Appendix

Equilibrium Analysis in Stage 2

Integrated Approach: Each district maximizes their individual surplus by choice of resources g_{Ij} and interoperability effort e_{Ij}:

$$\max_{g_{I1},\, e_{I1}} S_{I1} = m_1 \left[[1-\kappa][1-f_I] g_{I1} + \kappa \left[\frac{e_{I1}+e_{I2}}{\bar{e}} \right] g_{I2} \right] - p g_{I1}^2 - \delta e_{I1}^2 \text{ and}$$
$$\max_{g_{I2},\, e_{I2}} S_{I2} = m_2 \left[[1-\kappa][1-f_I] g_{I2} + \kappa \left[\frac{e_{I1}+e_{I2}}{\bar{e}} \right] g_{I1} \right] - p g_{I2}^2 - \delta e_{I2}^2,$$
$$\text{subject to } g_{Ij} \in [0, \overline{g}] \text{ and } e_{Ij} \in [0, \overline{e}].$$

Jointly solving for individual districts' optimal resources and interoperability effort, and assuming interior solutions to both, yields a Nash equilibrium:

$$g_{I1} = \frac{m_1[1-\kappa][1-f_I]}{2p}, g_{I2} = \frac{m_2[1-\kappa][1-f_I]}{2p},$$
$$e_{I1} = e_{I2} = \frac{m_1 m_2 \kappa [1-\kappa][1-f_I]}{4p\delta\bar{e}}.$$

We then compute the corresponding surplus for each district and social welfare in equilibrium:

$$S_{I1}^{eqm} = \frac{[1-f_I]^2 m_1^2 [1-\kappa]^2 [4\bar{e}^2 p\delta + 3m_2^2\kappa^2]}{16\bar{e}^2 p^2 \delta},$$
$$S_{I2}^{eqm} = \frac{[1-f_I]^2 m_2^2 [1-\kappa]^2 [4\bar{e}^2 p\delta + 3m_1^2\kappa^2]}{16\bar{e}^2 p^2 \delta},$$
$$S_I^{eqm} = S_{I1}^{eqm} + S_{I2}^{eqm} = \frac{[1-f_I]^2 [1-\kappa]^2 [2\bar{e}^2 [m_1^2 + m_2^2] p\delta + 3m_1^2 m_2^2 \kappa^2]}{8\bar{e}^2 p^2 \delta}.$$

Unified Approach: Each district maximizes their individual surplus by choice of resources g_{Uj} and interoperability effort e_{Uj}:

$$\max_{g_{U1},\, e_{U1}} S_{U1} = m_1 \left[[1-\kappa][1-f_U] g_{U1} + \kappa\beta_U \left[\frac{e_{U1}+e_{U2}}{\bar{e}} \right] g_{U2} \right] - p g_{U1}^2 - \delta e_{U1}^2 \text{ and}$$
$$\max_{g_{U2},\, e_{U2}} S_{U2} = m_2 \left[[1-\kappa][1-f_U] g_{U2} + \kappa\beta_U \left[\frac{e_{U1}+e_{U2}}{\bar{e}} \right] g_{U1} \right] - p g_{U2}^2 - \delta e_{U2}^2,$$
$$\text{subject to } g_{Uj} \in [0, \overline{g}] \text{ and } e_{Uj} \in [0, \overline{e}].$$

Jointly solving for individual districts' optimal resources and interoperability effort, and assuming interior solutions to both, yields a Nash equilibrium

$$g_{U1} = \frac{m_1[1-\kappa][1-f_U]}{2p}, g_{U2} = \frac{m_2[1-\kappa][1-f_U]}{2p},$$

$$e_{U1} = e_{U2} = \frac{m_1 m_2 \beta_U \kappa [1-\kappa][1-f_U]}{4p\delta\bar{e}}.$$

We then compute the corresponding surplus for each district and social welfare in equilibrium:

$$S_{U1}^{eqm} = \frac{m_1^2[1-\kappa]^2[1-f_U]^2[3m_2^2\beta_U^2\kappa^2 + 4p\delta\bar{e}^2]}{16p^2\delta\bar{e}^2},$$
$$S_{U2}^{eqm} = \frac{m_2^2[1-\kappa]^2[1-f_U]^2[3m_1^2\beta_U^2\kappa^2 + 4p\delta\bar{e}^2]}{16p^2\delta\bar{e}^2},$$
$$S_U^{eqm} = S_{U1}^{eqm} + S_{U2}^{eqm} = \frac{[1-f_U]^2[1-\kappa]^2[2\bar{e}^2[m_1^2+m_2^2]p\delta + 3m_1^2m_2^2\beta_U^2\kappa^2]}{8p^2\delta\bar{e}^2}.$$

Federated Approach: Each district maximizes their individual surplus by choice of resources g_{Fj} and interoperability effort e_{Fj}

$$\max_{g_{F1}, e_{F1}} S_{F1} = m_1\left[[1-\kappa]g_{F1} + \kappa\beta_F\left[\frac{e_{F1}+e_{F2}}{\bar{e}}\right]g_{F2}\right] - pg_{F1}^2 - \delta e_{F1}^2 \text{ and}$$

$$\max_{g_{F2}, e_{F2}} S_{F2} = m_2\left[[1-\kappa]g_{F2} + \kappa\beta_F\left[\frac{e_{F1}+e_{F2}}{\bar{e}}\right]g_{F1}\right] - pg_{F2}^2 - \delta e_{F2}^2,$$

subject to $g_{Fj} \in [0, \overline{g}]$ and $e_{Fj} \in [0, \overline{e}]$.

Jointly solving for individual districts' optimal resources and interoperability effort, and assuming interior solutions to both, yields a Nash equilibrium

$$g_{F1} = \frac{[1-\kappa]m_1}{2p},\ g_{F2} = \frac{[1-\kappa]m_2}{2p},$$

$$e_{F1} = e_{F2} = \frac{\kappa\beta_F[1-\kappa]m_1m_2}{4\bar{e}p\delta}.$$

We then compute the corresponding surplus for each district and social welfare in equilibrium:

$$S_{F1}^{eqm} = \frac{m_1^2[1-\kappa]^2[4\bar{e}^2p\delta + 3m_2^2\beta_F^2\kappa^2]}{16\bar{e}^2p^2\delta},$$
$$S_{F2}^{eqm} = \frac{m_2^2[1-\kappa]^2[4\bar{e}^2p\delta + 3m_1^2\beta_F^2\kappa^2]}{16\bar{e}^2p^2\delta},$$
$$S_F^{eqm} = S_{F1}^{eqm} + S_{F2}^{eqm} = \frac{[1-\kappa]^2[2\bar{e}^2[m_1^2+m_2^2]p\delta + 3m_1^2m_2^2\beta_F^2\kappa^2]}{8\bar{e}^2p^2\delta}.$$

Equilibrium Analysis in Stage 1

There are nine cases:

Case 1:

		District 2		
		$t_2 = I$	$t_2 = U$	$t_2 = F$
District 1	$t_1 = I$	I $\underline{\overline{(S_{I1}, S_{I2})}}$	U (S_{U1}, S_{U2})	F $\underline{(S_{F1}, S_{F2})}$
	$t_1 = U$	U (S_{U1}, S_{U2})	U (S_{U1}, S_{U2})	F $\underline{\overline{(S_{F1}, S_{F2})}}$
	$t_1 = F$	F $\overline{(S_{F1}, S_{F2})}$	F $\underline{\overline{(S_{F1}, S_{F2})}}$	F $\underline{\overline{(S_{F1}, S_{F2})}}$

Case 2:

		District 2		
		$t_2 = I$	$t_2 = U$	$t_2 = F$
District 1	$t_1 = I$	I $\underline{\overline{(S_{I1}, S_{I2})}}$	U (S_{U1}, S_{U2})	F $\underline{(S_{F1}, S_{F2})}$
	$t_1 = U$	U $\overline{(S_{U1}, S_{U2})}$	U $\overline{(S_{U1}, S_{U2})}$	F $\underline{(S_{F1}, S_{F2})}$
	$t_1 = F$	F $\overline{(S_{F1}, S_{F2})}$	F $\underline{\overline{(S_{F1}, S_{F2})}}$	F $\underline{\overline{(S_{F1}, S_{F2})}}$

Case 3:

		District 2		
		$t_2 = I$	$t_2 = U$	$t_2 = F$
District 1	$t_1 = I$	I $\underline{\overline{(S_{I1}, S_{I2})}}$	U $\underline{(S_{U1}, S_{U2})}$	F $\underline{(S_{F1}, S_{F2})}$
	$t_1 = U$	U $\overline{(S_{U1}, S_{U2})}$	U $\underline{\overline{(S_{U1}, S_{U2})}}$	F $\underline{(S_{F1}, S_{F2})}$
	$t_1 = F$	F $\overline{(S_{F1}, S_{F2})}$	F $\overline{(S_{F1}, S_{F2})}$	F $\underline{\overline{(S_{F1}, S_{F2})}}$

Case 4:

		District 2		
		$t_2 = I$	$t_2 = U$	$t_2 = F$
District 1	$t_1 = I$	I $\overline{(S_{I1}, S_{I2})}$	U $\underline{(S_{U1}, S_{U2})}$	F $\underline{(S_{F1}, S_{F2})}$
	$t_1 = U$	U $\underline{\overline{(S_{U1}, S_{U2})}}$	U $\underline{\overline{(S_{U1}, S_{U2})}}$	F $\underline{(S_{F1}, S_{F2})}$
	$t_1 = F$	F $\underline{\overline{(S_{F1}, S_{F2})}}$	F $\underline{\overline{(S_{F1}, S_{F2})}}$	F $\underline{\overline{(S_{F1}, S_{F2})}}$

Case 5:

		District 2		
		$t_2 = I$	$t_2 = U$	$t_2 = F$
District 1	$t_1 = I$	I (S_{I1}, S_{I2})	U $\underline{\overline{(S_{U1}, S_{U2})}}$	F $\underline{(S_{F1}, S_{F2})}$
	$t_1 = U$	U $\underline{\overline{(S_{U1}, S_{U2})}}$	U $\underline{\overline{(S_{U1}, S_{U2})}}$	F $\underline{(S_{F1}, S_{F2})}$
	$t_1 = F$	F $\underline{\overline{(S_{F1}, S_{F2})}}$	F $\underline{\overline{(S_{F1}, S_{F2})}}$	F $\underline{\overline{(S_{F1}, S_{F2})}}$

Case 6:

		District 2		
		$t_2 = I$	$t_2 = U$	$t_2 = F$
District 1	$t_1 = I$	I (S_{I1}, S_{I2})	U $\overline{(S_{U1}, S_{U2})}$	F $\underline{(S_{F1}, S_{F2})}$
	$t_1 = U$	U $\overline{(S_{U1}, S_{U2})}$	U $\overline{(S_{U1}, S_{U2})}$	F $\underline{(S_{F1}, S_{F2})}$
	$t_1 = F$	F $\underline{\overline{(S_{F1}, S_{F2})}}$	F $\underline{\overline{(S_{F1}, S_{F2})}}$	F $\underline{\overline{(S_{F1}, S_{F2})}}$

Case 7:

		District 2		
		$t_2 = I$	$t_2 = U$	$t_2 = F$
District 1	$t_1 = I$	I $\overline{(S_{I1}, S_{I2})}$	U (S_{U1}, S_{U2})	F $\underline{(S_{F1}, S_{F2})}$
	$t_1 = U$	U $\overline{(S_{U1}, S_{U2})}$	U $\overline{(S_{U1}, S_{U2})}$	F $\underline{(S_{F1}, S_{F2})}$
	$t_1 = F$	F $\overline{\underline{(S_{F1}, S_{F2})}}$	F $\overline{\underline{(S_{F1}, S_{F2})}}$	F $\overline{\underline{(S_{F1}, S_{F2})}}$

Case 8:

		District 2		
		$t_2 = I$	$t_2 = U$	$t_2 = F$
District 1	$t_1 = I$	I $\overline{(S_{I1}, S_{I2})}$	U (S_{U1}, S_{U2})	F $\underline{(S_{F1}, S_{F2})}$
	$t_1 = U$	U (S_{U1}, S_{U2})	U (S_{U1}, S_{U2})	F $\overline{\underline{(S_{F1}, S_{F2})}}$
	$t_1 = F$	F $\overline{\underline{(S_{F1}, S_{F2})}}$	F $\overline{\underline{(S_{F1}, S_{F2})}}$	F $\overline{\underline{(S_{F1}, S_{F2})}}$

Case 9:

		District 2		
		$t_2 = I$	$t_2 = U$	$t_2 = F$
District 1	$t_1 = I$	I (S_{I1}, S_{I2})	U (S_{U1}, S_{U2})	F $\overline{\underline{(S_{F1}, S_{F2})}}$
	$t_1 = U$	U (S_{U1}, S_{U2})	U (S_{U1}, S_{U2})	F $\overline{\underline{(S_{F1}, S_{F2})}}$
	$t_1 = F$	F $\overline{\underline{(S_{F1}, S_{F2})}}$	F $\overline{\underline{(S_{F1}, S_{F2})}}$	F $\overline{\underline{(S_{F1}, S_{F2})}}$

Proof of Lemma 1

For District 1: If $[\beta_U]^2 < \hat{\beta}_{U1}$ and $[\beta_F]^2 < \hat{\beta}_{F1}$, then this corresponds to the regions defined in Cases 1, 2, & 3 as shown in Figure 1. Under all three cases, the Integrated approach is preferred by District 1 as it provides higher surplus than the other two approaches, hence the Pareto efficient interoperability approach is Integrated. If $[\beta_U]^2 \geq max\{\hat{\beta}_{U1}, \hat{\beta}_{UF1}\}$, then this corresponds to the regions defined in Cases 4 & 5 as shown in Figure 1. Under both cases, the Unified approach provides higher surplus than the other two approaches, hence the Pareto efficient interoperability approach is Unified. If $[\beta_U]^2 < \hat{\beta}_{UF1}$ and $[\beta_F]^2 \geq \hat{\beta}_{F1}$, then this corresponds to the regions defined in Cases 6, 7, 8, & 9 as shown in Figure 1. Under all four cases, the Federated approach provides higher surplus than the other two approaches, hence the Pareto efficient interoperability approach is Federated.

For District 2: Similarly, if $[\beta_U]^2 < \hat{\beta}_{U2}$ and $[\beta_F]^2 < \hat{\beta}_{F2}$, then this corresponds to the regions defined in Cases 1, 2, 3, 4, 7, & 8 as shown in Figure 1. Under all six cases, the Integrated approach provides higher surplus than the other two approaches, hence the Pareto efficient interoperability approach is Integrated. If $[\beta_U]^2 \geq max\{\hat{\beta}_{U2}, \hat{\beta}_{UF2}\}$, then this corresponds to the regions defined in Cases 5 & 6 as shown in Figure 1. Under both cases, the Unified approach provides higher surplus than the other two approaches, hence the Pareto efficient interoperability approach is Unified. If $[\beta_U]^2 < \hat{\beta}_{UF2}$ and $[\beta_F]^2 \geq \hat{\beta}_{F2}$, then this corresponds to the regions defined in Case 9 as shown in Figure 1. Under Case 9, the Federated approach provides higher surplus than the other two approaches, hence the Pareto efficient interoperability approach is Federated.

Lemma 1 is obtained by combing the above results for both Districts.

Proof of Lemma 2

Lemma 2 is derived based on the definition of the equilibrium interoperability approach, which is determined by individual districts' preferences. The Integrated approach is the equilibrium if and only if both districts prefer the Integrated approach. The Unified approach is the equilibrium if both districts prefer the Unified approach or one district prefers Unified but the other prefers Integrated. The Federated approach is the equilibrium if either district prefers the Federated approach.

Proof of Proposition 1

The threshold values are given below:

$$\hat{\beta}_{U1} = \frac{[1-f_I]^2}{[1-f_U]^2} - \frac{4\bar{e}^2 p\delta[f_I - f_U][2-f_I-f_U]}{3[1-f_U]^2 {m_2}^2 \kappa^2},$$

$$\hat{\beta}_{F1} = 1 - \frac{1}{3}[2-f_I]f_I\left[3 + \frac{4\bar{e}^2 p\delta}{{m_2}^2\kappa^2}\right],$$

$$\hat{\beta}_{UF1} = \frac{1}{[1-f_U]^2}\hat{\beta}_{F1} + \frac{4\bar{e}^2 p\delta[2-f_U]f_U}{3[1-f_U]^2 {m_2}^2\kappa^2},$$

$$\hat{\beta}_{U2} = \frac{[1-f_I]^2}{[1-f_U]^2} - \frac{4\bar{e}^2 p\delta[f_I - f_U][2-f_I-f_U]}{3[1-f_U]^2 {m_1}^2 \kappa^2},$$

$$\hat{\beta}_{F2} = 1 - \frac{1}{3}[2-f_I]f_I\left[3 + \frac{4\bar{e}^2 p\delta}{{m_1}^2\kappa^2}\right],$$

$$\hat{\beta}_{UF2} = \frac{1}{[1-f_U]^2}\hat{\beta}_{F2} + \frac{4\bar{e}^2 p\delta[2-f_U]f_U}{3[1-f_U]^2 {m_1}^2\kappa^2}.$$

Comparing the values of the thresholds, we find:

$$\hat{\beta}_{U1} < \hat{\beta}_{U2}, \hat{\beta}_{F1} < \hat{\beta}_{F2} \text{ and } \hat{\beta}_{UF1} > \hat{\beta}_{UF2}. \text{ (As } m_1 > m_2\text{)}$$

If $[\beta_U]^2 < \hat{\beta}_{U1}$ & $[\beta_U]^2 < \hat{\beta}_{F1}$, then both Districts prefer the Integrated approach and Integrated is the equilibrium.

If $[\beta_U]^2 > \hat{\beta}_{U1}$ & $[\beta_U]^2 > \hat{\beta}_{UF1}$, then District 1 prefers the Unified approach, District 2 prefers either the Integrated (i.e.,$\hat{\beta}_{U1} < [\beta_U]^2 < \hat{\beta}_{U2}$) or Unified approach (i.e., $[\beta_U]^2 > \hat{\beta}_{U2}$). By definition Unified is the equilibrium.

If $[\beta_U]^2 > \hat{\beta}_{F1}$ & $[\beta_U]^2 < \hat{\beta}_{UF1}$, then District 1 prefers the Federated approach. By definition Federated is the equilibrium.

Based on the above, the Pareto efficient equilibrium is determined by District 1's preferences only. Hence $\hat{\beta}_U^{Eqm} = \hat{\beta}_{U1}$, $\hat{\beta}_F^{Eqm} = \hat{\beta}_{F1}$, and $\hat{\beta}_{UF}^{Eqm} = \hat{\beta}_{UF1}$.

Proof of Corollary 1

Comparative statics of f_I and f_U:

$$\frac{\partial \hat{\beta}_U^{Eqm}}{\partial f_I} = -\frac{2[1-f_I][4\bar{e}^2p\delta+3{m_2}^2\kappa^2]}{3[1-f_U]^2{m_2}^2\kappa^2} < 0,$$

$$\frac{\partial \hat{\beta}_U^{Eqm}}{\partial f_U} = \frac{2[1-f_I]^2[4\bar{e}^2p\delta+3{m_2}^2\kappa^2]}{3[1-f_U]^3{m_2}^2\kappa^2} > 0,$$

$$\frac{\partial \hat{\beta}_F^{Eqm}}{\partial f_I} = -\frac{2}{3}[1-f_I]\left[3+\frac{4\bar{e}^2p\delta}{{m_2}^2\kappa^2}\right] < 0,$$

$$\frac{\partial \hat{\beta}_F^{Eqm}}{\partial f_U} = 0,$$

$$\frac{\partial \hat{\beta}_{UF}^{Eqm}}{\partial f_I} = 0.$$

The slope of $\hat{\beta}_{UF}^{Eqm}$ is $\frac{1}{[1-f_U]^2}$, which increases with the increase of f_U.

The intercept of $\hat{\beta}_{UF}^{Eqm}$ is $\frac{4\bar{e}^2p\delta[2-f_U]f_U}{3[1-f_U]^2{m_2}^2\kappa^2}$ and $\frac{\partial \frac{4\bar{e}^2p\delta[2-f_U]f_U}{3[1-f_U]^2{m_2}^2\kappa^2}}{\partial f_U} = \frac{8\bar{e}^2p\delta}{3[1-f_U]^3{m_2}^2\kappa^2} > 0.$

Based on the signs of the comparative statics above, we obtain results about the impact of f_I and f_U as reported in Corollary 1(i).

Comparative statics of κ:

$$\frac{\partial \hat{\beta}_U^{Eqm}}{\partial \kappa} = \frac{8\bar{e}^2p\delta[2-f_I-f_U]}{3[1-f_U]^2{m_2}^2\kappa^3} > 0,$$

$$\frac{\partial \hat{\beta}_F^{Eqm}}{\partial \kappa} = \frac{8\bar{e}^2p\delta[2-f_I]}{3{m_2}^2\kappa^3} > 0,$$

$$\frac{\partial \hat{\beta}_{UF}^{Eqm}}{\partial \kappa} = -\frac{8\bar{e}^2p\delta[2-f_U]f_U}{3[1-f_U]^2{m_2}^2\kappa^3} < 0.$$

Based on the signs of the comparative statics above, we obtain results about the impact of κ as reported in Corollary 1(ii).

Proof of Lemma 3

The socially optimal interoperability approach is obtained by comparing social welfare across the three approaches. The social welfare of each interoperability approach is given below:

Integrated Approach:

$$\mathrm{SW}_I = S_{I1} + S_{I2} = \frac{[1-f_I]^2[1-\kappa]^2[2\bar{e}^2[m_1^2+m_2^2]p\delta + 3m_1^2m_2^2\kappa^2]}{8\bar{e}^2p^2\delta}.$$

Unified Approach:

$$\mathrm{SW}_U = S_{U1} + S_{U2} = \frac{[1-f_U]^2[1-\kappa]^2[2\bar{e}^2[m_1^2+m_2^2]p\delta + 3m_1^2m_2^2\beta_U^2\kappa^2]}{8\bar{e}^2p^2\delta}.$$

Federated Approach:

$$\mathrm{SW}_F = S_{F1} + S_{F2} = \frac{[1-\kappa]^2[2\bar{e}^2[m_1^2+m_2^2]p\delta + 3m_1^2m_2^2\beta_F^2\kappa^2]}{8\bar{e}^2p^2\delta}.$$

$\hat{\beta}_U^{SW}$ is obtained by comparing SW_I and SW_U (i.e., solve for $[\beta_U]^2$ such that $\mathrm{SW}_I = \mathrm{SW}_U$):

$$\hat{\beta}_U^{SW} = \frac{3[1-f_I]^2 + \dfrac{2\bar{e}^2[f_I-f_U][f_I+f_U-2][m_1^2+m_2^2]p\delta}{m_1^2m_2^2\kappa^2}}{3[1-f_U]^2}.$$

$\hat{\beta}_F^{SW}$ is obtained by comparing SW_I and SW_F (i.e., solve for $[\beta_F]^2$ such that $\mathrm{SW}_I = \mathrm{SW}_F$):

$$\hat{\beta}_F^{SW} = 1 - \frac{1}{3}[2-f_I]f_I\left[3 + \frac{2\bar{e}^2[m_1^2+m_2^2]p\delta}{m_1^2m_2^2\kappa^2}\right].$$

$\hat{\beta}_{UF}^{SW}$ is obtained by comparing SW_U and SW_F (i.e., solve for $[\beta_U]^2$ such that $\mathrm{SW}_U = \mathrm{SW}_F$):

$$\hat{\beta}_{UF}^{SW} = \frac{3\beta_F^2 + \dfrac{2\bar{e}^2[2-f_U]f_U[m_1^2+m_2^2]p\delta}{m_1^2m_2^2\kappa^2}}{3[1-f_U]^2}.$$

If $[\beta_U]^2 < \hat{\beta}_U^{SW}$ and $[\beta_F]^2 < \hat{\beta}_F^{SW}$, then $\mathrm{SW}_I > \mathrm{SW}_U$ and $\mathrm{SW}_I > \mathrm{SW}_F$, indicating that the socially optimal approach is Integrated.

If $[\beta_U]^2 \geq \max\{\hat{\beta}_U^{SW}, \hat{\beta}_{UF}^{SW}\}$, then $\mathrm{SW}_I < \mathrm{SW}_U$ and $\mathrm{SW}_F < \mathrm{SW}_U$, indicating that the socially optimal approach is Unified.

If $[\beta_U]^2 < \hat{\beta}_{UF}^{SW}$ and $[\beta_F]^2 \geq \hat{\beta}_F^{SW}$, then $\mathrm{SW}_U < \mathrm{SW}_F$ and $\mathrm{SW}_I < \mathrm{SW}_F$, indicating that the socially optimal approach is Federated.

Proof of Proposition 2

The results are obtained by comparing $\hat{\beta}_U^{SW}$ with $\hat{\beta}_U^{Eqm}$, $\hat{\beta}_F^{SW}$ with $\hat{\beta}_F^{Eqm}$, and $\hat{\beta}_{UF}^{SW}$ with $\hat{\beta}_{UF}^{Eqm}$.

$$\hat{\beta}_U^{Eqm} = \frac{3[1-f_I]^2 + \dfrac{4\bar{e}^2[f_I-f_U][f_I+f_U-2]p\delta}{m_2^2\kappa^2}}{3[1-f_U]^2},$$

$$\hat{\beta}_F^{Eqm} = 1 - \frac{1}{3}[2-f_I]f_I\left[3 + \frac{4\bar{e}^2p\delta}{m_2^2\kappa^2}\right],$$

$$\hat{\beta}_{UF}^{Eqm} = \frac{3\beta_F^2 + \dfrac{2\bar{e}^2[2-f_U]f_U[m_1^2+m_2^2]p\delta}{m_1^2m_2^2\kappa^2}}{3[1-f_U]^2}.$$

Comparing the socially optimal thresholds with equilibrium thresholds we obtain

$$\hat{\beta}_U^{SW} - \hat{\beta}_U^{Eqm} = \frac{2\bar{e}^2[f_I-f_U][2-f_I-f_U][m_1^2-m_2^2]p\delta}{3[1-f_U]^2m_1^2m_2^2\kappa^2} > 0,$$

$$\hat{\beta}_F^{SW} - \hat{\beta}_F^{Eqm} = \frac{2\bar{e}^2[2-f_I]f_I[m_1^2-m_2^2]p\delta}{3m_1^2m_2^2\kappa^2} > 0,$$

$$\hat{\beta}_{UF}^{SW} - \hat{\beta}_{UF}^{Eqm} = -\frac{2\bar{e}^2[2-f_U]f_U[m_1^2-m_2^2]p\delta}{3[1-f_U]^2m_1^2m_2^2\kappa^2} < 0.$$

From the above, we obtain $\hat{\beta}_U^{SW} > \hat{\beta}_U^{Eqm}$, $\hat{\beta}_F^{SW} > \hat{\beta}_F^{Eqm}$, and $\hat{\beta}_{UF}^{SW} < \hat{\beta}_{UF}^{Eqm}$.

Proof of Corollary 2

Comparative statics of f_I and f_U on the socially optimal interoperability approaches:

$$\frac{\partial \hat{\beta}_U^{SW}}{\partial f_I} = -\frac{2[1-f_I][2\bar{e}^2 p\delta[m_1{}^2+m_2{}^2]+3m_1{}^2m_2{}^2\kappa^2]}{3[1-f_U]^2 m_1{}^2 m_2{}^2\kappa^2} < 0,$$

$$\frac{\partial \hat{\beta}_U^{SW}}{\partial f_U} = \frac{2[1-f_I]^2[2\bar{e}^2 p\delta[m_1{}^2+m_2{}^2]+3m_1{}^2m_2{}^2\kappa^2]}{3[1-f_U]^3 m_1{}^2 m_2{}^2\kappa^2} > 0,$$

$$\frac{\partial \hat{\beta}_F^{SW}}{\partial f_I} = -\frac{2}{3}[1-f_I]\left[3+\frac{2\bar{e}^2 p\delta[m_1{}^2+m_2{}^2]}{m_1{}^2 m_2{}^2\kappa^2}\right] < 0,$$

$$\frac{\partial \hat{\beta}_F^{SW}}{\partial f_U} = 0,$$

$$\frac{\partial \hat{\beta}_{UF}^{SW}}{\partial f_I} = 0.$$

The slope of $\hat{\beta}_{UF}^{SW}$ is $\frac{1}{[1-f_U]^2}$, which increases with the increase of f_U.

The intercept of $\hat{\beta}_{UF}^{SW}$ is $\frac{2\bar{e}^2 p\delta f_U[2-f_U][m_1{}^2+m_2{}^2]}{3[1-f_U]^2 m_1{}^2 m_2{}^2\kappa^2}$ and $\frac{\partial \frac{2\bar{e}^2 p\delta f_U[2-f_U][m_1{}^2+m_2{}^2]}{3[1-f_U]^2 m_1{}^2 m_2{}^2\kappa^2}}{\partial f_U} = \frac{4\bar{e}^2 p\delta[m_1{}^2+m_2{}^2]}{3[1-f_U]^3 m_1{}^2 m_2{}^2\kappa^2} > 0.$

Based on the signs of the comparative statics above, we obtain the results about the impact of f_I and f_U as reported in Corollary 2(i).

Comparative statics of κ on the socially optimal interoperability approaches:

$$\frac{\partial \hat{\beta}_U^{SW}}{\partial \kappa} = \frac{4\bar{e}^2 p\delta[f_I-f_U][2-f_I-f_U][m_1{}^2+m_2{}^2]}{3[1-f_U]^2 m_1{}^2 m_2{}^2\kappa^3} > 0,$$

$$\frac{\partial \hat{\beta}_F^{SW}}{\partial \kappa} = \frac{4\bar{e}^2 p\delta f_I[2-f_I][m_1{}^2+m_2{}^2]}{3m_1{}^2 m_2{}^2\kappa^3} > 0,$$

$$\frac{\partial \hat{\beta}_{UF}^{SW}}{\partial \kappa} = -\frac{4\bar{e}^2 p\delta f_U[2-f_U][m_1{}^2+m_2{}^2]}{3[1-f_U]^2 m_1{}^2 m_2{}^2\kappa^3} < 0.$$

Based on the signs of the comparative statics above, we obtain the results about the impact of κ as reported in Corollary 2(ii).

Proof of Lemma 4

A social planner should find the optimal incentive mechanism that achieves social optimum while minimizing the total administrative effort under the federated approach. That is

$$\min_{x_{ij}}[|x_{F1}| + |x_{F2}|].$$

The incentive mechanism must satisfy the following conditions:

$$x_{I1} - x_{F1} = \frac{[2-f_I]f_I[m_1^2-m_2^2][1-\kappa]^2}{8p},$$

$$x_{U1} - x_{F1} = \frac{[2-f_U]f_U[m_1^2-m_2^2][1-\kappa]^2}{8p},$$

$$x_{I2} - x_{F2} = -\frac{[2 - f_I]f_I[m_1^2 - m_2^2][1 - \kappa]^2}{8p},$$

$$x_{U2} - x_{F2} = -\frac{[2 - f_U]f_U[m_1^2 - m_2^2][1 - \kappa]^2}{8p}.$$

When both tax and subsidy are available to a social planner, a district shall receive no tax as a penalty nor subsidy as an incentive if they choose the Federated approach (i.e., $x_{F1} = x_{F2} = 0$). This way the social planner avoids its involvement as much as possible. As for any non-zero tax/subsidy, a social planner must rely on extra funds or incur extra expenses to induce the social optimum, which is not desirable. Given that $x_{F1} = x_{F2} = 0$, we solve for the amount of tax/subsidy given to each district.

$$x_{I1} = \frac{[2 - f_I]f_I[m_1^2 - m_2^2][1 - \kappa]^2}{8p} > 0,$$

$$x_{U1} = \frac{[2 - f_U]f_U[m_1^2 - m_2^2][1 - \kappa]^2}{8p} > 0,$$

$$x_{I2} = -x_{I1} = -\frac{[2 - f_I]f_I[m_1^2 - m_2^2][1 - \kappa]^2}{8p} < 0,$$

$$x_{U2} = -x_{U1}\frac{[2 - f_U]f_U[m_1^2 - m_2^2][1 - \kappa]^2}{8p} < 0.$$

Proof of Proposition 3

Based on the results in Lemma 4, we have:

$$x_{I1} + x_{I2} = \frac{[2 - f_I]f_I[m_1^2 - m_2^2][1 - \kappa]^2}{8p} - \frac{[2 - f_I]f_I[m_1^2 - m_2^2][1 - \kappa]^2}{8p} = 0,$$

$$x_{U1} + x_{U2} = \frac{[2 - f_U]f_U[m_1^2 - m_2^2][1 - \kappa]^2}{8p} - \frac{[2 - f_U]f_U[m_1^2 - m_2^2][1 - \kappa]^2}{8p} = 0.$$

$$x_{F1} + x_{F2} = 0 + 0 = 0$$

Because $1 > f_I > f_U > 0$, simple algebra indicates $[2 - f_I]f_I > [2 - f_U]f_U$, hence we have $x_{I1} > x_{U1} > x_{F1} = 0$ and $x_{I2} < x_{U2} < x_{F2} = 0$.

Proof of Lemma 5

To induce the social optimum, the incentive mechanism must satisfy the same conditions as shown in the proof of Lemma 4. When a subsidy is the only available incentive mechanism, a social planner should find the optimal incentive mechanism that achieves social optimum while minimizing the total incentives provided to both districts under all three approaches. That is

$$\min_{x_{ij}}[x_{I1} + x_{I2} + x_{U1} + x_{U2} + x_{F1} + x_{F2}]$$

Subject to:

$$x_{I1} - x_{F1} = \frac{[2 - f_I]f_I[m_1^2 - m_2^2][1 - \kappa]^2}{8p}$$

$$x_{U1} - x_{F1} = \frac{[2 - f_U] f_U [m_1^2 - m_2^2][1 - \kappa]^2}{8p}$$

$$x_{I2} - x_{F2} = -\frac{[2 - f_I] f_I [m_1^2 - m_2^2][1 - \kappa]^2}{8p}$$

$$x_{U2} - x_{F2} = -\frac{[2 - f_U] f_U [m_1^2 - m_2^2][1 - \kappa]^2}{8p}$$

$$x_{I1},\ x_{I2}, x_{U1},\ x_{U2}, x_{F1}, x_{F2} \geq 0.$$

Solving the above minimization problem leads to the solutions we reported in Lemma 5:

$$x_{I1} = \frac{[2 - f_I] f_I [m_1^2 - m_2^2][1 - \kappa]^2}{8p},$$

$$x_{I2} = 0,$$

$$x_{U1} = \frac{[2 - f_U] f_U [m_1^2 - m_2^2][1 - \kappa]^2}{8p},$$

$$x_{U2} = \frac{\big[[2 - f_I] f_I - [2 - f_U] f_U\big][m_1^2 - m_2^2][1 - \kappa]^2}{8p},$$

$$x_{F1} = 0,$$

$$x_{F2} = \frac{[2 - f_I] f_I [m_1^2 - m_2^2][1 - \kappa]^2}{8p}.$$

Proof of Proposition 4

(i) is obtained based on the results we obtained in Lemma 5: i.e., $x_{I1} > 0$, $x_{U1} > 0$, $x_{F1} = 0$, and $x_{F2} > 0$, $x_{U2} > 0$, $x_{I2} = 0$.

(ii) is obtained based on the following:

$$x_{I1} + x_{I2} = \frac{[2 - f_I] f_I [m_1^2 - m_2^2][1 - \kappa]^2}{8p} > 0,$$

$$x_{U1} + x_{U2} = \frac{[2 - f_I] f_I [m_1^2 - m_2^2][1 - \kappa]^2}{8p} > 0,$$

$$x_{F1} + x_{F2} = \frac{[2 - f_I] f_I [m_1^2 - m_2^2][1 - \kappa]^2}{8p} > 0.$$

And hence $x_{I1} + x_{I2} = x_{U1} + x_{U2} = x_{F1} + x_{F2} > 0$.

(iii) Given that $0 < f_U < f_I < 1$, simple calculation indicates $x_{I1} > x_{U1} > x_{F1} = 0$ and $x_{F2} > x_{U2} > x_{I2} = 0$.

Proof of Lemma 6 and Proposition 5

Total subsidy needed for each approach to induce social optimum is given by:

$$x_{I1} + x_{I2} = x_{U1} + x_{U2} = x_{F1} + x_{F2} = \frac{[2 - f_I]f_I[m_1^2 - m_2^2][1 - \kappa]^2}{8p}.$$

We compare this amount to the social gain and see if a subsidy can be justified.

Case 1: When the social optimum is Integrated, but the equilibrium is Unified (the horizontally shaded area in Figure 4), the social gain is given by:

$$S_I(x_{I1}, x_{I2}) - S_U(x_{U1}, x_{U2}) = \frac{[1-\kappa]^2[2\bar{e}^2[f_I - f_U][f_I + f_U - 2][m_1^2 + m_2^2]p\delta + 3m_1^2m_2^2[[1-f_I]^2 - [1-f_U]^2\beta_U^2]\kappa^2]}{8\bar{e}^2p^2\delta}.$$

The separating thresholds for the cost effectiveness of the incentive mechanism is obtained by comparing the social gain with the total subsidy:

$$\hat{\beta}_U^{SG} = \frac{3[1-f_I]^2 + \frac{\bar{e}^2p\delta\left[[3[f_I - 2]f_I - 2[f_U - 2]f_U]m_1^2 + \left[[f_I - 2]f_I - 2[f_U - 2]f_U\right]m_2^2\right]}{m_1^2m_2^2\kappa^2}}{3[1-f_U]^2},$$

$S_I(x_{I1}, x_{I2}) - S_U(x_{U1}, x_{U2}) > x_{I1} + x_{I2}$ if $[\beta_U]^2 < \hat{\beta}_U^{SG}$

To show that the social gain is enough to cover the total subsidy, we need to show $[\beta_U]^2 < \hat{\beta}_U^{SG}$ of all valid values of $[\beta_U]^2$ within the horizontally shaded area of Figure 4. The necessary and sufficient condition for $[\beta_U]^2 < \hat{\beta}_U^{SG}$ is to show that $\hat{\beta}_U^{SG} > \hat{\beta}_U^{SW}$:

$$\hat{\beta}_U^{SG} - \hat{\beta}_U^{SW} = -\frac{\bar{e}^2[2-f_I]f_I[m_1^2 - m_2^2]p\delta}{3[1-f_U]^2m_1^2m_2^2\kappa^2} < 0.$$

This indicates that the social gain cannot justify the total subsidy for all valid values of $[\beta_U]^2$. To see if the social gain is enough to justify a portion of the horizontally shaded area in Figure 4, we then compare $\hat{\beta}_U^{SG}$ with $\hat{\beta}_U^{Eqm}$:

$$\hat{\beta}_U^{SG} - \hat{\beta}_U^{Eqm} = \frac{\bar{e}^2\left[[2-f_I]f_I - 2[2-f_U]f_U\right][m_1^2 - m_2^2]p\delta}{3[1-f_U]^2m_1^2m_2^2\kappa^2}$$

$\hat{\beta}_U^{SG} - \hat{\beta}_U^{Eqm} \geq 0$ if $[2-f_I]f_I - 2[2-f_U]f_U \geq 0$, otherwise $\hat{\beta}_U^{SG} - \hat{\beta}_U^{Eqm} < 0$.

If $\frac{f_I[2-f_I]}{f_U[2-f_U]} \geq 2$, then $\hat{\beta}_U^{Eqm} < \hat{\beta}_U^{SG} < \hat{\beta}_U^{SW}$, this in turn indicates that within the horizontally shaded region, total subsidy can be justified by social gain if $\hat{\beta}_U^{Eqm} < [\beta_U]^2 < \hat{\beta}_U^{SG}$. Otherwise, total subsidy cannot be justified by social gain.

Case 2: When social optimum is Integrated, but the equilibrium is Federated (the vertically shaded area in Figure 4), the social gain is given by:

$$S_I(x_{I1}, x_{I2}) - S_F(x_{F1}, x_{F2}) = \frac{[1-\kappa]^2[2\bar{e}^2[f_I - 2][m_1^2 + m_2^2]p\delta + 3m_1^2m_2^2[[1-f_I]^2 - \beta_F^2]\kappa^2]}{8\bar{e}^2p^2\delta}.$$

The separating thresholds for the cost effectiveness of the incentive mechanism is obtained by comparing the social gain with the total subsidy:

$$\hat{\beta}_F^{SG} = 1 - \frac{1}{3}[2-f_I]f_I\left[3 + \frac{\bar{e}^2[3m_1^2 + m_2^2]p\delta}{m_1^2m_2^2\kappa^2}\right]$$

$$S_I(x_{I1}, x_{I2}) - S_F(x_{F1}, x_{F2}) > x_{F1} + x_{F2} \text{ if } [\beta_F]^2 < \hat{\beta}_F^{SG}.$$

To show that the social gain is enough to cover the total subsidy, we need to show $[\beta_F]^2 < \hat{\beta}_F^{SG}$ for all valid values of $[\beta_F]^2$ within the vertically shaded area of Figure 4. The necessary and sufficient condition for $[\beta_F]^2 < \hat{\beta}_F^{SG}$ is to show that $\hat{\beta}_F^{SG} > \hat{\beta}_F^{SW}$:

$$\hat{\beta}_F^{SG} - \hat{\beta}_F^{SW} = -\frac{\bar{e}^2[2-f_I]f_I[m_1^2 - m_2^2]p\delta}{3m_1^2 m_2^2 \kappa^2} < 0.$$

This indicates that the social gain cannot justify the total subsidy for all values of $[\beta_F]^2$. To see if the social gain is enough to justify a portion of the vertically shaded area in Figure 4, we then compare $\hat{\beta}_F^{SG}$ with $\hat{\beta}_F^{Eqm}$:

$$\hat{\beta}_F^{SG} - \hat{\beta}_F^{Eqm} = \frac{\bar{e}^2[2-f_I]f_I[m_1^2 - m_2^2]p\delta}{3m_1^2 m_2^2 \kappa^2} > 0.$$

The above results suggest $\hat{\beta}_F^{Eqm} < \hat{\beta}_F^{SG} < \hat{\beta}_F^{SW}$ is always true. As a result, if $\hat{\beta}_F^{Eqm} < [\beta_F]^2 < \hat{\beta}_F^{SG}$, then the social gain is sufficient to cover the total subsidy, otherwise the social gain is not sufficient to cover the total subsidy.

Case 3: When social optimum is Unified, but the equilibrium is Federated (the diagonally shaded area in Figure 4), the social gain is given by:

$$S_U(x_{U1}, x_{U2}) - S_F(x_{F1}, x_{F2}) = \frac{[1-\kappa]^2[2\bar{e}^2[f_U - 2]f_U[m_1^2 + m_2^2]p\delta + 3m_1^2 m_2^2[[1-f_U]^2\beta_U^2 - \beta_F^2]\kappa^2]}{8\bar{e}^2 p^2 \delta}.$$

The separating thresholds for the cost effectiveness of the incentive mechanism is obtained by comparing the social gain with the total subsidy:

$$\hat{\beta}_{UF}^{SG} = \frac{3\beta_F^2 + \dfrac{\bar{e}^2 p\delta\left[[[2-f_I]f_I - 2[2-f_U]f_U]m_1^2 + [[f_I - 2]f_I - 2[f_U - 2]f_U]m_2^2\right]}{m_1^2 m_2^2 \kappa^2}}{3[1-f_U]^2}$$

$$S_U(x_{U1}, x_{U2}) - S_F(x_{F1}, x_{F2}) > x_{U1} + x_{U2} \text{ if } [\beta_U]^2 > \hat{\beta}_{UF}^{SG}.$$

To show that the social gain is enough to cover the total subsidy, we need to show $[\beta_U]^2 > \hat{\beta}_{UF}^{SG}$ for all valid values of $[\beta_U]^2$ within the diagonally shaded area of Figure 4. The necessary and sufficient condition for $[\beta_U]^2 > \hat{\beta}_{UF}^{SG}$ is to show that $\hat{\beta}_{UF}^{SG} < \hat{\beta}_{UF}^{SW}$:

$$\hat{\beta}_{UF}^{SG} - \hat{\beta}_{UF}^{SW} = \frac{\bar{e}^2[2-f_I]f_I[m_1^2 - m_2^2]p\delta}{3m_1^2 m_2^2 \kappa^2} > 0.$$

The above result indicates that the social gain cannot justify the total subsidy for all values of $[\beta_U]^2$. To see if the social gain is enough to justify a portion of the diagonally shaded area in Figure 4, we then compare $\hat{\beta}_{UF}^{SG}$ with $\hat{\beta}_{UF}^{Eqm}$:

$$\hat{\beta}_{UF}^{Eqm} - \hat{\beta}_{UF}^{SG} = -\frac{\bar{e}^2[[2-f_I]f_I - 2[2-f_U]f_U][m_1^2 - m_2^2]p\delta}{3[1-f_U]^2 m_1^2 m_2^2 \kappa^2}$$

$$\hat{\beta}_U^{Eqm} - \hat{\beta}_U^{SG} \geq 0 \text{ if } [2-f_I]f_I - 2[2-f_U]f_U \leq 0, \text{ otherwise } \hat{\beta}_U^{Eqm} - \hat{\beta}_U^{SG} < 0.$$

If $\frac{f_I[2-f_I]}{f_U[2-f_U]} \geq 2$, then the social gain is sufficient to cover the total subsidy when $\hat{\beta}_{UF}^{Eqm} < [\beta_U]^2 < \hat{\beta}_{UF}^{SG}$.

If $\frac{f_I[2-f_I]}{f_U[2-f_U]} < 2$, then the social gain is sufficient to cover the total subsidy when $\hat{\beta}_{UF}^{SG} < [\beta_U]^2 < \hat{\beta}_{UF}^{Eqm}$.

The social gain is not sufficient to cover the total subsidy for all other cases.

When we combine the results of Cases 1-3 together, we obtain Lemma 6 and Propositions 5.

Proof of Proposition 6

First, focusing on the interior solutions, we obtain the new threshold values under the impact of initial interoperability I_0:

$$\hat{\beta}_{U1-new} = \frac{[1-f_I]^2}{[1-f_U]^2} - \frac{4\bar{e}^2 p\delta[f_I-f_U][m_1[2-f_I-f_U][1-\kappa]+2I_0 m_2\kappa]}{3m_1 m_2{}^2[1-f_U]^2[1-\kappa]\kappa^2},$$

$$\hat{\beta}_{F1-new} = [1-f_I]^2 - \frac{4\bar{e}^2 p\delta f_I[m_1[2-f_I][1-\kappa]+2I_0 m_2\kappa]}{3m_1 m_2{}^2[1-\kappa]\kappa^2},$$

$$\hat{\beta}_{UF1-new} = \frac{1}{[1-f_U]^2}\hat{\beta}_{F1-new} + \frac{4\bar{e}^2 p\delta f_U[m_1[2-f_U][1-\kappa]+2I_0 m_2\kappa]}{3m_1 m_2{}^2[1-f_U]^2[1-\kappa]\kappa^2},$$

$$\hat{\beta}_{U2-new} = \frac{[1-f_I]^2}{[1-f_U]^2} - \frac{4\bar{e}^2 p\delta[f_I-f_U][m_2[2-f_I-f_U][1-\kappa]+2I_0 m_1\kappa]}{3m_1{}^2 m_2[1-f_U]^2[1-\kappa]\kappa^2},$$

$$\hat{\beta}_{F2-new} = [1-f_I]^2 - \frac{4\bar{e}^2 p\delta f_I[m_2[2-f_I][1-\kappa]+2I_0 m_1\kappa]}{3m_1{}^2 m_2[1-\kappa]\kappa^2},$$

$$\hat{\beta}_{UF2-new} = \frac{1}{[1-f_U]^2}\hat{\beta}_{F2-new} + \frac{4\bar{e}^2 p\delta f_U[m_2[2-f_U][1-\kappa]+2I_0 m_1\kappa]}{3m_1{}^2 m_2[1-f_U]^2[1-\kappa]\kappa^2}.$$

When we compare the new threshold values with the old threshold values (as given in the proof of Proposition 1) we obtain:

$$\hat{\beta}_{U1-new} < \hat{\beta}_{U1},\ \hat{\beta}_{F1-new} < \hat{\beta}_{F1}, \text{ and } \hat{\beta}_{UF1-new} > \hat{\beta}_{UF1};$$

$$\hat{\beta}_{U2-new} < \hat{\beta}_{U2},\ \hat{\beta}_{F2-new} < \hat{\beta}_{F2}, \text{ and } \hat{\beta}_{UF2-new} > \hat{\beta}_{UF2}.$$

This indicate the horizontal line moves down; the vertical line moves left, and the diagonal line moves up towards the upper left corner.

When we compare the new threshold values between District 1 and District 2 we obtain:

$$\hat{\beta}_{U1-new} < \hat{\beta}_{U2-new}, \hat{\beta}_{F1-new} < \hat{\beta}_{F2-new} \text{ and } \hat{\beta}_{UF1-new} > \hat{\beta}_{UF2-new}.$$

Recall that when the initial interoperability is not considered, we also have:

$$\hat{\beta}_{U1} < \hat{\beta}_{U2}, \hat{\beta}_{F1} < \hat{\beta}_{F2} \text{ and } \hat{\beta}_{UF1} > \hat{\beta}_{UF2}.$$

This indicates that the relative positions between the two districts' preferences remain the same with or without the influence of initial interoperability. This proves all the results reported in Proposition 6.

Next, we focus on the case of boundary solution (i.e., the initial interoperability is large), we then solve for the optimal e_{ij} and g_{ij} under the new constraint $I_0 + \beta_i\left[\frac{e_{i1}+e_{i2}}{\bar{e}}\right] = 1$. Here we use the maximization problem for unified approach as a demonstration, the results for the other two interoperability approaches can be derived in a similar way.

Under the unified approach, the individual district's decision problem is

$$\max S_{Uj}\left(g_{Uj}, e_{Uj}\right) = m_j\left[[1-\kappa][1-f_U]g_{Uj} + \kappa\left[I_0 + \beta_U\left[\frac{e_{Uj}+e_{U\backslash j}}{\bar{e}}\right]\right]g_{U\backslash j}\right] - pg_{Uj}^2 - \delta e_{Uj}^2$$

Subject to $0 \le e_{Uj} \le \bar{e}$, $0 \le g_{Uj} \le \bar{g}$, $0 \le I_0 + \beta_U\left[\frac{e_{Uj}+e_{U\backslash j}}{\bar{e}}\right] = 1$, where $j \in \{1, 2\}$.

Solve for the above maximization problem under the new binding constraint, we obtain:

$$g_{U1} = \frac{m_1(1-f_U)(1-\kappa)}{2p}, g_{U2} = \frac{m_2(1-f_U)(1-\kappa)}{2p}, e_{U1} = e_{U2} = \frac{\bar{e}(1-I_0)}{2\beta_U}.$$

Here we can see that if the initial interoperability is very high (e.g., $I_0 = 1$), then the optimal effort level should be zero for both districts.

The boundary solution surplus for each district is

$$S_{U1-B} = \frac{1}{4}\left[\frac{m_1[1-f_U][1-\kappa][m_1[1-f_U][1-\kappa]+2m_2\kappa]}{p} - \frac{\bar{e}^2\delta[1-I_0]^2}{{\beta_U}^2}\right],$$

$$S_{U2-B} = \frac{1}{4}\left[\frac{m_2[1-f_U][1-\kappa][m_2[1-f_U][1-\kappa]+2m_1\kappa]}{p} - \frac{\bar{e}^2\delta[1-I_0]^2}{{\beta_U}^2}\right].$$

Here we use subscript B to denote Boundary solution.

The interior solution surplus for each district is

$$S_{U1} = \frac{m_1[1-f_U][1-\kappa]\left[3m_1m_2^2\beta_U^2[1-f_U][1-\kappa]\kappa^2 + 4\bar{e}^2p\delta[m_1[1-f_U][1-\kappa]+2I_0m_2\kappa]\right]}{16\bar{e}^2p\delta},$$

$$S_{U2} = \frac{m_2[1-f_U][1-\kappa]\left[3m_1^2m_2\beta_U^2[1-f_U][1-\kappa]\kappa^2 + 4\bar{e}^2p\delta[m_2[1-f_U][1-\kappa]+2I_0m_1\kappa]\right]}{16\bar{e}^2p\delta}.$$

Comparing the boundary solution's surpluses with interior solution's surpluses for each district, we derive the conditions for boundary solution. We find that the boundary solution's condition is the same for both districts (i.e., both districts prefer the boundary solution or both districts prefer the interior solution simultaneously), in other words, the case whereby one district prefers the boundary solution and the other district prefers the interior solution does not exist. Specifically, when $I_0 \leq 1 - \frac{3m_1m_2\beta_i^2\kappa(1-\kappa)(1-f_i)}{2\bar{e}p\delta}$, then the interior solution provides higher surplus to both districts and hence is the equilibrium, otherwise, the boundary solution is the equilibrium.