and delivery date for shipments, and then stock

their freight at the 3PL's warehouse until the freight is delivered. The 3PL transports the customers' shipments by trucks. The 3PL's transportation network includes multiple lanes each of which connects to a city in a neighboring state/ province. For each lane, there are a certain number of trucks running between the destination city and the home city. The 3PL operates its transportation lanes with its business partners, usually other 3PLs located in the destination cities. When the 3PL operates a lane with a partner, it shares the trucks running on that lane and the warehouses at both ends of the lane. The trucks usually belong to independent transporters, and for each lane, the 3PL signs long-term contracts with these transporters. If the 3PL temporarily increases the number of trucks for a lane, the temporarily added trucks will charge higher transportation costs than the long-term contracted trucks. Thus, for any lane, the daily transportation capacity, which is determined by the number of long-term contracted trucks, is fixed, and if the freight scheduled to be transported in a day is more than the 3PL's daily transportation capacity, the 3PL incurs a penalty as a consequence of hiring temporary capacity. With the daily arrival of hundreds of shipping orders from different customers with different quantities and delivery date preferences, it is difficult for the 3PL to satisfy all the customers' first-best delivery date requirements with the limited transportation capacity.

We develop a dynamic pricing strategy (DPS) that links the price for each shipping order with the 3PL's capacity usage and inventory holding cost. In our DPS, different delivery dates are priced differently. Similar practices can often be found for firms offering standard orders and rush orders with different prices. However, this simple price differentiation in practice usually cannot promise an exact delivery date. Rather, at best each type of order is promised an expected leadtime or a maximum leadtime that is computed based on theory of priority queues. Because no exact delivery date is promised, this pricing strategy can only be applied in a business environment where customers have delivery date flexibility rather than strict requirements for just-in-time (JIT) delivery. For the 3PLs, the pricing problem is more complex when considering the 3PL's transportation capacity, inventory cost and the customers' delivery date requirements that are increasingly strict and unpredictable. Thus, a well-designed DPS has the potential to increase the profit of the 3PL and its customers, which consequently increases the profit of the entire supply chain.

We compute the optimal price for each delivery-date option through stochastic nonlinear programming (SNLP). In our SNLP model, we use a multinomial logit (MNL) function-based method to estimate customers' choices of delivery dates, and the maximum-likelihood estimation (MLE) is employed for regressing the MNL function. The MNL function has been extensively used in marketing science to predict customers' choices from multiple substitutable alternatives. The MNL function and MLE-based method is applicable to our problem for two reasons. First, because customers can obtain different value by choosing to deliver their freight on different delivery dates, we can treat the multiple options of delivery dates as alternatives differentiated in 'quality'. Second, because 3PLs usually receive hundreds of orders from local customers each day, large enough samples can be collected for regression.

In a MNL function, the term "taste coefficients" is often used to describe the weights a consumer puts on different dimensions of an alternative, and a consumer's taste coefficients determine its utility from choosing different alternatives (Train 2003). We use taste coefficients to describe the weights that determine the choices of the 3PL's customers although the customers are usually firms. One difficulty in using a standard MNL function is the assumption that customers are homogeneous in their taste coefficients. In our problem customers are heterogeneous in their taste coefficients. That is, individual customers may put different valuations on delivery dates and prices when they choose from delivery date options based on their individual requirements. A mixed logit function is often employed to solve a discrete choice problem when customers' taste coefficients are heterogeneous. In a mixed logit function, the probability of choosing an alternative is computed by integrating the standard logit probability over the density of the taste coefficients. Thus, the employment of a mixed logit function requires the distributions of all taste coefficients to be known. However, because in our problem, the factors that affect customer choices are outside the information available to the 3PLs, the distributions of customers' taste coefficients are unknown. Hence, we develop a new variant of a MNL function for our problem where customers have heterogeneous taste coefficients with unknown distributions.

Our pricing model can also be employed in other manufacturing or service-providing

settings which have similar features as the 3PL studied in this work. For example, a make-to-order company produces goods or parts for its customers that are heterogeneous in their delivery date preferences, and developing a DPS incorporating its inventory and production status can increase its profit and optimize the allocation of its resources to inventory and production capacities.

1.1. Background Literature

DPS and its impact on a supply chain have attracted increasing interest in recent literature. For example, Jia and Zhang (2013) and Zhu (2015) studied the cooperation of a manufacturer and a retailer both employing DPS. We study the DPS that enables a 3PL to coordinate its transportation schedule with its customers' production schedule, which is a marketing-based solution for *supply chain scheduling* as first defined by Hall and Potts (2003). Chen (2010) provides a review of research studying supply chain scheduling between manufacturers and distributors. However, little research in supply chain scheduling studies cooperation between manufacturers and 3PLs that provide comprehensive warehousing and LTL transportation services. Moreover, in review articles by Maloni and Carter (2006), Selviaridis and Spring (2007) and Marasco (2008), there is no work that covers 3PL pricing. Lukassen and Wallenburg (2010) reviewed research related to pricing strategies for 3PL services but only in the context of the pricing of logistics services where the 3PL and its client are involved in a long-term contract. Carter et al. (1995) studied how the pricing in LTL transportation industry affects the manufacturers' lot-sizing decision but not scheduling. Ülkü and Bookbinder (2012a) studied dynamic pricing for a manufacturer which provides delivery with guaranteed delivery dates and Ülkü and Bookbinder (2012b) extends the pricing method for a 3PL which provides both transportation and warehousing services. In their DPS, the price for each order depends on the order's time of arrival, but customers do not have multiple delivery date options to choose from. Similar DPS is can be also found in Feng et al. (2011) which studies a GI/M/1 system.

By adopting our DPS to maximize profit while incorporating its holding cost and transportation capacity, the 3PL induces some flexible customers to choose other delivery dates if there is not enough capacity on their first-best delivery dates. Thus, the DPS is actually

a priority dispatching strategy when the 3PL's transportation capacity is limited. Priority pricing relates to the literature on pricing/queuing models defined by Hall et al. (2009), literature that originates from the research on priority pricing in a queuing system. To the best of our knowledge, Kleinrock (1967) was the first to study priority queues in which the priorities are associated with prices paid by customers. However, he focuses on the customers' behavior rather than on the priority pricing scheme. Well-cited research on priority pricing in priority queues can be found in Dolan (1978), Mendelson and Whang (1990) and Rao and Petersen (1998). In these articles, the authors propose a priority pricing mechanism based on self-revelation theory so that each arriving customer is dispatched with the proper priority, and the target of the proposed priority pricing mechanism is to maximize either social welfare (Dolan 1978, Mendelson and Whang 1990) or the firm's profit (Rao and Petersen 1998). In the research on priority pricing in priority queues, the firm does not guarantee the delivery dates, and customers make decisions only based on the expected leadtime. Because there is no delivery date guarantee, the firm does not need to control the number of customer choices for each delivery date. Thus, the pricing methods proposed by the work on priority pricing are only applicable in the case where there is no need to induce customer choices or to control the arrival rate. Hence, for customers that specify their delivery date requirements and their freight has to be delivered on specified delivery dates 3PLs cannot apply the priority pricing mechanism. Moreover, in priority-queue based studies, a shorter leadtime is assumed to be more attractive to all customers, which is not true in a business-to-business (B2B) context.

With increased demand for guaranteed delivery dates in B2B E-commerce, the literature has started to focus on problems with leadtime guarantees, where the leadtime for processing an order cannot exceed the maximum leadtime specified by the customer. So and Song (1998) and Ata and Olsen (2009) studied leadtime pricing with leadtime guarantees, but did not consider the heterogeneity of customer orders. Liu et al. (2007) and Shang and Liu (2011) assume customers sensitivity in price and leadtime, but they also do not consider customers heterogeneity and the leadtime and price quotes are static regardless of the system congestion. Plambeck (2004), Zhang et al. (2012) and Ata and Olsen (2013) studied the pricing for different leadtime options when the firm faces customers that are heterogeneous both in price sensitivity and leadtime sensitivity. Due to problem complexity, they only

studied the case where the firm faces two classes of customers and offers two different leadtime options. A number of recent articles study leadtime pricing with guaranteed leadtime for an arbitrary number of customer classes. Çelik and Maglaras (2008) studied leadtime pricing for multiple customer classes where they guarantee the promised leadtime using expediting at an extra cost. Akan et al. (2012) studied leadtime pricing for a single-server queue to maximize social welfare. Similar to leadtime pricing studies based on priority queues, this work assumes that a shorter leadtime is more attractive to all customers. Another limitation of the literature on leadtime pricing with leadtime guarantees is that simplified queuing models cannot be employed in a more complex settings such as when there is an inventory cost.

From the articles cited above the superiority of dynamic pricing has been proven extensively using analytical methods. However, there are two common weaknesses in the analyses using analytical methods. First, in general the model formulations are too simple to be used in practical settings. Second, in these formulations the firm (3PL in our case) requires certain private information of customers and this private information is usually hard or unrealistic to obtain. Hence, we focus on designing a dynamic pricing method that overcomes these two weaknesses. The techniques we employ are supported by the fact that a number of articles use logit functions in pricing problems such as price optimization for multiple substitutable products or services. Aydin and Ryan (2000), Hopp and Xu (2005), and Maddah and Bish (2007) studied the assortment and pricing problem where the customer choices of substitutable products are modeled by logit functions. Aydin and Porteus (2008) and Dong et al. (2009) studied pricing and inventory control where the customer choices of products are modeled by logit functions. Huang (2002) studied the pricing of a simple two-mode transportation system where the customer choices of transportation modes are modeled by a MNL function. Ferrer et al. (2010) studied the pricing of product or service bundles where the customer choices of bundles is modeled by a MNL function. Li and Huh (2011) studied a pricing problem by modeling the profit as function of prices and the customer choices of products are modeled by a nested logit function. Rodríguez and Aydın (2011) studied the pricing of configurable products where the customers' preference on each configuration is modeled by a logit function. As far as we know, the literature studying pricing problems

with logit models does not consider customer heterogeneity in tastes, and thus only standard multinomial logit functions or nested logit functions are employed. A review of research using mixed logit function to model customer choices when the customers are heterogeneous in taste can be found from the survey conducted by Hensher and Greene (2003). The mixed logit research that focuses on price optimization is limited because it is usually hard to identify the distribution of customers' taste coefficients. Bastin et al. (2010) developed a B-spline-based approach to model the non-parametric distribution of taste coefficients so that the mixed logit function can be formed, but the B-spline approximation requires a large number of samples for regression and the obtained mixed logit model is too complex to be employed in a stochastic program for price optimization.

The remainder of our paper is organized as follows. In Section 2 we describe the problem and provide notation for our pricing model. In Section 3, we study the pricing model and the solution method. In Section 4, we design a simulation experiment based on a LTL and warehousing service provider to test the performance of the proposed pricing method. In Section 5, we compare the DPS with a static pricing strategy (SPS) in terms of the 3PL's profit, customer and social welfare, and the shadow prices of transportation capacity. Section 6 contains our concluding remarks.

2. PROBLEM DESCRIPTION

We suppose that the 3PL's business is managed in days. In each day, the 3PL receives shipping orders from N different arriving customers, each of which has a freight to ship. The 3PL decides prices for different delivery date options at the beginning of each day and the prices are not changed during the day. Thus, in the pricing phase, the 3PL only optimizes the price for each delivery date option based on the distribution of relevant order features obtained from historical sales records. Freight quantity can be scaled by weights, sizes, etc., which we take as an independent and identically distributed (i.i.d.) random variable denoted by $q \in \mathbb{R}^+$. In practice, it is common that a 3PL can categorize its customers into different classes based on similar characteristics such as freight quantity, patience, price sensitivity and so on. Because the pricing for the case with multi-class customers is only an extension

of pricing for single-class customers, we focus the single-class case herein, and an extension modeling pricing for multi-class customers is provided in Appendix B.

The 3PL offers T delivery date options. If a customer chooses delivery date $t : t \in \{1, \dots, T\}$, then its freight should be delivered at the t th day after it arrives at the 3PL. The 3PL quotes different freight rates for different delivery date options, and the freight rate for a delivery date is defined as the price for delivering a unit quantity of freight at that delivery date including the storage and transportation. The charge for a customer order can be obtained by multiplying the freight rate by the quantity of freight. We define a freight quote as a list of freight rates for all the delivery date options, and a freight quote is denoted by $\vec{p} : \vec{p} \in \mathbb{R}^T$ where the t th element, p_t , is the freight rate for delivery date t . Because in our DPS the freight quote is not changed during a day, the customer is able to compare prices between different 3PLs.

We assume that the 3PL incurs a holding cost, denoted by $h : h \in \mathbb{R}^+$, for each freight unit stocked in its warehouse for each day. The 3PL can dynamically adjust the setting of h based on its inventory status. The 3PL manages its business for each lane independently. It stocks freight for the same lane in the same area, and freight quotes for different lanes are different. Thus, we can simplify the problem to a single-lane problem.

In the single lane problem, we define the 3PL's daily transportation capacity, denoted by $c : c \in \mathbb{R}^+$, as the maximum quantity of freight that can be transported each day. The 3PL's daily transportation capacity is fixed because the number of trucks going to the destination city every day is fixed. If freight scheduled to be transported in a day is more than the 3PL's daily transportation capacity, the 3PL incurs a penalty of $\omega : \omega \in \mathbb{R}^+$ for each freight unit over the daily transportation capacity. The penalty accounts for the 3PL's loss from hiring temporary transportation capacity (i.e., trucks) or violating promised delivery dates. As there might be freight received in earlier days already scheduled to be shipped or preserved capacity for future arriving orders on a certain date, the available transportation capacity for each delivery date option may vary. We assume that at the beginning of each day, the 3PL counts the available transportation capacity for each delivery date option. We denote the available transportation capacities for delivery date t by c_t , and $\vec{c} : \vec{c} \in \mathbb{R}^T$ represents

the c_t 's for all $t \in \{1, \dots, T\}$.

Following the trend in the literature, we use the term ‘taste coefficients’ to refer to the weights a customer gives to the factors which determine its utility from choosing an alternative. That is, the taste coefficients determine a customer’s preference for each alternative. We assume that the 3PL’s customers have two taste coefficients for each delivery date option: valuation and price sensitivity. We define a customer’s valuation of a delivery date option t , denoted by v_t , as its valuation for transporting a unit of its freight on delivery date t , and its valuation, denoted by $\vec{v} : \vec{v} \in \mathbb{R}^T$, represents the v_t ’s for all $t \in \{1, \dots, T\}$. Similarly, we define a customer’s price sensitivity for delivery date t , denoted by α_t , as its marginal disutility caused by a unit increase of freight rate for delivery date t , and its price sensitivity, denoted by $\vec{\alpha} : \vec{\alpha} \in \mathbb{R}^T$, represents the α_t ’s for all $t \in \{1, \dots, T\}$. We assume that the values of $\vec{v}$ and $\vec{\alpha}$ for each customer are determined by details outside the information available to the 3PL. Hence, we assume that for each customer, each element in $\vec{v}$ and $\vec{\alpha}$ is an i.i.d. random variable with an unknown distribution. We set a customer’s $\vec{v}$ and $\vec{\alpha}$ to be independent from its freight quantity because valuations and price sensitivity can be higher for large or small shipments.

We define a customer’s utility, or profit, from choosing leadtime t , denoted by $\xi(t)$, as

$$\xi(t) = v_t - \alpha_t p_t + \epsilon,$$

where $\epsilon : \epsilon \in \mathbb{R}$ is a random component of a customer’s profit, possibly as a result of unobserved variables or decision makers’ errors and biases (Su 2008). According to Guadagni and Little (1983), ϵ is distributed with a standard double exponential (Gumbel extreme value) distribution. A more general form of a customer’s profit would include a further option-associated parameter and a scale parameter. However, any option-associated parameter, even one dependent on t , can be absorbed into v_t without loss of generality, and the scaling of profit is arbitrary because the scale is eliminated when computing the probability of choosing a delivery date using a MNL function. A customer chooses the delivery date option which maximizes its profit, or rejects without purchase if $\xi(t) < 0$ for all $t \in \{1, \dots, T\}$. If a customer rejects, then its profit is 0. Because a customer’s profit from choosing delivery date t is random and depends on the freight quote $\vec{p}$, we denote the probability of a customer

choosing delivery date t by $\mathbb{P}_t(\vec{p})$.

For clarity, we summarize the notation list in Appendix

3. MODELING AND SOLUTION METHODS

In this section, we first present our adjusted MNL model. Then we develop a leadtime-pricing model based on an actual LTL shipping case.

3.1. The Adjusted MNL Model

We use a logit function to model the probability of a customer choosing each delivery date. Because the standard MNL model requires customers to be homogeneous in their taste coefficients, it is not applicable in our problem where customers' taste coefficients, $\vec{v}$ and $\vec{\alpha}$, are heterogeneous. In the family of logit functions, the mixed logit function is often employed in the heterogeneous case. However, for our problem, the mixed logit function is not applicable because the customers' taste coefficients follow unknown distributions. In addition, when employing a mixed logit function, the probability of choosing a delivery date option should be approximated through simulation by repetitively drawing the taste coefficients from their distributions (Hensher and Greene 2003). Because we need to use a heuristic method to solve our SNLP and the distribution of customer choices has to be computed under each freight quote obtained from the iteration, the procedure of approximating a mixed logit function makes it impossible to solve the optimal freight quote within a reasonable computing time. Based on this analysis, we develop an *adjusted MNL model* for our problem where customers are heterogeneous in their taste coefficients.

We first use a standard MNL function to model customer choices by treating heterogeneous customers as homogeneous ones whose valuation and price sensitivity are deterministic, denoted by $\vec{v}^S$ and $\vec{\alpha}^S$, respectively. Then, based on $\vec{v}^S$ and $\vec{\alpha}^S$, we use the standard MNL to obtain an approximate probability of a customer choosing delivery date t under freight quote $\vec{p}$, denoted by $\mathbb{P}_t^S(\vec{p})$, as

$$\mathbb{P}_t^S(\vec{p}) = \frac{e^{v_t^S - \alpha_t^S p_t}}{1 + \sum_{i=1}^T e^{v_i^S - \alpha_i^S p_i}} \quad t = 1, 2, \dots, T, \quad (1)$$

where $\vec{v}^S$ and $\vec{\alpha}^S$ can be estimated using MLE. When using MLE, we suppose that $\mathbb{P}_t^S(\vec{p})$ is equal to the true probability of a customer choosing delivery date t under freight quote $\vec{p}$. Then if v_t^S and α_t^S are estimated based on the customer choices within K recent days, where K is a positive integer, the likelihood, denoted by $\mathcal{L}$, is constructed as

$$\mathcal{L} = \prod_{k=1}^K \left[\left[1 - \sum_{t=1}^T \mathbb{P}_t^S(\vec{p}_k) \right]^{N_{0k}} \prod_{t=1}^T \mathbb{P}_t^S(\vec{p}_k)^{N_{tk}} \right], \quad (2)$$

where $\vec{p}_k$ is the freight quote in the k th day in K days, N_{tk} is the number of customers arriving in the k th day with delivery date choice t , and N_{0k} is the number of customers arriving in the k th day rejecting all delivery date options. To simplify the form of likelihood, we take natural logs on both sides of (2). After simple algebra, we can obtain

$$\ln \mathcal{L} = \sum_{k=1}^K \left[N_{0k} \ln \left[1 - \sum_{t=1}^T \mathbb{P}_t^S(\vec{p}_k) \right] + \sum_{t=1}^T N_{tk} \ln \mathbb{P}_t^S(\vec{p}_k) \right]. \quad (3)$$

In (3), substituting $\mathbb{P}_t^S(\vec{p}_k)$ with the forms of v_t^S and α_t^S as defined in (1), we can have $\ln \mathcal{L}$ as a function of v_t^S and α_t^S for all $t \in \{1, \dots, T\}$. Then we find the optimal setting of v_t^S and α_t^S which maximizes $\ln \mathcal{L}$. Because of the complexity of solving the first-order condition of (3), the optimal setting of v_t^S and α_t^S for each $t \in \{1, \dots, T\}$ is obtained using numerical methods.

Because customers are actually heterogeneous in their taste coefficients, we can expect that $\mathbb{P}_t^S(\vec{p})$ computed in (1) may not equal to the true probability of a customer choosing delivery date t given freight quote $\vec{p}$ and the estimations of v_t^S and α_t^S for all $t \in \{1, \dots, T\}$. Thus, after running the MLE with likelihood in (3), we use the adjusted MNL function to compute the probability of a customer choosing delivery date t in our SNLP. Let $\mathbb{P}_t^A(\vec{p}_k)$ be the probability obtained using the adjusted MNL function, and we set $\mathbb{P}_t^A(\vec{p}_k)$ to be

$$\mathbb{P}_t^A(\vec{p}) = \frac{e^{v_t^S - \alpha_t^S p_t + r_t(p_t)}}{1 + \sum_{i=1}^T e^{v_i^S - \alpha_i^S p_i + r_i(p_i)}} \quad t = 1, 2, \dots, T. \quad (4)$$

In (4), we add an adjusting term, $r_t(p_t)$, to the customers' valuation for each delivery date option to mitigate the errors of prediction caused by customers' heterogeneity. Note that in our adjusted MNL function, we set the adjusting term for delivery date option t to be a function of p_t by ignoring the impacts from the freight rates for other delivery date options.

From the simulation experiments in Section 4, we can see that this treatment does not affect the accuracy of our adjusted MNL function in predicting customer choices.

The probability of a customer rejecting all delivery date options can be obtained as

$$1 - \sum_{i=1}^T \mathbb{P}_i^A(\vec{p}) = \frac{1}{1 + \sum_{i=1}^T e^{v_i^S - \alpha_i^S p_i + r_i(p_i)}}. \quad (5)$$

From (4) and (5), we have

$$e^{v_t^S - \alpha_t^S p_t + r_t(p_t)} = \frac{\mathbb{P}_t^A(\vec{p})}{1 - \sum_{i=1}^T \mathbb{P}_i^A(\vec{p})}.$$

Then we can obtain $r_t(p_t)$ as

$$r_t(p_t) = \ln \left[\frac{\mathbb{P}_t^A(\vec{p})}{1 - \sum_{i=1}^T \mathbb{P}_i^A(\vec{p})} \right] - v_t^S + \alpha_t^S p_t. \quad (6)$$

In (6), by supposing that $\mathbb{P}_t^A(\vec{p})$ computes the true probability of a customer choosing delivery date t , we substitute $\mathbb{P}_t^A(\vec{p})$ and $1 - \sum_{i=1}^T \mathbb{P}_i^A(\vec{p})$ with N_{tk} and N_{0k} , respectively, for all $k \in \{1, \dots, K\}$. Then, we can obtain the realized values of $r_t(p_t)$ for all days in the sales record as

$$r_{tk} = \ln \left[\frac{N_{tk}}{N_{0k}} \right] - v_t^S + \alpha_t^S p_{tk} \quad k = 1, 2, \dots, K,$$

where p_{tk} is the freight rate for delivery date option t in the k th day, and r_{tk} is computed based on N_{tk} and p_{tk} , for the k th day. With p_{tk} and r_{tk} for all $k \in \{1, \dots, K\}$ known, we can regress $r_t(p_t)$ when the form of $r_t(p_t)$ is given with a series of to-be-determined coefficients.

In order to regress $r_t(p_t)$ for all $t \in \{1, 2, \dots, T\}$, we first need to select the form of $r_t(p_t)$. $r_t(p_t)$ can be set to any form depending on the curve of $r_t(p_t)$ from simulation results. We set $r_t(p_t)$ to a polynomial because according to the Weierstrass approximation theorem, within a closed domain, any continuous function can be approximated by a polynomial as closely as desired. When $r_t(p_t)$ is set to a polynomial, the optimal order setting should depend on the fitness of the regressed function. From simulation experiments, we observe that $r_t(p_t)$ usually can fit closely to the realized values when set to be a cubic function as

$$r_t(p_t) = \beta_{t0} + \beta_{t1}p_t + \beta_{t2}p_t^2 + \beta_{t3}p_t^3, \quad (7)$$

where β_{t0} , β_{t1} , β_{t2} and β_{t3} are the coefficients of $r_t(p_t)$ to be determined through regression. Note that according to Weierstrass approximation theorem, the regressed polynomial can

only approximate $r_t(p_t)$ when p_t is within the interval where p_{tk} is distributed. Thus, when using the adjusted MNL function to predict customer choices, p_t should also be constrained in the same interval. We use a simplified simulation as an example to show the regression result of $r_t(p_t)$ when the form of $r_t(p_t)$ goes from linear to cubic.

Example

In our example, the 3PL only offers a sole delivery date option ($T = 1$). Thus, we eliminate the subscript ‘ t ’ for clarity. We generate the sales record of 30 days, each of which includes 100 customers each with freight quantity drawn from $\mathcal{U}(0.5, 1.5)$. For each of the 30 days, the freight rate is drawn from $\mathcal{U}(0, 8)$. We test the cases where a customer’s valuation, V , and price sensitivity, α , follow different distributions:

Case 1: The customers are homogeneous, i.e., $V = 5$ and $\alpha = 1$.

Case 2: The customers are heterogeneous in V , i.e., $V \sim \mathcal{U}(0, 10)$ and $\alpha = 1$.

Case 3: The customers are heterogeneous in α , i.e., $V = 5$, $\alpha \sim \mathcal{U}(0, 2)$.

Case 4: The customers are heterogeneous in V and α , i.e., $V \sim \mathcal{U}(0, 10)$, $\alpha \sim \mathcal{U}(0, 2)$.

In Figure 1, we display the realized values of $r(p)$ and the regressed curve in each case. We observe that with homogeneous customers, $r(p) \approx 0$ for all p , which indicates that the standard MNL can accurately predict the probability of customer choices when the customers are homogeneous. In Cases 2-4, it is obvious that $r(p)$ is not always 0 at different p . The regression curves for linear regression, quadratic regression and cubic regression are displayed in each chart. The cubic regression obtains a close fit to the realized values of $r(p)$ in all the cases.

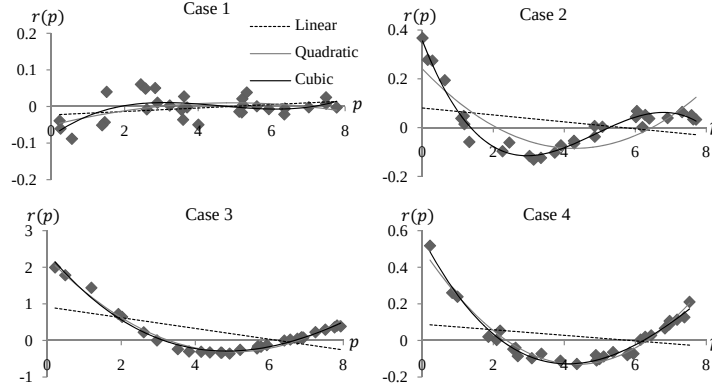


Figure 1: The curves of regressed $r(p)$ in different cases

With the adjusted MNL model, we increase the prediction accuracy of customer choices of different delivery dates under different freight quotes $\vec{p}$, which is critical for computing the optimal freight quote using our SNLP pricing model.

3.2. Dynamic Pricing Model based on an Actual LTL Shipping Case

We study a 3PL that provides LTL shipping and warehousing services to demonstrate our pricing model, noting that our model is also applicable when there are no warehousing services. A LTL freight quote is usually determined by many factors such as the transport distance, the weight and volume of the freight and other requirements, e.g., if the freight is fragile, frozen, or needs special packaging. To implement our pricing method, the weight and volume of the freight are taken as inputs, and the total cost incurred for other requirements are computed separately and added to the computed price afterwards.

Following the convention in the LTL industry, the weight of a freight order used herein refers to the *dimensional weight*, which is a calculation of a theoretical weight of a package. This theoretical weight is the weight of the package at a minimum density chosen by the freight carrier. If the package is below this minimum density, then the actual weight is irrelevant as the freight carrier charges for the volume of the package as if it were of the chosen density (what the package would weigh at the minimum density). The weight of a freight order used in our model is the greater of the actual weight and dimensional weight of the freight.

We name our dynamic pricing model as DPM. The constraint $qN_t \leq c_t$ has to be made for all $t \in \{1, \dots, T\}$ to ensure that the LTL provider has enough available capacity to guarantee that the customers' freight can be delivered on the chosen delivery dates. If $qN_t > c_t$ for some t , then a penalty, ω , is incurred for each unit of capacity shortage. Let Q_t be the total quantity of freight from customers choosing delivery date option t after pricing. The total penalty for capacity shortage, denoted by $\Omega(\vec{p})$, can be obtained as

$$\Omega(\vec{p}) = \omega \sum_{t=1}^T \left[\int_{c_t}^{+\infty} f_t^Q(Q_t, \vec{p}) [Q_t - c_t] dQ_t \right] \quad (8)$$

where $f_t^Q(Q_t, \vec{p})$ is the pdf of Q_t under $\vec{p}$. Then with $\Omega(\vec{p})$, DPM can be obtained as

$$\mathbf{DP:} \max_{\vec{p}} E(q)E(N) \sum_{t=1}^T [p_t - ht] \mathbb{P}_t(\vec{p}) - \Omega(\vec{p}), \quad \ni p_t \geq 0 \quad t = 1, 2, \dots, T.$$

In DP, $E(q)$ and $E(N)$ are the expected values of q and N , respectively. ht computes the holding cost incurred by a freight unit if delivery date of the freight is t . The objective function is to maximize the LTL provider's expected profit subject to a nonnegative freight rate for each delivery date option. Because the 3PL can dynamically adjust the holding cost, h , and the available transportation capacity for all delivery date options, $\vec{c}$, it can compute the dynamic freight quote based on its current warehouse and transportation status. In order to solve DP, we need to determine the forms of $f_t^Q(Q_t, \vec{p})$ for all $t \in \{1, \dots, T\}$ in (8).

Because the value of Q_t is mutually determined by the number of customers, the quantity of each customer's order and the choices of delivery dates when N and q are random, deriving the analytical form of $f_t^Q(Q_t, \vec{p})$ is complex. In the literature, histogram or Kernel density estimation can be a general approach to determine $f_t^Q(Q_t, \vec{p})$ when N and q are both random (Green et al. 1988). In this general approach, given any setting of $\vec{p}$, the realized values of Q_t are obtained from large number of repeated simulations and the curve of $f_t^Q(Q_t, \vec{p})$ in Q_t is obtained based on the distribution of the realized values of Q_t . This general approach can obtain the approximate curve of $f_t^Q(Q_t, \vec{p})$ in Q_t given any setting of $\vec{p}$ when N and q follow any distributions. However, because the curve of $f_t^Q(Q_t, \vec{p})$ changes with $\vec{p}$, we have to re-draw the curve of $f_t^Q(Q_t, \vec{p})$ every time $\vec{p}$ is changed. Thus, using this general approach may result in unreasonably long computing time when we search the optimal $\vec{p}$ for DPM

using heuristics where $\vec{p}$ is iteratively changed and the curve of $f_t^Q(Q_t, \vec{p})$ has to be drawn for each instance of $\vec{p}$. In order to solve DPM within reasonable computing time, we develop a method to find an asymptotic form of $f_t^Q(Q_t, \vec{p})$. Because our method requires a normally distributed N , here we only examine the realistic case where N is normally distributed, i.e., $N \sim \mathcal{N}(\mu_N, \sigma_N^2)$ where μ_N and σ_N are the mean and the standard deviation of N , respectively.

Because for any t we can use a Bernoulli distributed variable (0 or 1) to represent if a customer chooses delivery date t , then it is straightforward that when N is random, the number of customers choosing delivery date t under $\vec{p}$, N_t , is the sum of a random number of i.i.d. Bernoulli distributed random variables. According to Robbins (1948), when N is normally distributed, N_t asymptotically follows the normal distribution as

$$N_t \sim \mathcal{N}\left(\mu_{N_t}, \sigma_{N_t}^2\right),$$

where

$$\mu_{N_t} = \mu_N \mathbb{P}_t(\vec{p}) \text{ and } \sigma_{N_t}^2 = \mu_N \mathbb{P}_t(\vec{p})[1 - \mathbb{P}_t(\vec{p})] + \sigma_N^2 [\mathbb{P}_t(\vec{p})]^2.$$

Similarly, the total quantity of freight from the customers choosing delivery date t , Q_t , can be treated as the sum of a random number (N_t) of i.i.d. random variables (q). Let μ_q and σ_q be the mean and standard deviation of q . Again, we can have Q_t asymptotically following the normal distribution as

$$Q_t \sim \mathcal{N}\left(\mu_{Q_t}, \sigma_{Q_t}^2\right), \tag{9}$$

where

$$\mu_{Q_t} = \mu_q \mu_{N_t} = \mu_q \mu_N \mathbb{P}_t(\vec{p})$$

$$\text{and } \sigma_{Q_t}^2 = \mu_{N_t} \sigma_q^2 + \mu_q^2 \sigma_{N_t}^2 = \mu_N \mathbb{P}_t(\vec{p}) \sigma_q^2 + \mu_q^2 [\mu_N \mathbb{P}_t(\vec{p})[1 - \mathbb{P}_t(\vec{p})] + \sigma_N^2 [\mathbb{P}_t(\vec{p})]^2].$$

From (9), we can obtain the asymptotic forms of $f_t^Q(Q_t, \vec{p})$ for all $t \in \{1, \dots, T\}$, and then DPM is solvable. We simplify the problem by treating Q_t as an independently distributed random number. This may incur an error due to the correlation between N_t for different $t \in \{1, \dots, T\}$. However, as can be seen in the simulation experiment in Section 4, this error is absorbed by the regressed MNL function and so it does not affect the accuracy of the proposed model and estimation methods.

Based on DPM, we provide the model for pricing multiple customer classes in Appendix B.

4. SIMULATION EXPERIMENT AND RESULTS

In our simulation experiment, we first test the accuracy of the proposed adjusted MNL model in predicting customer choices. Then we compare the 3PL's profits obtained using the adjusted MNL model and the standard MNL model. After that, we compare the 3PL's long-term profits obtained by employing the DPS and the SPS. Finally, we show how a 3PL can use the proposed pricing method to optimize its long-term decision.

We propose an online quoting system based on the proposed pricing method as in Figure 2, and our simulation experiment is designed according to the same procedure except for that the market analyzer and the other cost estimator are omitted. All simulations are programmed in Matlab and the relevant nonlinear constrained programs are solved using the built-in solver 'fmincon' with the interior point algorithm.

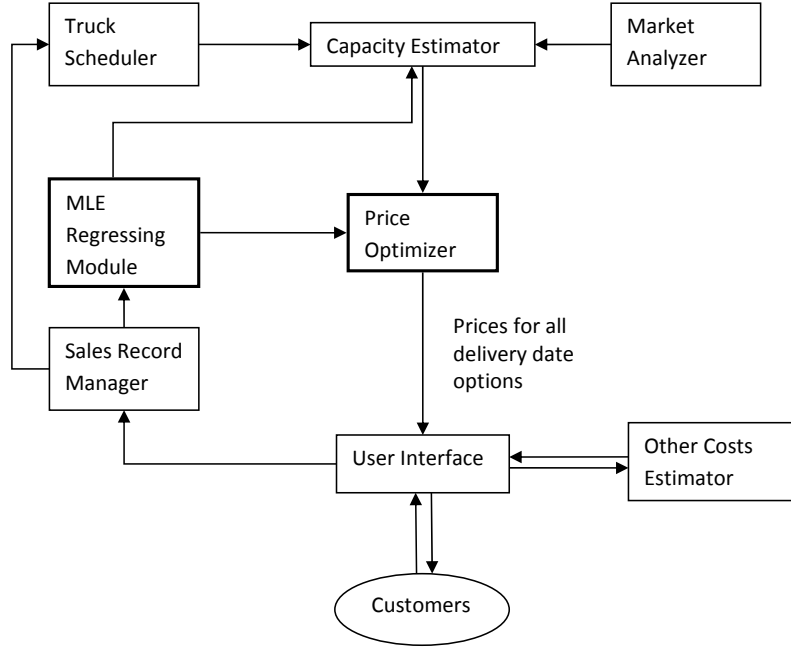


Figure 2: A proposed online quoting system for

We scale the parameters in our simulation to reflect typical settings in practice. We

consider a 3PL that operates a lane with a daily transportation capacity of 50 tons. The number of arriving customers, N , is distributed from $\mathcal{N}(500, 50^2)$ and the freight weight, q , is distributed from $\mathcal{N}(200, 30^2)$ in kg. The 3PL is set up to offer five delivery date options, i.e., $T = 5$, to simulate pricing for orders scheduled for the next 5 business days (1 week). Each simulated customer has a first-best delivery date, denoted by t^* , and the probabilities of $t^* = 1, t^* = 2, \dots, t^* = 5$ are set to be 0.4, 0.3, 0.2, 0.05 and 0.05, respectively. We ran our simulations by setting the customers' first-best delivery dates to follow different distributions, even including the case where $t^* = 1$ for all customers. However, because the results for different settings are qualitatively the same, for brevity we omit the results for other settings.

In our simulation, for each delivery date t , a customer's price sensitivity, α_t , is distributed from $\mathcal{N}(1.5, 0.2^2)$ and its valuation, v_t , is determined by

$$v_t = v^* - \tau |t^* - t| + \epsilon.$$

In the above equation $v^* : v^* \in \mathbb{R}^+$ is the customer's valuation if the shipment is delivered at its first-best delivery date, $\tau : \tau \in \mathbb{R}^+$ is the constant marginal cost incurred by a unit deviation of t from the customer's first best delivery date, and $\epsilon : \epsilon \in \mathbb{R}$ is the Gumbel error term which simulates a customer's bounded rationality (Su 2008). We name τ as the customer's delivery-date sensitivity. In the simulation, we set v^* to be distributed from $\mathcal{N}(1, 0.2^2)$ in \$/kg. For the setting of τ (in \$/day·kg), we consider 4 different cases, in each of which the mean and the variance of τ are high or low compared to v^* .

Case HH: The mean and the variance of τ are both high, i.e., $\tau \sim \mathcal{U}(0.1, 0.2)$.

Case HL: The mean of τ is high, but the variance of τ is low, i.e., $\tau \sim \mathcal{U}(0.15, 0.2)$.

Case LH: The mean of τ is low, but the variance of τ is high, i.e., $\tau \sim \mathcal{U}(0, 0.1)$.

Case LL: The mean and the variance of τ are both Low, i.e., $\tau \sim \mathcal{U}(0, 0.5)$.

The performance of our pricing method is not affected when the distribution of v^* and τ is scaled to a different magnitude because, as we stated in Section 2, the scale effect is

eliminated in the MNL function. Thus, we use this four-case simulation to illustrate the performance of our pricing model when the 3PL faces different situations in practice. As for the other parameters for the simulation, we set the 3PL's holding cost for each unit freight for each day, h , to be 0.1\$/day·kg, and set the penalty for a unit capacity shortage, ω , to be 5\$/day·kg.

To test the accuracy of the adjusted MNL, we first use the method introduced in Section 3.1 to estimate customers' valuation and price sensitivity for all delivery date options, $\vec{v}^S$ and $\vec{\alpha}^S$. We generate the sales records for 90 days ($K = 90$), and in each day, the price for each delivery date option is a random real number drawn from $\mathcal{U}(1.5, 3)$ in \$/kg. The simulation is repeated 100 times. In Table 1, the averages of the estimated v_t^S and α_t^S for all $t \in \{1, \dots, T\}$ are displayed with the standard deviations in parentheses. We observe from Table 1 that based on our settings, the standard deviations are small compared to the average of the estimations for each parameter. This shows that the 3PL can obtain stable estimates for each parameter based on a 30-day sales record. In the remainder of our simulation, we use the averages of v_t^S and α_t^S for all $t \in \{1, \dots, T\}$ to compute the optimal freight quote in each case.

Table 1: $\vec{v}^S$ and $\vec{\alpha}^S$ estimated by MLE with standard MNL

Case	Taste coefficient	$t = 1$	2	3	4	5
HH:	v_t^S	0.74 (0.029)	0.76 (0.031)	0.70 (0.030)	0.59(0.030)	0.46 (0.027)
	α_t^S	1.41(0.044)	1.41 (0.051)	1.41 (0.048)	1.41(0.049)	1.41 (0.053)
HL:	v_t^S	0.72 (0.028)	0.75 (0.029)	0.67 (0.029)	0.54 (0.027)	0.40 (0.022)
	α_t^S	1.40 (0.044)	1.41 (0.053)	1.40 (0.049)	1.41 (0.049)	1.41 (0.053)
LH:	v_t^S	0.83 (0.028)	0.84 (0.031)	0.83 (0.030)	0.78 (0.030)	0.74 (0.028)
	α_t^S	1.41 (0.044)	1.41 (0.051)	1.41 (0.047)	1.40 (0.048)	1.40 (0.050)
LL:	v_t^S	0.66(0.031)	0.68 (0.026)	0.59 (0.028)	0.46 (0.025)	0.24 (0.018)
	α_t^S	1.40 (0.045)	1.40 (0.052)	1.40 (0.048)	1.40 (0.050)	1.40 (0.060)

After estimating $\vec{v}^S$ and $\vec{\alpha}^S$, we regress $r_t(p_t)$ following the steps introduced in Section 3.1. The average coefficients of the regressed polynomial, are displayed in Table 2.

Table 2: The regressed $r_t(p_t)$ for each delivery date option

Case	Coefficient	$t = 1$	2	3	4	5
HH:	β_{t0}	-0.342	0.448	-0.092	0.036	0.642
	β_{t1}	0.559	-0.634	0.139	-0.087	-0.907
	β_{t2}	-0.296	0.290	-0.071	0.055	0.415
	β_{t3}	0.049	-0.045	0.010	-0.013	-0.064
HL:	β_{t0}	-0.169	0.440	-0.128	0.339	0.771
	β_{t1}	0.314	-0.634	0.189	-0.549	-1.097
	β_{t2}	-0.183	0.296	-0.093	0.284	0.506
	β_{t3}	0.032	-0.046	0.013	-0.049	-0.079
LH:	β_{t0}	-0.222	0.745	0.277	0.171	0.107
	β_{t1}	0.384	-1.079	-0.392	-0.274	-0.159
	β_{t2}	-0.214	0.507	0.177	0.139	0.075
	β_{t3}	0.036	-0.079	-0.028	-0.024	-0.013
LL:	β_{t0}	-0.273	0.179	-0.183	0.260	0.679
	β_{t1}	0.451	-0.239	0.265	-0.399	-0.977
	β_{t2}	-0.241	0.100	-0.127	0.197	0.456
	β_{t3}	0.039	-0.015	0.018	-0.034	-0.072

With the obtained adjusting term $r_t(p_t)$ for all $t \in \{1, \dots, T\}$ in each case, we can compute the adjusted probabilities of customer choices with the adjusted MNL function from (4). In order to test the superiority of the adjusted MNL model, we first randomly generate a number of freight quotes. Then under the generated freight quotes, we compare the simulated distribution of customer choices and the distributions predicted using the two MNL models by R^2 values. Table 3 compares R^2 values of both models for each day and each case. We can observe that by using the adjusted MNL, the R^2 value increased, indicating that the adjusted MNL can predict customer choices better when customers have heterogeneous taste coefficients.

Table 3: R^2 values of both models for each day and each case.

Case	Model	$t = 1$	2	3	4	5
HH:	S†	0.9774	0.9767	0.9720	0.9795	0.9720
	A	0.9938	0.9928	0.9898	0.9901	0.9921
HL:	S	0.9835	0.9763	0.9756	0.9773	0.9814
	A	0.9940	0.9929	0.9903	0.9889	0.9924
LH:	S	0.9826	0.9731	0.9823	0.9708	0.9766
	A	0.9937	0.9933	0.9931	0.9905	0.9928
LL:	S	0.9803	0.9764	0.9774	0.9794	0.9753
	A	0.9938	0.9920	0.9906	0.9907	0.9877

†S - Standard MNL; A - Adjusted MNL

Because the inaccuracy of DPM is caused by customer heterogeneity rather than the setting of available capacity, we only show the results when the available capacity, $\vec{c}$, is randomly set to $\vec{c} = (10, 15, 20, 30, 40)$ in tons, which simulates the practical case where the transportation capacity is more heavily utilized for delivery dates closer to the current date. When repeating the simulation with different settings of available capacities, we obtain the same evidence that shows the superiority of adjusted MNL model. The optimal freight quotes obtained by the DPM with the standard MNL and the adjusted MNL, denoted by DPM(S) and DPM(A), respectively, are displayed in Table 4.

Table 4: The freight quotes computed by DPM(S) and DPM(A)

Model	Case	$t = 1$	2	3	4	5
DPM(S)	HH:	1.91	1.67	1.61	1.71	1.81
	HL:	1.90	1.66	1.60	1.70	1.80
	LH:	1.95	1.70	1.66	1.76	1.86
	LL:	1.88	1.63	1.58	1.68	1.78
DPM (A)	HH:	1.90	1.66	1.62	1.69	1.81
	HL:	1.89	1.65	1.60	1.68	1.80
	LH:	1.94	1.69	1.67	1.75	1.85
	LL:	1.87	1.63	1.59	1.67	1.78

We use $\vec{p}^S$ and $\vec{p}^A$ to denote the optimal freight quotes obtained by DPM(S) and DPM(A), respectively, and $Q_t^{\vec{p}^S}$ and $Q_t^{\vec{p}^A}$ are the quantity of freight with delivery date t under $\vec{p}^S$ and $\vec{p}^A$, respectively. We first simulate the true expected quantity of freight with delivery date t , denoted by $E(Q_t^{\vec{p}^S})$ and $E(Q_t^{\vec{p}^A})$, and then we compare $\mu_q \mu_N \mathbb{P}_t^S(\vec{p}^S)$ and $\mu_q \mu_N \mathbb{P}_t^A(\vec{p}^A)$ with $E(Q_t^{\vec{p}^S})$ and $E(Q_t^{\vec{p}^A})$ for all $t \in \{1, \dots, T\}$. If the standard MNL function and the adjusted MNL function can both accurately predict the probabilities of delivery date choices, then we should have $\mu_q \mu_N \mathbb{P}_t^S(\vec{p}^S)$ and $\mu_q \mu_N \mathbb{P}_t^A(\vec{p}^A)$ statistically equal to $E(Q_t^{\vec{p}^S})$ and $E(Q_t^{\vec{p}^A})$, respectively. We calculate $E(Q_t^{\vec{p}^S})$ and $E(Q_t^{\vec{p}^A})$ by averaging $Q_t^{\vec{p}^S}$ and $Q_t^{\vec{p}^A}$ obtained from 100 runs of the simulation program in which random customers are generated for a typical day and face freight quotes of $\vec{p}^S$ and $\vec{p}^A$, respectively. The results of comparing the accuracies of standard MNL function and the adjusted MNL function in predicting the probabilities of customer choices are displayed in Table 5. From Table 5, we observe that based on t-tests reflecting a 95% confidence level, many $E(Q_t^{\vec{p}^S})$ values are significantly different from the paired values computed by $\mu_q \mu_N \mathbb{P}_t^S(\vec{p}^S)$, whereas almost all the $E(Q_t^{\vec{p}^A})$ values are not significantly different from the paired values computed by $\mu_q \mu_N \mathbb{P}_t^A(\vec{p}^A)$. From this result, we

conclude that when customers are heterogeneous our adjusted MNL function yields better predictions of delivery date choices.

Table 5: Comparing the accuracies of standard MNL function and the adjusted MNL function in predicting the quantities of freight (kg) with different delivery choices [†]

Model	Case	$t = 1$	2	3	4	5
DPM(S)	HH:	*(7810, 7688)	(11155, 10703)	*(11341, 11039)	*(8805, 8720)	(6728, 6335)
	HL:	*(7812, 7715)	(11170, 10720)	*(11317, 11184)	*(8599, 8827)	(6437, 6760)
	LH:	(7795, 7438)	(11073, 10512)	(11440, 10523)	(9582, 8299)	(7998, 6441)
	LL:	*(7820, 7806)	*(11181, 10931)	*(11055, 11390)	(8069, 8989)	(5951, 6880)
DPM(A)	HH:	*(7822, 7709)	*(11258, 11222)	*(11270, 10849)	*(9010, 8742)	*(6731, 6690)
	HL:	*(7824, 7753)	*(11278, 11251)	*(11234, 10994)	*(8823, 8826)	*(6408, 6386)
	LH:	*(7804, 7485)	*(11165, 10977)	*(11389, 11353)	(9725, 8348)	(8052, 6818)
	LL:	*(7836, 7885)	*(11300, 11490)	*(10892, 11121)	*(8720, 8887)	(6103, 6479)

[†] In each parentheses, the first value is $\mu_N \mu_q \mathbb{P}_t(\vec{p}^S)$ (or $\mu_N \mu_q \mathbb{P}_t(\vec{p}^A)$), and the second value is the simulated $E(Q_t^{\vec{p}^S})$ (or $E(Q_t^{\vec{p}^A})$).

* Indicates that $\mu_N \mu_q \mathbb{P}_t(\vec{p}^S)$ (or $\mu_N \mu_q \mathbb{P}_t(\vec{p}^A)$) is not significantly different from the paired simulated value of $E(N_t^{\vec{p}^S})$ (or $E(N_t^{\vec{p}^A})$) based on t-test with 95% confidence level.

Table 6 compares the 3PL's expected profits under the freight quotes computed from DPM(S) and DPM(A). Denoting the 3PL's true expected profits under $\vec{p}^S$ and $\vec{p}^A$ by $E(\pi(\vec{p}^S))$ and $E(\pi(\vec{p}^A))$, respectively, we obtain $E(\pi(\vec{p}^S))$ and $E(\pi(\vec{p}^A))$ by averaging the profits obtained from 100 runs of the simulation program in which random customers are generated for a typical day and face freight quotes of $\vec{p}^S$ and $\vec{p}^A$, respectively. In Table 6, we observe that by increasing the prediction accuracy of the distribution of customer choices, DPM(A) can more accurately predict the 3PL's expected profit and consequently computes a better freight quote. In contrast, because DPM(S) less accurately predicts the 3PL's expected profit, it computes a worse freight quote although it obtains a higher objective value in each case.

Table 6: Comparing the 3PL's expected profits (\$) under the freight quotes computed from DPM(S) and DPM(A)[†]

Case	DPM(S)	DPM(A)
HH:	(66161, 64014)	*(65356, 65332)
HL:	(64863, 64029)	*(64208, 64267)
LH:	(70995, 68781)	*(70157, 69300)
LL:	(62690, 60043)	*(61605, 61245)

[†] In each parenthesis, the first value is the expected profit computed from the objective function DPM(S) (or DPM(A)), and the second value is the simulated $E(\pi(\vec{p}^S))$ (or $E(\pi(\vec{p}^A))$).

* Indicates that the expected profit computed from the objective function DPM(S) (or DPM(A)) is not significantly different from the paired simulated value of $E(\pi(\vec{p}^S))$ (or $E(\pi(\vec{p}^A))$) based on t-test with 95% confidence level.

Because the model for multiple customer classes as provided in Appendix B is a simple extension of DPM, for simplicity the repetitive numerical results are not shown.

5. COMPARING THE DPS AND SPS

We demonstrate the superiority of our DPS using the proposed pricing model by comparing it with a common SPS where the 3PL does not dynamically change the freight rate for each delivery date option. In the SPS, the 3PL sets a fixed freight rate that maximize the long-term profit as defined later in Section 5.1. A method to determine the preserved capacity for each delivery date option to handle customers arriving in future days.

5.1. Computing the Steady-State Preserved Transportation Capacity

In order to seek a the global optimum for the long-run, the 3PL has to preserve enough transportation capacity for each delivery date option to handle orders from customers arriving in future days. We focus on the case where the customers' arrival rate and delivery date preference is i.i.d. in each day, and the 3PL does not change the setting of transportation capacity for each delivery date in each day. We define this unchanged setting of preserved transportation capacity as the steady-state preserved transportation capacity.

Let Q_{tk} be the quantity of freight received in day k with delivery date choice t . Then the total transport quantity in day k can be obtained as $\sum_{t=1}^T Q_{t,k-t}$ which is constrained by the daily transportation capacity, c . It is straightforward that because the customers' arrival rate and delivery date preference is i.i.d. in each day, we can substitute $Q_{t,k-t}$ with Q_{tk} for all $t \in \{1, \dots, T\}$. Hence, let Q_t be the value of Q_{tk} for any day k in the steady state. Then, we have the penalty incurred due to transportation capacity shortage for each day in the steady state, denoted by $\Omega^{SS}(\vec{p})$, as:

$$\Omega^{SS}(\vec{p}) = \omega \left[\int_c^{+\infty} f^Q(Q^{\vec{p}}, \vec{p}) [Q^{\vec{p}} - c] dQ^{\vec{p}} \right] \quad (10)$$

where $Q^{\vec{p}} = \sum_{t=1}^T Q_t$ and $f^Q(Q^{\vec{p}}, \vec{p})$ is the pdf of $Q^{\vec{p}}$ under $\vec{p}$. Then with $\Omega^{SS}(\vec{p})$, the optimal setting of $\vec{p}$ can be obtained by

$$\max_{\vec{p}} E(q)E(N) \sum_{t=1}^T [p_t - ht] \mathbb{P}_t(\vec{p}) - \Omega^{SS}(\vec{p}), \quad \ni p_t \geq 0 \quad t = 1, 2, \dots, T. \quad (11)$$

Note that because $\vec{p}$ maximizes the 3PL's profit in the steady state, $\vec{p}$ is the optimal freight quote in our SPS that maximizes long-term profit. Similar to Section 3.2, we approximate the form of $f^Q(Q^{\vec{p}}, \vec{p})$ only for the case where $N \sim \mathcal{N}(\mu_N, \sigma_N^2)$. Because we can use a Bernoulli distributed variable (1 or 0) to represent if a customer places an order (choosing any delivery date) or rejects, then when N is random, the number of customers that place orders under $\vec{p}$ is denoted by $N^{\vec{p}}$ such that $N^{\vec{p}} = \sum_{t=1}^T N_t$ is the sum of a random number of i.i.d. Bernoulli distributed random variables. Let $\mathbb{P}(\vec{p}) = \sum_{t=1}^T \mathbb{P}_t(\vec{p})$. Again, from Robbins (1948), when N is normally distributed, $N^{\vec{p}}$ asymptotically follows the normal distribution as

$$N^{\vec{p}} \sim \mathcal{N}\left(\mu_{N^{\vec{p}}}, \sigma_{N^{\vec{p}}}^2\right),$$

where

$$\mu_{N^{\vec{p}}} = \mu_N \mathbb{P}(\vec{p}) \text{ and } \sigma_{N^{\vec{p}}}^2 = \mu_N \mathbb{P}(\vec{p}) [1 - \mathbb{P}(\vec{p})] + \sigma_N^2 \mathbb{P}_t(\vec{p})^2.$$

Similarly, the total quantity of freight received in a day under $\vec{p}$, $Q^{\vec{p}}$, can be treated as the sum of a random number ($N^{\vec{p}}$) of i.i.d. random variables (q). Let μ_q and σ_q be the mean and standard deviation of q . Again, we can have $Q^{\vec{p}}$ asymptotically normal as

$$Q^{\vec{p}} \sim \mathcal{N}\left(\mu_{Q^{\vec{p}}}, \sigma_{Q^{\vec{p}}}^2\right), \quad (12)$$

where

$$\mu_{Q^{\vec{p}}} = \mu_q \mu_{N^{\vec{p}}} = \mu_q \mu_N \mathbb{P}(\vec{p})$$

$$\text{and } \sigma_{Q^{\vec{p}}}^2 = \mu_{N^{\vec{p}}} \sigma_q^2 + \mu_q^2 \sigma_{N^{\vec{p}}}^2 = \mu_N \mathbb{P}(\vec{p}) \sigma_q^2 + \mu_q^2 [\mu_N \mathbb{P}(\vec{p}) [1 - \mathbb{P}(\vec{p})] + \sigma_N^2 \mathbb{P}(\vec{p})^2].$$

From (12), we can obtain the asymptotic form of $f^Q(Q^{\vec{p}}, \vec{p})$ and then (11) is solvable.

We use c_t^P to denote the capacity preserved for $Q_t^{\vec{p}}$ in the steady state and define $\vec{c}^P = (c_1^P, \dots, c_T^P)$. When $\vec{p}$ is obtained from (11), we compute the optimal setting of $\vec{c}^P$ from

$$\min_{\vec{c}^P} \sum_{t=1}^T \left[\int_{c_t^P}^{+\infty} f_t^Q(Q_t, \vec{p}) [Q_t - c_t^P] dQ_t \right], \ni \sum_{t=1}^T c_t^P \leq c. \quad (13)$$

By solving (13), we optimize the setting of preserved capacities for all delivery date options to minimize the total overflow quantity given that the sum of the preserved capacities for all delivery date options does not exceed the 3PL's daily transportation capacity. In (13),

$f_t^Q(Q_t, \vec{p})$, which is the pdf of Q_t under $\vec{p}$, can be approximated as in Section 3.2. When $\vec{c}^P$ is obtained, the available transportation capacity for any delivery date option t , c_t , can be obtained by subtracting from c the total preserved capacity for the future $t - 1$ days and the quantity already scheduled for delivery date t , denoted by A_t , as

$$c_t = \max \left\{ c - \sum_{\hat{t}=1}^t c_{\hat{t}}^P + c_t^P - A_t, 0 \right\}, \quad t \in \{1, \dots, T\}. \quad (14)$$

We test the superiority of our proposed DPS over the long-run with the available capacity computation method as shown in (14).

5.2. Comparing the 3PL's Expected Profit, Customer Welfare and Social Welfare

In our simulation experiment, we simulate 100 days of the 3PL's business. The difference between the DPS and the SPS is not significant when the system traffic is low. This is because when system traffic is low the 3PL can deliver all the freight orders at customers' ideal delivery dates without breaking the capacity constraints, which means that the 3PL does not need a DPS to influence the distribution of customer choices. In the literature of dynamic pricing, it is common to assume a congested system, e.g., Çelik and Maglaras (2008). It is also reasonable for the 3PL to avoid over-investment in its transportation capacity by increasing its capacity utilization.

As in the previous simulation experiments, we set the 3PL to provide five delivery date options ($T = 5$). The parameters of the 3PL's unit holding cost, unit penalty for capacity shortage and the distributions of customers' parameters, are set the same as in Section 4.

The results of the simulation experiment are displayed in Table 7. From the simulation, we first obtained the optimal freight rate settings from the SPS. Then we simulated 100 days of the 3PL's business and compare the utilization of its transportation capacity and the profits under the two pricing strategies. We use π^{100} to denote the profit obtained by the 3PL over 100 days. We see that using the DPS, the utilization of the 3PL's transportation capacity is substantially increased, and consequently its profit is increased. The superiority of the DPS increases when there are more customers with high delivery-date sensitivity. The 3PL's expected profit is increased most using the DPS in the HL case where the customers mainly have high delivery-date sensitivity.

Table 7: Comparing the 3PL's expected profit obtained from the two pricing strategies

Case	Static pricing strategy		Dynamic pricing strategy	
	Capacity utilization	$\pi^{100}(\$)$	Capacity utilization	$\pi^{100}(\$)$
HH	79.3%	5593k	94.10%	5960k (+6.56%)
HL	78.3%	5532k	94.00%	5898k (+6.62%)
LH	80.2%	5744k	95.00%	6084k (+5.92%)
LL	79.8%	5464k	93.50%	5726k (+4.80%)

We compare customer and social welfare under the SPS and the DPS in Table 8. We sum the net gain of all the customers arriving over 100 days. The social welfare is obtained by adding up the simulated customer welfare and the 3PL's profit in Table 7. We observe that with the DPS, the customer and social welfare are substantially increased. Hence, with the DPS, the 3PL, customers and society are better off.

Table 8: Comparing customer welfare and social welfare under two pricing strategies

Case	Static pricing strategy		Dynamic pricing strategy	
	Customer welfare	Social welfare	Customer welfare	Social welfare
HH	4312k	9905k	5259k (+6.56%)	11219k (+21.97%)
HL	4277k	9809k	5233k (+6.62%)	11131k (+22.34%)
LH	4242k	9986k	5128k (+5.92%)	11212k (+20.89%)
LL	4113k	9577k	5062k (+4.80%)	10788k (+23.06%)

5.3. Comparing the Incentives of Increasing Transportation Capacity

We compare the incentives of increasing transportation capacity in the two pricing strategies to analyze how the different pricing strategies influence the 3PL's decisions about long-term investment.

We define the transportation capacity shadow price as the increase in daily profit that is caused by a unit increase in the 3PL's daily transportation capacity. In our simulation, we increase the setting of daily transportation capacity, c , from 40 tonnes to 80 tonnes with the step, denoted by Δc , of $\Delta c = 10$ tonnes. We simulate the 3PL's profits under different settings of c and then compute the transportation capacity shadow price when the current daily transportation capacity is c , denoted by $\gamma^C(c)$, from the simulated profits when the transportation capacity are c and $c + \Delta c$. Using $\pi^{100}(c)$ to denote the 3PL's total profit over 100 days obtained from the simulation when the daily transportation capacity is c , we

obtain $\gamma^C(c)$ as

$$\gamma^C(c) = \frac{\pi^{100}(c + \Delta c) - \pi^{100}(c)}{\Delta c} \times \frac{1}{100}.$$

The simulation results are displayed in Figure 3. We observe that when daily production capacity increases, the 3PL's profit increases with both the DPS and the SPS, but the transportation capacity shadow price is lower with the DPS. This indicates that with the DPS, the 3PL has a lesser incentive to increase its daily transportation capacity. This is because the utilization of transportation capacity is higher with the DPS which means that the 3PL quotes lower prices to most customers and more orders are placed. When transportation capacity is increased, the 3PL further reduces the freight rate to attract more orders in both pricing strategies. However, with the DPS, because the freight rates are already lower, the price elasticity of demand is lower, which means fewer orders are attracted with the DPS with the same reductions in freight rates. Thus, when the daily transportation capacity is increased, the 3PL obtains a lower increase in its profit using the DPS, which means that the 3PL has lower incentive to invest in transportation capacity with the DPS.

6. CONCLUSION

We develop and study a DPS for a 3PL that provides comprehensive logistic services including warehousing and LTL transportation. We use SNLP models to solve the near optimal freight quote which includes freight rates for multiple delivery date options. Of the two models, the DPM model solves the freight quote for the case where the total quantity of the freight received each day is deterministic, whereas the DPM model is for the more realistic case where the total quantity of the freight received each day is random.

We use the MNL function to model customer choices of delivery date options. We developed an adjusted MNL function to model customer choices. Through a series of simulation experiments designed based on a LTL and warehousing service provider, we show that the adjusted MNL function can accurately predict the distribution of customer choices when customers are heterogeneous and customer utility functions are unknown. Consequently, we show that the 3PL can obtain a higher expected profit when computing freight quotes using our SNLP with the adjusted MNL function. We also design simulation experiments to

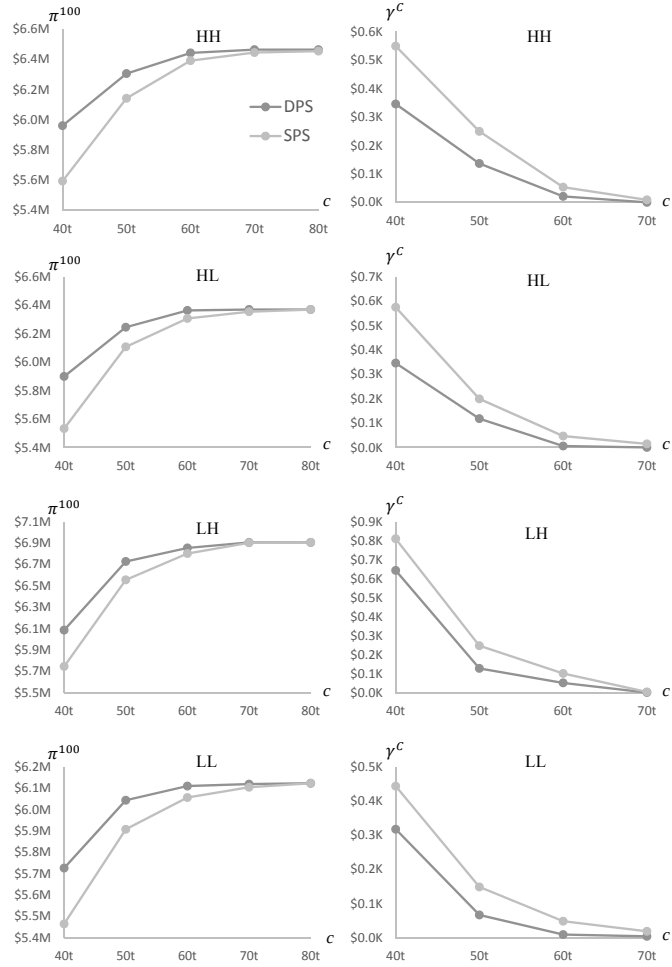


Figure 3: The profit (π^{100}) and the shadow price of increasing transportation capacity (γ^c) when the current daily transportation capacity (c) is at different values in DPS and SPS

compare the proposed DPS with a SPS. We show that with our proposed DPS, both the 3PL and its customers are better off and the 3PL has different capacity investment incentives for the longer-term.

Our work makes three contributions. First, by adopting our DPS, the 3PL optimizes its resource allocation to customers accounting for its current capacity utilization. As described in Section 1, by modifying the objective function and constraints while keeping the same underlying structure, the proposed pricing model can be applied in other manufacturing or service-providing settings that have similar features as the 3PL we study. Second, we show that the adjusted MNL function we develop is capable of modelling customer choices when customers are heterogeneous, which indicates that our adjusted MNL function is good replacement for the mixed MNL function in the cases where the mixed MNL function is not applicable. In our simulation we implement the DPS method using Matlab, but practitioners can simply incorporate our proposed model and parameter estimation method into their decision support system so that the system can compute the optimal price automatically after receiving information about historical transactions and system capacity.

Finally, our comparisons of pricing strategies provide management insight for practitioners regarding decisions about employing a DPS. That is, by comparing the 3PL's profit in DPS and SPS, we show why practitioners should employ the proposed DPS to increase profit. By comparing the shadow prices of transportation capacity, we show how practitioners should adjust their long-term investments after adopting DPS. Through simulation, we demonstrated that with our proposed DPS, the 3PL not only increases its own profit, but increases its customers' profit as well. As the customers of the 3PL are usually manufacturers, distributors and retailers, we conclude that the profit of entire supply chain is increased when the proposed DPS is employed by the 3PL – essentially improving coordination across the supply chain through dynamic pricing.

As a suggestion for future study, a quantity discount policy in which discounting is used to encourage customers to make orders with larger freight quantities, could be incorporated in the DPS. The quantity discount policy has been well-studied in the literature, but it is still an open problem when integrated into a DPS that accounts for both holding cost and

the transportation capacity utilization.

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Appendix A. List of Notation

N : The number of customers arriving at each day.

q : The quantity of a freight order.

T : The number of delivery date options.

$t \in \{1, \dots, T\}$: The index of a delivery date option.

$\vec{p}$: A freight quote for T delivery date options, and the t th element, p_t , denotes the quote for delivery date option t .

h : The cost for holding a freight unit for a day.

c : The 3PL's daily transportation capacity.

$\vec{c}$: The available transportation capacities, and the t th element, c_t , denotes the available capacity of delivery date option t .

ω : The unit penalty for violating daily transportation capacity.

$\vec{v}$: A customer's valuation for transporting a unit of its freight, and the t th element, v_t , denotes the valuation for delivery date option t .

$\vec{\alpha}$: A customer's price sensitivity, and the t th element, α_t , denotes the price sensitivity for delivery date option t .

$\xi(t)$: A customer's utility, or profit, from choosing delivery date option t .

$\mathbb{P}_t(\vec{p})$: The probability of a customer choosing delivery date t given freight quote $\vec{p}$.

$\mathbb{P}_t^S(\vec{p})$, $\vec{v}^S$, $\vec{\alpha}^S$: The value of $\mathbb{P}_t(\vec{p})$, $\vec{v}$ and $\vec{\alpha}$ computed using the standard MNL.

$\mathbb{P}_t^A(\vec{p})$: The value of $\mathbb{P}_t(\vec{p})$ computed using the adjusted MNL.

$r_t(p_t)$: The adjusting term for delivery date t .

β_{t0} , β_{t1} , β_{t2} , β_{t3} : The polynomial coefficients of $r_t(p_t)$.

Q_t : The total quantity of freight from customers choosing delivery date option t after pricing.

$f_t^Q(Q_t, \vec{p})$: The pdf of Q_t under $\vec{p}$.

μ_N , σ_N : The mean and the standard deviation of N .

N_t : The number of customers choosing delivery date t .

μ_{N_t} , σ_{N_t} : The mean and the standard deviation of N_t .

Appendix B. Pricing with multi-class customers

Suppose that there are M classes of customers and we use a superscript $m : m \in \{1, \dots, M\}$ to denote that a symbol is associated with a customer of class m . Then the price optimization model can be modified as

$$\begin{aligned} \max_{\vec{p}^m} \quad & \sum_{m=1}^M E(q^m) E(N^m) \sum_{t=1}^T [p_t^m - ht] \mathbb{P}_t^m(\vec{p}^m) - \Omega^R(\vec{p}^1, \vec{p}^2, \dots, \vec{p}^M) \\ \ni \quad & p_t^m \geq 0 \quad \forall m \in \{1, \dots, M\} \text{ and } \forall t \in \{1, 2, \dots, T\}, \end{aligned} \quad (\text{B.1})$$

where

$$\Omega^R(\vec{p}^1, \vec{p}^2, \dots, \vec{p}^M) = \omega \sum_{t=1}^T \left[\int_{c_t}^{+\infty} f_t^Q \left(\sum_{m=1}^M Q_t^m, \vec{p}^m \right) \left[\sum_{m=1}^M Q_t^m - c_t \right] d \sum_{m=1}^M Q_t^m \right]$$

Because Q_t asymptotically follows the normal distribution as shown in (12), we can obtain that $\sum_{m=1}^M Q_t^m$ asymptotically follows the normal distribution as

$$\sum_{m=1}^M Q_t^m \sim \mathcal{N} \left(\sum_{m=1}^M \mu_{Q_t^m}, \sum_{m=1}^M \sigma_{Q_t^m}^2 \right),$$

Based on the distribution of $\sum_{m=1}^M Q_t^m$, we can obtain the density function of $\sum_{m=1}^M Q_t^m$, $f_t^Q(\sum_{m=1}^M Q_t^m, \vec{p}^m)$, and then (B.1) is solvable.