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Join Up or Stay Away? Coalition Formation for Critical IT Infrastructure

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We consider districts that invest in critical IT infrastructure which spillover to other districts. IT Infrastructure is considered critical if its disruption can cause significant damage to security, the economy, public health, or safety. Districts choose whether to participate in a coalition noncooperatively and the coalition subsequently makes resource investment decisions cooperatively. Districts 'inside' the coalition have superior interoperability in the spillovers relative to 'outside' districts. Inside districts also benefit from a coalition economy of scale, a discounted resource investment cost, and face diseconomies of scope in the number of coalition members and their investment levels. Coalition structures include grand, partial (varying in size), minimal (two members), and singleton, and we consider the formation of only one coalition. We find that inside districts' resource levels decrease with coalition size. Even with homogeneous districts, any size coalition can be an equilibrium depending on the coalition economy of scale and the relative interoperability of resources in inside versus outside districts. The equilibrium coalition size is increasing in the economy of scale and decreasing in relative interoperability. Similarly, any size coalition can be socially optimal depending on the coalition economy of scale and relative interoperability. The socially optimal coalition size is also increasing in the economy of scale and decreasing in relative interoperability. In most cases the socially optimal coalition size is larger than the equilibrium coalition. A subsidy or tax can incentivize the equilibrium coalition size and district resource levels to be socially optimal, providing a general solution to the provisioning of critical IT infrastructure. We use the European Union's Digital COVID Certificate program providing vaccine status information and the United States Government's Direct Project that supports the establishment of nationwide health information exchanges to illustrate elements of our model.

Key words: Coalition formation, Critical IT infrastructure, Organizational economics, Interoperability, Public policy.

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1. Introduction

Critical IT infrastructure differs from basic IT infrastructure in how damaging the consequences of the infrastructure's disruption are. Consequences include *... a debilitating effect on security, national economic security, national public health or safety, or any combination thereof.* (Cybersecurity and Infrastructure Security Agency 2021a). Disruptions to IT infrastructure can spillover to other infrastructures that are critical and even spillover across borders (Gallais and Filiol 2017). For example, in Canada, critical IT infrastructure can be interconnected and interdependent within and across provinces, territories and national borders. (Government of Canada 2009).

IT Infrastructure is often classified as 'hard' or 'soft' where the former is physical such as fiber-optic cables for high-speed Internet and the latter is institutional such as data sharing among emergency services and government departments. A conceptual view of IT infrastructure is an interconnected set of platforms for activities where the seamless functioning of IT infrastructure from connected jurisdictions depends on compatibility and interoperability. Crucial to compatibility and interoperability are technical standards and supporting services, which in turn are essential for IT infrastructure to fulfill its potential as a public good (NCOIC 2012).

The more technologically complex the IT infrastructure is, the less well-defined technical standards and supporting services are. As a result, compatibility and interoperability can be compromised. In this case centralized decisions about critical IT infrastructure investment, where standards and supporting services are uniform, can take advantage of positive externalities and benefit individual jurisdictions and society. Obtaining this benefit requires coordinating decisions among a set of jurisdictions such that they behave as a coalition whereby the coalition can enforce agreements with coalition members.

In brief, our objectives are to construct a model of coalition formation where different jurisdictions that we refer to as districts decide whether to participate noncooperatively but the coalition invests cooperatively based on spillovers, interoperability, and cost elements; and to characterize what coalition structures can occur in equilibrium, how these would differ from coalition structures that are socially optimal, and whether there is a direct incentive that would cause equilibrium coalition structures to be socially optimal. As such our work is normative in that we build a model to represent a generic institutional setting, identify the goals of different participants and how these lead to different coalition structures, and design a direct mechanism to achieve the policy objective.

Illustrative Examples: We use two examples to help explain our model, recognizing our goal is to illustrate elements of our analysis but not to model the institutional details of each example directly. First, we use the European Union's Digital COVID Certificate (EUDCC) program that provides certified information about an individual's vaccination, testing, and recovery status during the COVID-19 pandemic. The certificates allow for travel between participating Member States as well as access to venues and services within Member States based on an individual's certificate status and the individual Member State's chosen restrictions. The EUDCC program has technical specifications delineated by the eHealth Network, a cooperative European Commission organization consisting of Member States and observer representation.[1] The EUDCC program involves an IT infrastructure "gateway" that stores and shares cryptographic public keys across participating Member States used for verification of certificates. Member States have autonomy regarding restrictions that require an EUDCC and what certificate types are accepted for certain restrictions.

These open-source specifications include technical specifications of approved regulations and implementing decisions,[2] the architecture and information relations using a central management system Gateway or Digital COVID Certificate Gateway (DCCG),[3] an interoperable 2D code detailing the use of an encoding mechanism, QR barcode,[4] specifications for transferring shared informa-

[1] The European Commission is the executive of the EU responsible for initiating and enforcing EU laws, and managing EU policies.

[2] https://health.ec.europa.eu/system/files/2022-02/digital-covid-certificates_v1_en.pdf.

[3] https://health.ec.europa.eu/system/files/2022-07/digital-covid-certificates_v2_en.pdf.

[4] https://health.ec.europa.eu/system/files/2022-02/digital-covid-certificates_v3_en_0.pdf.

tion,[5] Public Key Certificate Governance which details the exchange of cryptographic information within a region and with the Gateway,[6] and EUDCC validation rules.[7]

Our second illustrative example is the Direct Project, a U.S. government initiative that is part of the Nationwide Health Information Network. The Direct Project's goal is to support health information exchange (HIE) activities by providing a common policy framework specifying *... a secure, scalable, standards-based way to establish universal health addressing and transport for participants (including providers, laboratories, hospitals, pharmacies, and patients) to send encrypted health information directly to cryptographically validated recipients over the Internet.* (Direct Project 2019). In practice the Direct Project provides technical standards and services that enable secure and interoperable exchange of health information by neutral third-party organizations that facilitate information exchange called HIEs or alternatively called health information organizations (HIOs) (Vest and Gamm 2010). The role of HIOs is to facilitate health information exchange such as access to and retrieval of electronic health information among health service providers including hospitals, clinics, and other health care providers. Many are regional HIOs (then called RHIOs) such as Indiana HIE, Washington State's OneHealthPort, Rhode Island Quality Institute, and Mississippi Coastal Health Information Exchange.

In their role as facilitators of health information exchange, HIOs invest in health information technology (HIT) resources including hardware, telecommunications, software, and policies to ensure the speed and safety of accessing health information. The Office of the National Coordinator for Health Information Technology (ONC) evaluates the applicability of standards chosen by individual HIOs with respect to specifications developed by the Direct Project.[8]

[5] https://health.ec.europa.eu/system/files/2022-07/digital-covid-certificates_v4_en.pdf.

[6] https://health.ec.europa.eu/system/files/2022-03/digital-covid-certificate_v5_en.pdf.

[7] https://health.ec.europa.eu/system/files/2022-02/eu-dcc_validation-rules_en.pdf

[8] The Direct Project is an example of critical IT infrastructure in the US Healthcare and Public Health sector, one of 16 sectors defined as critical infrastructure sectors. As the US Government Cybersecurity & Infrastructure Security Agency states *... Because the vast majority of the sector's assets are privately owned and operated, collaboration and information sharing between the public and private sectors is essential to increasing resilience of the nation's Healthcare and Public Health critical infrastructure.* (Cybersecurity and Infrastructure Security Agency 2021b).

In part of its broader mission, the World Health Organization's Digital Implementation Investment Guide (DIIG) describes a setting for Digital Document of COVID-19 Certificate Vaccination Status (DDCC:VS) that uses a multi-healthcare-service related OpenHIE architecture with mechanisms for interoperability across different applications in the DIIG. Thus, there is a conceptual level where our two examples converge.

Research Questions and Results Preview: Using a district to represent a jurisdiction as a decision-making unit, we address a set of related research questions. The first question is whether a coalition of districts other than the grand coalition can be an equilibrium in which no districts 'inside' the coalition prefer to leave and those 'outside' the coalition do not want to participate, especially when districts are homogeneous. In the context of our examples this means whether some but not all Member States (HIOs) choose to participate in the EUDCC program (Direct Project) in equilibrium. The second question is whether a coalition of districts other than the grand coalition can be socially optimal. In our examples this is whether it is socially optimal for some but not all Member States (HIOs) to choose to participate in the EUDCC program (Direct Project). The third question is whether an equilibrium coalition size can also be socially optimal, and if not then whether there is a simple mechanism that can induce the equilibrium coalition size to be socially optimal. In our examples this is whether the number of Member States (HIOs) that choose to participate is also socially optimal, and if not then whether there is a simple and implementable incentive that the EUDCC program (Direct Project) can employ to induce the socially optimal number of Member States (HIOs) to participate.

To answer these questions, we develop a model for the provisioning of critical IT infrastructure across a fixed number of districts, where each district invests in infrastructure resources, and where these infrastructure resources spillover to other districts. Districts choose whether to participate in a coalition noncooperatively where infrastructure investments of districts that choose to participate are decided cooperatively and centrally by a coalition coordinator. The coalition coordinator's role is to maximize the surplus of the coalition through the choice of infrastructure investments for participating districts. There is perfect interoperability for resource spillovers within the coalition, and the coalition benefits from a coalition economy of scale in infrastructure investment but suffers diseconomies of scope in the number of districts in the coalition and their levels of investment. Districts outside the coalition make infrastructure investments as stand-alone entities and their infrastructure spillovers with other districts, including those inside the coalition, are reduced due to lower levels of interoperability.

In our examples, in the EUDCC program the eHealth Network provides technical specifications for the EUDCCs, districts that participate are Member States that use a national "backend" IT system as well as issuer, holder (wallet), and verifier applications.[9] Thus, the coalition coordinator that decides the specifications of IT infrastructure for participating Member States – which are effectively the IT infrastructure investments, is the eHealth Network. In the Direct Project the ONC for HIT provides technical standards and support services, districts are HIOs engaged in health information exchange, and infrastructure investments are HIT resources including hardware, telecommunications, software, and policies. Thus, the ONC for HIT acts as coalition coordinator through its Interoperability Standards Advisory. Table 1 systematically lists examples of critical IT infrastructures in different industry sectors. For each example, we specify the coalition coordinator and district, which illustrate the connections between our key modeling components and real-world applications.

[9] As we describe in the Conclusion, non-EU European Economic Area countries also participated to some extent.

Table 1 Critical IT Infrastructure Examples

Sector	Critical IT infrastructure	Coalition Coordinator	District
Healthcare	European Union's Digital COVID Certificate (EUDCC) program	eHealth Network	European Union member states
Healthcare	Direct Project as a part of Nationwide Health Information Network	Office of the National Coordinator for Health Information Technology (ONC)	Health Information Exchange (HIE) or Health Information Organizations (HIOs)
Security	National Cybersecurity Protection System (NCPS)	Cybersecurity and Infrastructure Security Agency (CISA)	Federal Civilian Executive Branch (FCEB) agencies
Transportation	Electronic toll collection (ETC) system	E-ZPass Interagency Group (IAG)	Individual states' toll collection agencies
Transportation	Positive Train Control (PTC) system	American Railway Engineering and Maintenance-of-Way Association (AREMA)	Amtrak, commuter railroads, and freight railroads
Finance	Society for Worldwide Interbank Financial Telecommunication (SWIFT)	SWIFT operations center	Financial institutions
Energy	Public Electric Vehicle Charging Infrastructure	Electric Vehicle Charging Association (EVCA)	Automotive manufactures, private charging station providers, and local municipalities

Notes: Except for the EUDCC and SWIFT examples, all other cases are located in the United States.

It is important to note that the social planner and coalition coordinator have different roles in these contexts. In the context of the EUDCC program, the social planner is the European Commission and the coalition coordinator is the eHealth Network. As a social planner the European Commission's role is to maximize social welfare that includes all member states in the EU. In contrast, the eHealth network acting as the coalition coordinator primarily focuses on maximizing the benefits of the members states that choose to participate the EUDCC program. In the context of the Direct Project, the social planner is the U.S. Department of Health and Human Services (HHS) and the coalition coordinator is the ONC. HHS is responsible for all HIOs whether they participate in the Direct Project or not. In contrast, ONC is directed by HHS to focus on HIOs that participate in the Direct Project. In the E-ZPass example, the coordinator's role is to maximize the benefits of the E-ZPass system for this specific coalition, focusing on solutions and improvements that address the coalition's particular needs and challenges. In contrast, a social planner would aim to optimize the toll collection system's benefits across all states and jurisdictions. In the PTC example, a social planner, e.g., Federal Railroad Administration, would be concerned with the nationwide implementation, operation, and benefits of this system. Their focus would be to maximize safety and efficiency of all rail networks across the country. In contrast, a coalition coordinator, e.g., American Railway Engineering and Maintenance-of-Way Association (AREMA), would focus on a specific group of rail companies or regions that have decided to work together on PTC issues. The coalition coordinator's role would be to optimize the benefits of the PTC system for this specific coalition. They would work on coordinating implementation schedules, sharing best practices, and addressing shared challenges or needs.

In our formulation if all districts choose to invest in infrastructure resources individually (i.e., no district chooses to participate in a coalition), then we call the resulting coalition structure a singleton. If only two districts choose to participate in a coalition, then we call it a minimal coalition. If at least three districts choose to participate, then we call it either a partial coalition when a strict subset of districts choose to participate or a grand coalition when all districts choose to participate.

Analyzing our coalition formation model leads to the following main results. First, we show that all sizes of coalitions other than the grand coalition can arise in equilibrium even when districts are homogeneous. Next, we determine the equilibrium coalition size and social optimum coalition size, and then detail their properties such as how the different coalition structures become the equilibrium structure with changes in the underlying parameters. Third, we compare the equilibrium coalition size and social optimum coalition size showing when they overlap and when they do not, and we derive a simple incentive mechanism that can induce the equilibrium coalition size to also be socially optimal. The social planner determines the incentive mechanism to achieve the socially optimal size maximizing welfare across districts.

We also extend our analysis in two directions. First, in Appendix B, we consider districts with different resource investment costs and examine the impact of such heterogeneity across districts. We find that, similar to when districts are homogeneous, all coalition structures (sizes) can arise in equilibrium. Moreover, the pattern of transitions between equilibrium coalition structures as the underlying parameters change is the same as with homogeneous districts. Second, in Appendix C, we relax our assumption on the range of interoperability between districts outside the coalition with those inside and find that when interoperability is low certain coalition sizes cannot be socially optimal.

2. Literature Review

Our work is related to two research streams – provisioning of critical IT infrastructure and coalition formation. In this section, we first review these two research streams and then discuss our contribution to the existing literature.

The first research stream is that of provisioning critical IT infrastructure. Prior studies in this area focus on the provision of different types of critical IT infrastructures such as Internet (Cheng et al. 2011), public safety networks (Liu et al. 2017), cloud computing (Guo et al. 2019), intelligent transportation systems (Cheng et al. 2020), and disaster management systems (Guo et al. 2021).

Cheng et al. (2011) investigate the impact of net neutrality on broadband service providers' incentive to invest in their Internet infrastructure and find that the incentive is likely to be lower in the absence of net neutrality. Nault and Zimmermann (2019) study openness and prioritization in a two-tier Internet and find that to maintain the quality of service of the open Internet and increase welfare a policy mechanism is necessary to motivate broadband provider investment in Internet infrastructure. Liu et al. (2017) compare centralized, decentralized, and mixed organization forms in the provision of public safety networks. The focus is on individual districts' incentives to opt-in or opt-out of the centralized organization form. Guo et al. (2019) study cloud service providers' provisioning strategies of virtual infrastructure resources under cloud service agreements. Online optimization algorithms are developed for both periodic and aperiodic policies. Besides the investment in transportation infrastructure, Cheng et al. (2020) offer empirical evidence that IT-enabled intelligent traffic systems are effective in mitigating traffic congestion. Guo et al. (2021) capture key characteristics of provisioning disaster management systems and propose a framework that reflects the differences among three interoperability approaches – integrated, unified and federated. They further analyze the equilibrium and socially optimal interoperability approach in a two-district setting.

Related to this work is research on the effect of IT infrastructure capability on cross-unit coordination. Firms with higher IT infrastructure capability, capability that includes infrastructure services that cross organizational boundaries and reach constituencies inside and outside the firm to transfer information and process transactions, are better able to adopt and implement changes in processes quickly (Broadbent et al. 1999). In the context of supply chains, IT infrastructure integration is the degree to which firms establish information systems for consistent and rapid transfer of information within and across their boundaries. Such integrated IT infrastructure allows firms to separate information and physical flows, and to share information with partners to improve planning for the staging and movement of physical products (Rai et al. 2006). And there is evidence that with higher levels of environmental uncertainty (e.g., volatility in the environment and heterogeneity among outside entities), such as would be case with critical IT infrastructure, centralized IT infrastructure governance has advantages both in terms of sharing and coordination of resources and of economies of scale (Xue et al. 2011).

The second research stream our work is related to is coalition structures in the literature on coalition formation. For a recent survey of the general topic of coalition formation refer to Ray and Vohra (2015). Here we focus on reviewing prior studies on coalition structures. Ray and Vohra (1999) characterize equilibrium coalition structures in a general context with widespread externalities and binding agreements among agents within a coalition. Ray and Vohra (2001) study the provision of public goods when agents may form coalitions through agreements among themselves and make rational predictions of the coalition structure. Thijssen et al. (2004) consider a coalition structure with one coalition and a group of outside agents and analyze a spillover game that captures the spillovers from the coalition to outside agents. Nagarajan and Sošić (2007) examine agents forming coalitions dynamically in competitive markets and analyze the stability of coalition structures. Hu et al. (2013) study the revenue-sharing schemes in airline alliances combining cooperative and noncooperative game theory. Their focus is on deriving a revenue-sharing rule that induces the grand coalition and that ensures the individual airlines achieve the same revenues as a central planner managing the global alliance network.

In the coalition formation literature there are different solution concepts which can be broadly categorized into noncooperative solution concepts and cooperative solution concepts (Carraro

2003). A cooperative solution may require that no subset of agents have an incentive to break away from the coalition, i.e., to eliminate the possibility that a subset of agents may collude and break away together from the coalition. In a noncooperative game of coalition formation all agents simultaneously decide whether to join the coalition and individual agents make their participation decisions independently.

Our contribution to the existing literature is threefold. First, we model the essential generic features of critical IT infrastructure as opposed to a particular type of critical IT infrastructure studied in the literature. Specifically, individual districts enjoy resource spillover benefits from other districts and the spillover is moderated by the interoperability between the two districts' infrastructure. Second, we characterize unique features of provisioning critical IT infrastructure through coalition formation, where districts simultaneously make decisions to join the coalition noncooperatively. Moreover, provisioning critical IT infrastructure cooperatively through the coalition exhibits both an economy of scale and diseconomies of scope; and the interoperability between inside and outside districts is lower compared to that among the inside districts. Finally, we contribute to the literature by being the first to analyze the provision of critical IT infrastructure involving more than two districts whereas most of the existing literature on critical IT infrastructure considers only two districts. Modeling more than two districts enables the analysis of different equilibrium coalition structures (singleton, minimal coalition, partial coalition, and grand coalition) and the corresponding coalition size. We further design a general incentive mechanism to induce the socially optimal coalition structure.

3. Model Setup

In this section, we build a coalition formation game to model the provisioning of interoperable critical IT infrastructure. Following d'Aspremont et al. (1983) and Thijssen et al. (2004), we model the formation of a single coalition where districts outside the coalition are a group of singletons – effectively a noncooperative game of coalition formation whereby individual districts choose whether to join the coalition. This is the case with the EUDCC program where Member States decide whether to participate to provide certified vaccination information for their residents, and Member States that do not participate do not use the EUDCC program. Similarly, HIOs that participate in the Direct Project are members of the coalition and those HIOs that do not participate instead exchange information with idiosyncratic protocols. This structure also mirrors that of a government, an alliance building a public network, and a cartel (Thijssen et al. 2004).

Once the coalition forms, districts in the coalition enter a binding agreement to centralize district IT infrastructure resource investment decisions, allocating these decisions to a single designated coalition coordinator. The coalition coordinator is an external party that can enforce the centralized resource investment decisions, making district resource investment a cooperative game. In the EUDCC program the eHealth Network as coalition coordinator directs resource investment decisions of participating Member States through its technical specifications that define the necessary capabilities which in turn determine the investments required to meet these capabilities. In the Direct Project the ONC as coalition coordinator directs resource investment decisions by evaluating standards chosen by participating HIOs with technical standards and services developed by the Direct Project. Again, these standards determine the investments required to meet the capabilities resulting from these standards.

Thus, in our model setup districts decide whether to participate in the coalition as a noncooperative game – a coalition coordinator cannot mandate participation – but districts that participate allocate resource investment decision rights to an external party whereby districts in the coalition play a cooperative game to maximize the welfare of the coalition.

3.1. Coalition Structures

We consider a finite number, n, of homogeneous districts, $i \in \{1, 2, ..., n\}$, that provide a system of critical IT infrastructure. In our examples the candidate districts are well-defined so that the potential number of districts n is known. The districts invest resources in their critical IT infrastructure and there are positive externalities between districts. We consider districts that can invest in resources to support infrastructure either through coalition or by themselves, where there are interoperability benefits for those districts that participate in a coalition. In Appendix B, we consider heterogeneous districts with different resource investment costs and examine the impact of such heterogeneity across districts.

When a coalition forms, district IT infrastructure resource investment decisions are centralized to a single designated coalition coordinator. As Xue et al. (2011) find, there are advantages provided by centralized IT infrastructure governance for the sharing and coordination of resources. When there is no coalition (i.e., singleton), or for districts that do not participate in the coalition, then the resource investment decision is decentralized to the individual districts.

Let $j \in \{2...n\}$ denote the number of districts that choose resource levels to support infrastructure together hence forming a coalition. Without loss of generality we assume districts $\{1, 2, ..., j\}$ are the districts that form the coalition and districts $\{j+1, j+2, ..., n\}$ are the districts that are not part of the coalition. The grand-coalition case of $j = n$ corresponds to all districts choosing to invest in infrastructure resources as a coalition. The partial-coalition case of $2 < j < n$ corresponds to some districts choosing to invest in infrastructure resources through coalition while others do not. A minimal coalition corresponds to the case when only two districts form a coalition to invest in infrastructure resource levels, $j = 2$. Finally, a singleton is where each district invests in its infrastructure resource level without concern for other districts.

Infrastructure resources are distributed across districts but can be accessible to all. This causes an individual district to value infrastructure resource investments in the other districts as well as valuing these resource investments in its own district, giving rise to positive externalities. Using subscripts to identify individual districts, let $g_i \in [0, \bar{g}]$ denote the levels of infrastructure resource investment made by each district and let $p \in R^+$ denote the cost per unit of resource investment.

Following prior work in public finance (Oates 1972) and provisioning of critical IT infrastructure (Liu et al. 2017, Guo et al. 2021), for a given district, we use $\kappa \in [0, 0.5]$ to denote the weight of resources in the other districts that spillover and provide value, and use $1 - \kappa$ to denote the weight of resources in its own district. Defining κ this way ensures local infrastructure resources always have a higher weight: when $\kappa = 0$, a district only values resources in its own district; when $\kappa = 0.5$, a district values resources in its own and other districts equally. We interpret κ as the degree of spillover of resources between districts such that a higher κ represents a higher cross-district value from resources, that is, greater positive externalities.

In our main model with homogeneous districts, a common weighting for local infrastructure resources is inherently consistent. In our extension to heterogeneous districts in Appendix B, each district would know their κ (e.g., κ is implied by policy in the EUDCC example) or can accurately estimate κ (especially if the policy was public).

Regarding the EUDCC program, spillovers can result from both personal and business travel as well as logistics supporting inter-Member State trade. For the Direct Project, benefits from spillovers result from different HIOs sharing health information in order to provide health care service to different participants including patients, hospitals, pharmacies, laboratories, etc.

3.2. Assumptions

We use the terms 'inside' and 'outside' to identify those districts inside and outside the coalition, respectively. Our first assumption differentiates the interoperability efficiency of the resource spillover for those districts inside versus outside the coalition.

Assumption 1. *Compared to districts inside the coalition, those districts outside the coalition have a relative interoperability of between 50% and 100%.*

We use $\beta \in (1/2, 1]$ to represent the overall relative interoperability for outside districts compared to inside districts where the smaller β is the lower is the relative interoperability. We place a lower limit on relative interoperability such that $\beta > 1/2$ to ensure social welfare is concave in coalition size. It is also reasonable that relative interoperability is at least 50%. In our second extension in Appendix C, we relax this lower limit.

When $\beta = 1$, resources are fully interoperable across all districts regardless of whether the district is inside or outside the coalition. When $\beta \in (1/2, 1)$, a district within the coalition derives more value from resources in other districts within the coalition than from resources in districts outside the coalition.

In our EUDCC program example Member States that participate and use the DCC's technical specifications are our inside districts, and those that do not are outside districts. Residents of Member States that participate in the program can use the DCC for seamless travel between participating Member States (conditional on local restrictions) for personal and business purposes including the movement of goods for trade. Residents of non-participating States and countries face delays and even exclusion from the participating Member States. The EUDCC program requires that participating Member States implement a universal system of standards. Consequently, interoperability is known and perfect among the Member States that participate (the interoperability among all inside districts is normalized to 1 in our model). Moreover, as these standards are open source, non-participants can determine their interoperability β with the coalition participants, and we take this β as the same for all non-participants.

Mapping the Direct Project example to our model, participants of the Direct Project correspond to inside districts and non-participants correspond to outside districts. All participants can leverage specified protocols and standards to access and retrieve health information from each other. However, non-participants may have to exert extra effort, such as significantly redesigning their clinical workflow, to communicate with other health information organizations. Interoperability issues in critical IT infrastructure are common, for example, in disaster management systems (Guo et al. 2021).

Using the elements we defined above we can detail the benefits each district receives from resources based on whether they are inside or outside the coalition. We take the benefits as additive and use $B_i(j)$ to denote the benefits of district i given a coalition size j:

$$B_i(j) = \begin{cases} [1-\kappa]\, g_i + \kappa \left[\sum_{l=1}^{i-1} g_l + \sum_{l=i+1}^{j} g_l\right] + \kappa\beta \sum_{l=j+1}^{n} g_l, & \text{for inside district } i \in \{1,...,j\} \\ [1-\kappa]\, g_i + \kappa\beta \left[\sum_{l=1}^{i-1} g_l + \sum_{l=i+1}^{n} g_l\right], & \text{for outside district } i \in \{j+1,...,n\}. \end{cases} \quad (1)$$

The difference in benefit $B_i(j)$ for inside districts relative to outside districts is that they have full interoperability of the resource spillovers from other inside districts (first line, second term) but they have only relative interoperability of the resource spillovers from outside districts (first line, third term). In contrast, outside districts have only relative interoperability for all resource spillovers (second line, second term). Thus, the benefit of being inside is the relative interoperability of resource spillovers from other inside districts. Our setup using κ and β has the same effect as using a different κ for inside and outside districts with the additional advantage of parameterizing the difference. There is little loss in generality of modeling benefit as additive so long as benefits aggregate, and aggregation for outside districts is tempered by β.

Cost Structure: For simplicity and tractability we use a quadratic form for the costs of resources. For any district outside the coalition, costs are:

$$C_i = pg_i^2 \text{ for } i \in \{j+1, ..., n\}.$$

Formulating the total costs across all inside districts in the coalition we develop a cost function form that captures both a coalition economy of scale and diseconomies of scope. We expect a cost economy of scale would occur for the accumulation of district resources across the coalition with the removal of duplicate resources, learning-by-doing across districts using similar resources, etc. We expect cost diseconomies of scope that would result from challenges of interoperability, integration, and coordination of resources from different districts in the coalition. Our second assumption implements these scale and scope economies.

ASSUMPTION 2. *There are separate effects of scale and scope on the accumulated resource costs faced by the coalition:*

(a) A coalition economy of scale decreases the accumulated resource costs faced by the coalition;

(b) Diseconomies of scope increase the accumulated resource costs faced by the coalition.

We use the parameter α, where $\alpha \in (0,1)$, to capture the benefits of a coalition economy of scale when multiple districts form the coalition. Effectively α is a proportional resource cost reduction for a minimal-or-greater coalition that dampens the accumulated resource costs faced by the coalition. To keep our model tractable, we model α as a constant across the different possible sizes of the coalition where our main focus is to capture a cost reduction as a consequence of being inside the coalition, and when considering joining the coalition the districts would be informed about α. The smaller α is, the greater the coalition economy of scale. In other words, α is a reverse measure of the coalition economy of scale. Meanwhile, as the coalition size grows, there are more districts and resources to coordinate, hence the total cost to maintain the effectiveness of resources in the coalition increases. To capture diseconomies of scope, we use the square of the sum (which is greater than the sum of the squares) for the total cost the coalition faces. Our form for the total cost of resources in a coalition of size j is

$$TC(j) = \alpha p \left[g_1 + ... + g_j\right]^2 \quad \text{for} \quad j \in \{2, ..., n\}. \tag{2}$$

Note that in the singleton case, where no coalition is formed, all districts have the same cost structure as the outside district, pg_i^2.

To summarize, the above cost structure captures both a cost economy of scale and cost diseconomies of scope. When districts form a coalition, we operationalize cost advantages of being an inside district via an economy of scale (α), and cost disadvantages of being an inside district via diseconomies of scope (square of the sum). When there is no coalition (singleton case), there are no economies of scale or diseconomies of scope in each district's resource cost. Qualitatively there is little loss of generality in our choosing a quadratic form for costs – what is necessary is a form of cost convexity that yields diseconomies of scope.

In the EUDCC program, there is both a coalition economy of scale and diseconomies of scope. On one hand, because the issuer, holder, and verifier applications have common specifications there is a coalition economy of scale across participating Member States as coding can be shared. On the other hand, each Member State provides their national"backend" that connects to the DCC "gateway" for certificate information and may be integrated with their own health care systems where specifications are likely idiosyncratic creating diseconomies of scope. In our Direct Project example, both an economy of scale and diseconomies of scope exist. As more HIOs participate in the Direct project, thanks to the economy of scale, participants enjoy proportionally lower resource costs for data storage, data protection, and data sharing from cloud-based solutions. At the same time, participants also suffer diseconomies of scope with higher costs of integration such as tracking and analyzing larger numbers of electronic health record (EHR) systems.

3.3. Coalition Welfare and Outside District Surplus

We use $CW(j)$ to denote the coalition welfare for a coalition of size j, which is the sum of total benefits from infrastructure resources of all inside districts minus the total cost of resources to the coalition. When the coalition is minimal, partial, or grand, the coalition welfare is given by:

$$CW(j) = \sum_{i=1}^{j} B_i(j) - TC(j) = [1-\kappa] \sum_{i=1}^{j} g_i + \kappa [j-1] \sum_{i=1}^{j} g_i + \kappa\beta j \sum_{i=j+1}^{n} g_i - \alpha p \left[\sum_{i=1}^{j} g_i\right]^2 .$$

The first term on the right-hand side is resources from inside districts, the second term is resource spillovers inside the coalition, the third term is resource spillovers from districts outside the coalition with relative interoperability β, and the last term is costs capturing our coalition economy of scale α and diseconomies of scope (square of the sum).

The surplus for each district outside the coalition is given by:

$$S_{j+1}(j) = B_{j+1}(j) - C_{j+1} = [1-\kappa] g_{j+1} + \kappa\beta \left[\sum_{i=1}^{n} g_i - g_{j+1}\right] - p g_{j+1}^2$$

$$\vdots$$

$$S_n(j) = B_n(j) - C_n = [1-\kappa] g_n + \kappa\beta \sum_{i=1}^{n-1} g_i - p g_n^2,$$

where the terms on the right-hand side are a district's own resources, spillovers from other districts with relative interoperability β, and full costs.

Timing: The timing of our coalition formation game is as follows. In Stage 1, each district chooses whether to participate in the coalition. The resulting coalition structure is the grand coalition with all n districts choosing to participate, a partial coalition with $j > 2$ districts choosing to participate, a minimal coalition with only two districts choosing to participate, or a singleton with no district choosing to participate. In Stage 2, each outside district chooses the resource level of its own critical IT infrastructure. Simultaneously, the coalition coordinator chooses the resource level for each inside district.

4. Optimal District Resources

We solve our coalition formation game through backward induction. Given the coalition structure, in Stage 2 the resource levels for districts inside the coalition are determined through a cooperative game where the coalition coordinator chooses resource levels for inside districts to maximize coalition welfare and outside districts choose their resource levels to maximize their surpluses.

4.1. Resource Level for Inside Districts

For districts that choose to build the infrastructure system cooperatively through the coalition, the coalition centralizes the choice of resource levels for each district to maximize the coalition welfare, $CW(j)$. The formulation in (3) below presents the decision problem for the coalition coordinator:

$$\max_{g_1,\dots,g_j} CW(j) = \max_{g_1,\dots,g_j} \{[1-\kappa]\sum_{i=1}^{j} g_i + \kappa[j-1]\sum_{i=1}^{j} g_i + \kappa\beta j \sum_{i=j+1}^{n} g_i - \alpha p\left[\sum_{i=1}^{j} g_i\right]^2\} \tag{3}$$

$$\text{Subject to: } \ 0 \le g_i \le \bar{g} \text{ for } i \in \{1,\dots,j\}.$$

Solving the above maximization problem for districts inside the coalition, where subscript in corresponds to district in inside the coalition, thus $in \in \{1,\dots,j\}$, we find that the optimal resource level is given by:

$$g_1^*(j) = \dots = g_j^*(j) = g_{in}(j) = \frac{1+[j-2]\kappa}{2\alpha jp}, \tag{4}$$

where $g_{in}(j)$ depends on the coalition size, j, given the degree of spillover (κ), the coalition economy of scale parameter (α), and our resource cost parameter (p). It is straightforward that the optimal resource level for an inside district is increasing in the coalition economy of scale, $\partial g_{in}(j)/\partial\alpha < 0$, where a greater coalition economy of scale corresponds to a lower α.

As districts are homogeneous, the optimal resource level provided by each inside district is the same. Therefore, the benefits for each inside district are the same and are given by:

$$B_1^*(j) = \dots = B_j^*(j) = B_{in}(j) = \frac{1}{2jp\alpha}\left[\kappa j^2[\kappa - \alpha\beta[1-\kappa]] + \kappa j[\alpha\beta[1-\kappa]n + 2 - 4\kappa] + [1-2\kappa]^2\right]. \tag{5}$$

Notice our relative interoperability parameter, β, only matters directly in the benefit function above and not through the optimal resource level.

4.2. Resource Level for Outside Districts

If a district chooses to build its infrastructure system outside the coalition, then it makes the choice of resource level to maximize the total surplus for its own district. The formulation in (6) below presents outside district i's decision problem:

$$\max_{g_i} S_i(j) = \max_{g_i}\{[1-\kappa]\, g_i + \kappa\beta \left[\sum_{l=1}^{j} g_l(j) + \sum_{l=j+1}^{n} g_l - g_i\right] - p g_i^2\}, \;\; \ni \; 0 \le g_i \le \bar{g}. \tag{6}$$

The optimization in (6) is solved for each district outside the coalition, $i = j+1, ..., n$. Solving the individual district's maximization problem for districts outside the coalition, where subscript *out* denotes a district outside the coalition, thus $out \in \{j+1, ..., n\}$, the optimal resource level is:

$$g_{j+1}^* = ... = g_n^* = g_{out} = \frac{1-\kappa}{2p}, \tag{7}$$

where the optimal resource level for an outside district does *not* depend on the coalition size. Although an outside district's optimal resource level, g_{out}, does not depend on the coalition size, an outside district's surplus, $S_{out}(j)$, does. This can be seen by restating the optimization in (6),

$$\max_{g_{out}} S_{out}(j) = \max_{g_{out}}\{[1-\kappa] g_{out} + \kappa\beta \Big[j g_{in}(j) + [n-j-1] g_{out}\Big] - p g_{out}^2\}, \;\; \ni \; 0 \le g_{out} \le \bar{g},$$

where $g_{in}(j)$ depends on j from (4). Thus, there is an indirect effect of coalition size on $S_{out}(j)$ through $g_{in}(j)$.

Based on the above analysis, the optimal resource level for all inside districts are the same, that is, g_{in}, and the optimal resource level for all outside districts are the same, that is, g_{out}. Consequently, all outside districts have the same surplus as given by:

$$S_{out}(j) = \frac{1}{4p\alpha}\left[2\beta\kappa\left[1+[j-2]\,\kappa\right] + \alpha\,[1-\kappa]\left[1-\kappa+2\beta\kappa\,[n-j-1]\right]\right]. \tag{8}$$

Equation (8) shows that the surplus for an outside district does depend on the coalition size because the outside district surplus includes a spillover from districts inside the coalition, although with an interoperability efficiency loss.

For the special case of singleton, where no coalition is formed, the surplus for each district is the same:

$$S_{single} = \frac{[1-\kappa]\,[\kappa\,[2\beta\,[n-1]-1]+1]}{4p}. \tag{9}$$

4.3. Optimal Resource Levels and Coalition Size

Our first proposition shows how optimal district resource levels change as the coalition size changes, and how the resource levels for inside districts compare to those of outside districts. Proofs of all propositions, lemmas, and corollaries are relegated to Appendix A.

PROPOSITION 1 **(Properties of Optimal Resource Levels)**. *Optimal resource levels have the following properties:*

(a) Inside districts' resource level $g_{in}(j)$ decreases in the coalition size j;

(b) Outside districts' resource level g_{out} is independent of the coalition size j;

(c) There exists a threshold coalition size, $\check{j} \equiv \frac{1-2\kappa}{\alpha-\kappa[1+\alpha]}$, defined by $g_{in}(\check{j}) = g_{out}$. If the coalition size is smaller than the threshold $j \le \check{j}$, then inside districts' resource levels are higher than those of outside districts $g_{in}(j) \ge g_{out}$; otherwise, outside districts' resource levels are higher $g_{in}(j) < g_{out}$.

In Proposition 1(a) the optimal resource level provided by each inside district decreases as the size of the coalition increases. This is because resources are fully interoperable within the coalition and it is more expensive to integrate resources together as the coalition size grows due to diseconomies of scope (Assumption 2(b)). This can be seen directly from the derivative of (4) in the proof, and the effect results from the last term in (3) being squared.

Furthermore, in Proposition 1(b) the optimal resource level for an outside district is independent of the coalition size j because for a district outside the coalition its choice of resource level, $g_i, i > j$ is additively separable in (6) from all the other districts' resource levels. Finally, Proposition 1(c) follows directly from the definition of the threshold coalition size that results from equating resource levels of inside and outside districts, together with Proposition 1(a).

COROLLARY 1. *The threshold coalition size, $\check{j}$, from Proposition 1(c) decreases in α and increases in κ.*

In Corollary 1, as the economy of scale decreases (α increases), the cost of resources for inside districts increases and g_{in} decreases. As g_{in} also decreases with coalition size from Proposition 1(a), the threshold coalition size decreases, and there is a smaller range of coalition sizes where optimal resource level of inside districts is greater than those of outside districts. In contrast, a greater spillover increases optimal resource levels of inside districts and decreases those of outside districts, thereby increasing the threshold coalition size and increasing the range of coalition sizes where optimal resource levels of inside districts are greater than those of outside districts.

COROLLARY 2. *The benefit for each inside district, $B_{in}(j)$, and the surplus for each outside district, $S_{out}(j)$, decrease in α and increase in β.*

Similar to Corollary 1, in Corollary 2 as the coalition economy of scale decreases (α increases), the cost of resources for inside districts increases and g_{in} decreases. This directly reduces the benefit for inside districts, and indirectly reduces the benefit and hence the surplus of outside districts through the reduced spillover from inside districts. An increase in relative interoperability (an increase in β) increases the spillover benefit for all districts. In this way the inside and outside districts are connected.

In our examples, benefits to Member States (HIOs) that participate and not both increase as the coalition economy of scale increases and as relative interoperability increases. The latter is because the ability of EU residents to travel across borders and access venues in the case of non-participants in the EUDCC program, and the ability to exchange health information with non-participating HIOs in the Direct Project, increases.

5. Equilibrium Analysis Under Equal Cost Sharing

With homogeneous districts in the coalition we use equal cost sharing for the cost of the coalition resources. The surplus for each district inside the coalition is given by:

$$S_{in}(j) = B_{in}(j) - \frac{TC(j)}{j},$$

where the total cost of coalition $TC(j)$ is defined in (2).

The grand coalition is an equilibrium, where all districts are inside the coalition with $j^*_{eqm} = n$, if and only if:

$$S_{in}(j = n) \geq S_{out}(j = n - 1), \tag{10}$$

where given our noncooperative solution concept, we only need to consider when only one district can change.

A partial coalition with size $j^*_{eqm} \in \{3, ..., n-1\}$ is an equilibrium if and only if:

$$S_{in}(j = j^*_{eqm}) \geq S_{out}(j = j^*_{eqm} - 1), \text{ and} \tag{11}$$

$$S_{out}(j = j^*_{eqm}) \geq S_{in}(j = j^*_{eqm} + 1). \tag{12}$$

The condition in (11) ensures that given other districts' coalition participation choices, a district inside the coalition does not have any incentive to deviate and leave the coalition. The condition in (12) ensures that given other districts' coalition participation choices, a district outside the coalition does not have any incentive to deviate and participate in the coalition. Thus, (11) and (12) are effectively incentive compatibility (IC) conditions for inside districts and outside districts, respectively.

The minimal coalition is an equilibrium with only two districts inside the coalition, $j^*_{eqm} = 2$, if and only if:

$$S_{in}(j = 2) \geq S_{single}, \text{ and} \tag{13}$$

$$S_{out}(j = 2) \geq S_{in}(j = 3). \tag{14}$$

The conditions in (13) and (14) play the same role for the minimal coalition as (11) and (12) do for the partial coalition. They are also effectively IC conditions for the inside districts and outside districts, respectively.

Singleton is an equilibrium if and only if:

$$S_{single} \geq S_{in}(j = 2), \tag{15}$$

where this is an IC condition for each district.

Note that obtaining the equilibrium for the partial coalition and the minimal coalition involves solving two conditions simultaneously, i.e., conditions (11) and (12) for the partial coalition and conditions (13) and (14) for the minimal coalition. This results in a solution for the equilibrium coalition size $j^*_{eqm} \in \{2, ..., n-1\}$ that we provide in Appendix A.

We use our coalition economy of scale parameter, α, to determine transitions between our different coalition structures based on coalition size. Setting inequalities (15) and (11) to equalities and solving for α in each equality yields equilibrium thresholds $\alpha_{eqm1}(\beta)$ and $\alpha_{eqm3}(\beta)$, respectively. Setting inequality (12) when $j^*_{eqm} = 2$ to equality and solving for α yields equilibrium threshold $\alpha_{eqm2}(\beta)$. These equilibrium thresholds are effectively solutions for our coalition economy of scale parameter α in terms of our relative interoperability parameter β:

$$\alpha_{eqm1}(\beta) = \frac{1}{2 - 4\kappa + 4\beta\kappa + 2\kappa^2 - 4\beta\kappa^2}, \quad \alpha_{eqm2}(\beta) = \frac{1 + \kappa\left[2 - 6\beta + \kappa\right]}{3\left[1 - \kappa\right]^2},$$

$$\text{and} \quad \alpha_{eqm3}(\beta) = \frac{\left[1 + \left[n-2\right]\kappa\right]^2 - 2n\beta\kappa\left[1 + \left[n-3\right]\kappa\right]}{n\left[1 - \kappa\right]^2}.$$

The following lemma shows that these equilibrium thresholds are strictly ordered.

Lemma 1. *The equilibrium thresholds are strictly ordered as follows:* $\alpha_{eqm3}(\beta) < \alpha_{eqm2}(\beta) < \alpha_{eqm1}(\beta)$.

The following proposition outlines the possible equilibrium coalition structures.

Proposition 2 **(Equilibrium Coalition Structures)**. *All four coalition structures are possible equilibrium:*

(a) When $\alpha < \alpha_{eqm3}(\beta)$*, the equilibrium is the grand coalition with* $j^*_{eqm} = n$*;*

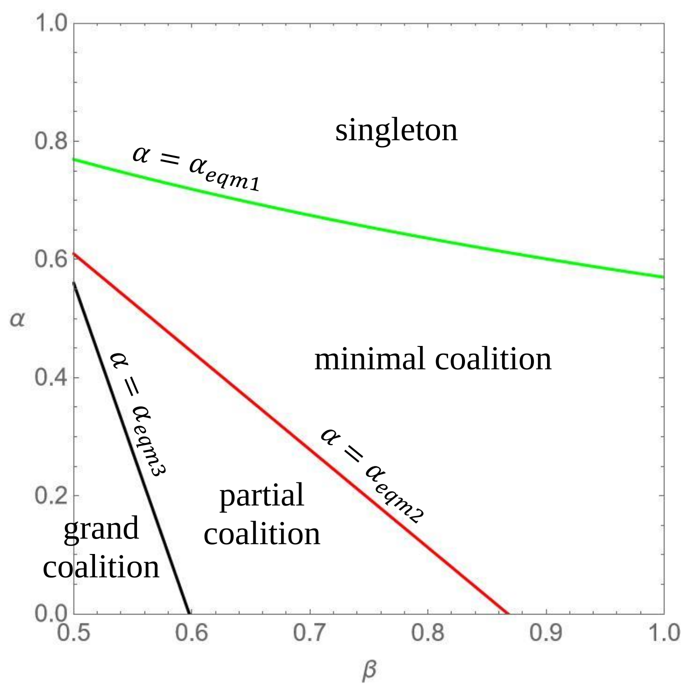
(b) When $\alpha_{eqm3}(\beta) < \alpha < \alpha_{eqm2}(\beta)$*, the equilibrium is a partial coalition with* $j^*_{eqm} = \lfloor j_{eqm} \rfloor$*;*

(c) When $\alpha_{eqm2}(\beta) < \alpha < \alpha_{eqm1}(\beta)$*, the equilibrium is the minimal coalition with* $j^*_{eqm} = 2$*;*

(d) When $\alpha > \alpha_{eqm1}(\beta)$*, the equilibrium is a singleton.*

Here, j_{eqm} is a continuous approximation of the equilibrium coalition size in case of the partial coalition, which solves the first-order conditions for the surplus maximization problem. We note that the equilibrium results are unique – that is, multiple equilibria do not exist given a pair of α and β. Details are available from the authors.

Our results from Proposition 2 are displayed in Figure 1. They show different equilibrium coalition structures in different parameter regions based on our coalition economy of scale parameter α and relative interoperability parameter β. What this shows in an example like the Direct Project is that without supplemental incentives any number of HIOs may participate, and many choose to not participate.



Notes: The figure shown here is based on parameter values of $n = 10$ and $\kappa = 0.35$. Figures based on other parameter values remain qualitatively the same.

Figure 1 Equilibrium Coalition Structures

A greater coalition economy of scale (lower α) and lesser relative interoperability (lower β) both favor larger sized coalition structures, while diseconomies of scope (the square of the sum of total costs) favor smaller size coalition structures. As α and β become smaller, the impact of the coalition economy of scale and relative interoperability outweighs the impact of diseconomies of scope, hence a coalition structure with a greater number of districts is the equilibrium. This is illustrated in Figure 1: as α and β move from the upper right corner to the lower left corner, the equilibrium coalition structure changes from no coalition (singleton) to minimal coalition to partial coalition and eventually to the grand coalition.

A closer examination on the properties of the equilibrium thresholds reveal that they all decrease in the relative interoperability β, corresponding to the decreasing boundary lines in Figure 1. For example, considering that $\alpha_{eqm3}(\beta)$ decreases with β, moving from a point within the partial coalition region to the right does not result in reaching the grand coalition region. This is because increasing β corresponds to the provision of additional spillover benefits to the outside districts, which essentially reduces their incentives to join the coalition. Therefore, to achieve the grand coalition, it is necessary to simultaneously reduce the resource costs faced by the coalition (lower α) as β increases.

Next we present how various parameters affect the equilibrium coalition size in the following proposition.

PROPOSITION 3 **(Properties of the Equilibrium Coalition Size)**. *The equilibrium coalition size has the following properties:*

(a) The equilibrium coalition size is increasing in the coalition economy of scale: the smaller is α*, the larger is* j^*_{eqm}*;*

(b) The equilibrium coalition size is decreasing in relative interoperability: the smaller is β*, the larger is* j^*_{eqm}*;*

(c) The equilibrium coalition size is weakly increasing in the total number of districts, n*.*

In Proposition 3(a), an increase in the coalition economy of scale increases optimal resource levels for inside districts, increasing spillovers. As spillovers are higher for inside districts than for outside districts due to their full interoperability, more districts benefit from being in the coalition. In Proposition 3(b), a decrease in relative interoperability directly reduces the spillover benefits all districts can enjoy from other districts. This effect is stronger for the outside districts as relative interoperability affects outside districts more than inside districts, leading to a larger coalition size. In Proposition 3(c), an increase in the total number of districts (e.g., from n to $n+1$), has no impact on the optimal resource levels for neither inside nor outside districts. The added district has the same impact on any existing district's surplus regardless of whether the district is inside or outside the coalition. As a result, tradeoffs between participating in the coalition or not are the same for all districts. Therefore, the equilibrium coalition size remains unchanged for most cases. The only exception is when grand coalition is the equilibrium, and the resulting equilibrium coalition size increases from n to $n+1$.

As many of the application specifications are open source in the EUDCC program and could easily be shared among adopting Member States, for example the wallet (holder) and verifier applications, there is a substantial coalition economy of scale favoring a larger coalition size whereby many, if not all, the Member States participate. Similarly, the lower is the relative interoperability from outside the EUDCC participants, the greater are the payoffs from participating in the EUDCC program allowing for unimpeded travel and trade.

6. Socially Optimal Coalition Structure

In this section we derive the socially optimal coalition structure and examine properties of the socially optimal coalition size. Here, district participation is decided noncooperatively and resource investment is decided cooperatively, where the social planner is an external party that can incentivize participation. In the context of the EUDCC program, the social planner is the European Commission that manages EU policies and promotes the EUDCC program to Member States, and the socially optimal structure and size is the level of participation of Member States. In the context of the Direct Project, the social planner is the U.S. Department of Health and Human Services (HHS) that advocates for potential members to participate in the Direct Project, and the socially optimal structure and size is analogous to the level of HIO participation that would be preferred by HHS. Our social optimality is distinct from first-best where the social planner could dictate coalition participation and resource investment for all districts. First-best would require substantial information and enforcement capabilities that do not exist in our examples.

Substituting the optimal resource levels back into the surplus functions for all districts, we obtain the coalition welfare $CW(j) = S_1(j) + ... + S_j(j) = jS_{in}(j)$ for all inside districts and $S_{out}(j)$ from (8) for each outside district. Specifically, for coalition welfare we have:

$$CW(j) = \frac{1}{4p\alpha}\left[1 + 2\left[j - 2 + j\left[n - j\right]\alpha\beta\right]\kappa + \left[\left[j - 2\right]^2 - 2j\left[n - j\right]\alpha\beta\right]\kappa^2\right].$$

By adding the surpluses of all n districts together, we obtain the overall social welfare for the n districts: $SW(j) = CW(j) + [n - j]\,S_{out}(j)$ if a coalition is formed; or $SW_{singleton} = nS_{singleton}$ if no coalition is formed.

A social planner's objective is to find the optimal coalition structure and size when that structure is a partial coalition such that the overall social welfare is maximized:

$$\max\{SW_{singleton}, SW(j)\}, \tag{16}$$

where $SW(j) = \{CW(j) + [n - j]\,S_{out}(j)\}$ for $j \in \{2, ..., n\}$.

Using our Assumption 1 that defines the range of the parameter we use to model relative interoperability, $\frac{1}{2} < \beta < 1$, social welfare $SW(j)$ is a concave quadratic function. Therefore, the necessary and sufficient conditions for the socially optimal coalition structure are as follows:

The grand coalition is socially optimal, i.e., $j^*_{sw} = n$, if and only if:

$$SW(j = n) \geq SW(j = n - 1). \tag{17}$$

A partial coalition is socially optimal, i.e., $j^*_{sw} \in \{3, ..., n - 1\}$, if and only if:

$$SW(j = j^*_{sw}) \geq SW(j = j^*_{sw} + 1), \text{ and} \tag{18}$$

$$SW(j = j^*_{sw}) \geq SW(j = j^*_{sw} - 1). \tag{19}$$

A minimal coalition is socially optimal, i.e., $j^*_{sw} = 2$, if and only if:

$$SW(j = 2) \geq SW(j = 3), \text{ and} \tag{20}$$

$$SW(j = 2) \geq SW_{singleton}. \tag{21}$$

A singleton is socially optimal if and only if:

$$SW_{singleton} \geq SW(j=2). \tag{22}$$

Similar to the way we determined equilibrium thresholds based on our coalition economy of scale parameter α in the prior section where the thresholds represent transitions between coalition structures, we set inequalities (22), (20), and (17) to equalities and solve for α yielding three welfare thresholds $\alpha_{sw1}(\beta)$, $\alpha_{sw2}(\beta)$, and $\alpha_{sw3}(\beta)$, respectively, as functions of our interoperability efficiency parameter, β:

$$\alpha_{sw1}(\beta) = \frac{1+2[n-2]\beta\kappa}{2[1-\kappa][1+[2[n-1]\beta-1]\kappa]}, \quad \alpha_{sw2}(\beta) = \frac{\kappa[2+\kappa+2\beta[[n-3]\kappa-1]]}{[1-\kappa][1+[2[n-1]\beta-1]\kappa]},$$

$$\text{and} \quad \alpha_{sw3}(\beta) = \frac{\kappa[[2n-5]\kappa-2\beta[1+[n-3]\kappa]+2]}{[1-\kappa][1+[2[n-1]\beta-1]\kappa]}.$$

With the help of these three thresholds, we next present the socially optimal coalition structures. To begin, the following lemma shows that these welfare thresholds are strictly ordered.

LEMMA 2. *The welfare thresholds are strictly ordered as follows:* $\alpha_{sw3}(\beta) < \alpha_{sw2}(\beta) < \alpha_{sw1}(\beta)$.

We characterize the possible socially optimal coalition structures in the following proposition.

PROPOSITION 4 **(Socially Optimal Coalition Structure)**. *There are four possible socially optimal coalition structures:*

(a) When $\alpha < \alpha_{sw3}(\beta)$*, the grand coalition with size* $j^*_{sw} = n$ *is the social optimum;*

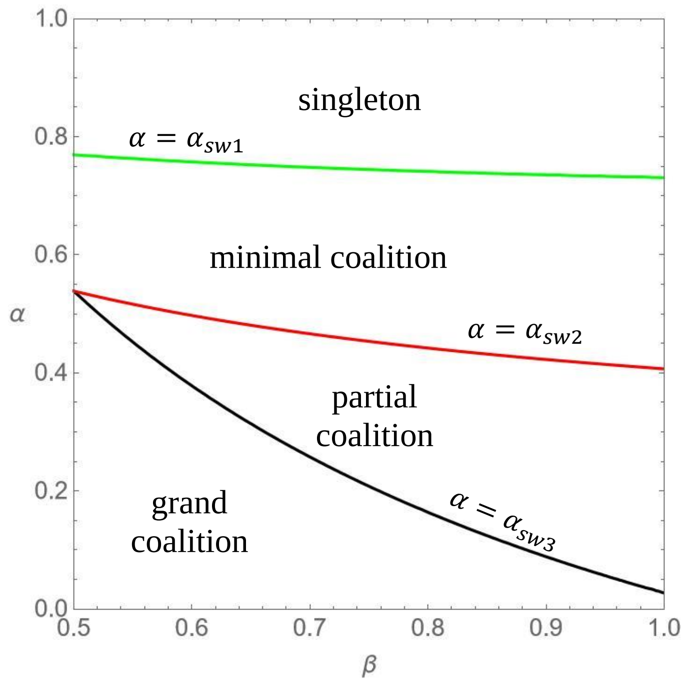
(b) When $\alpha_{sw3}(\beta) < \alpha < \alpha_{sw2}(\beta)$*, a partial coalition with size* $j^*_{sw} = \lceil j_{sw} \rceil$ *or* $\lfloor j_{sw} \rfloor$ *is the social optimum, depending on which yields a higher social welfare;*

(c) When $\alpha_{sw2}(\beta) < \alpha < \alpha_{sw1}(\beta)$*, the minimal coalition with size* $j^*_{sw} = 2$ *is the social optimum;*

(d) When $\alpha > \alpha_{sw1}(\beta)$*, a singleton is the social optimum.*

Here, j^*_{sw} is a continuous approximation of the socially optimal coalition size which solves the first-order condition of the social welfare maximization problem. Using results from Proposition 4, similar to Figure 1 with equilibrium coalition structures, Figure 2 shows different socially optimal coalition structures in different parameter regions based on our coalition economy of scale parameter

α and relative interoperability parameter β. As illustrated in Figure 2, the socially optimal coalition structure changes from no coalition (singleton) to minimal coalition to partial coalition and to the grand coalition when moving from the upper right corner to the lower left corner. This pattern is similar to the equilibrium coalition structure results because a greater coalition economy of scale (lower α) and lesser relative interoperability (lower β) both favor larger size coalition structures. However, as we show below, the social planner prefers larger size coalition structures in a larger parameter region than do individual districts when deciding their equilibrium coalition structure (e.g., the grand coalition region in Figure 2 is larger than that in Figure 1).



Notes: This figure is based on parameter values of $n = 10$ and $\kappa = 0.35$.

Figure 2 Socially Optimal Coalition Structures

Further examination on the properties of the social welfare thresholds reveal that they all decrease in the relative interoperability β, corresponding to the decreasing boundary lines in Figure 2. This indicates that for an existing partial coalition, no matter how much extra spillover can be generated (larger β), a grand coalition is never possible to be socially optimal when the coalition resource costs remain the same. In practice, enhancing data integration and interoperability not necessarily leads to the formation of a grand coalition, the cost associated with coalition formation, such as data privacy protection and maintenance, must also be taken into account.

Next, we present how various parameters affect the socially optimal coalition size.

PROPOSITION 5 **(Properties of the Socially Optimal Coalition Size)**. *The socially optimal coalition size, j^*_{sw}, has the following properties:*

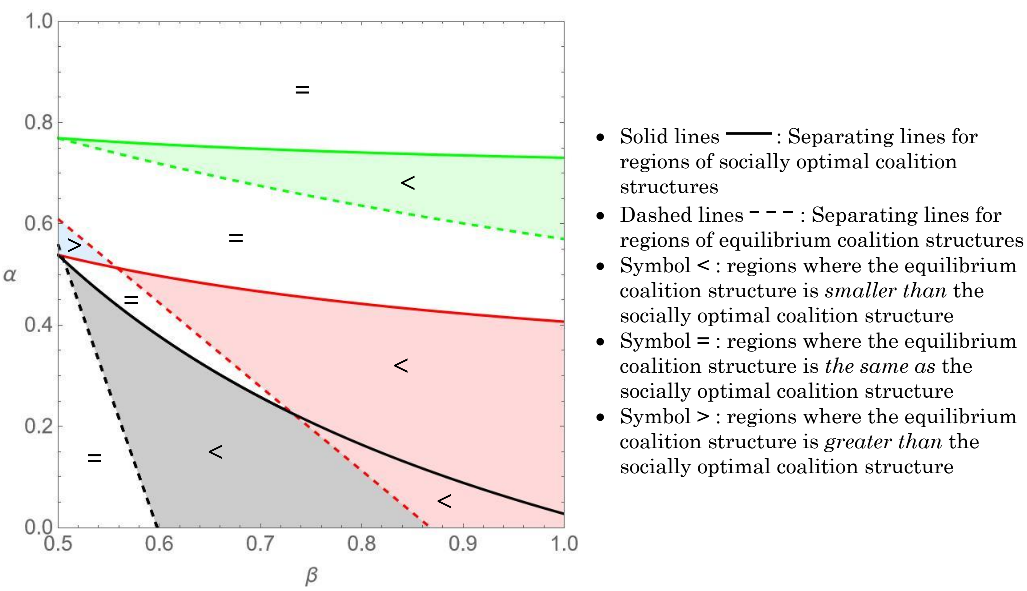
(a) The socially optimal coalition size is increasing in the coalition economy of scale: the smaller the α the larger is j^*_{sw};

(b) The socially optimal coalition size is decreasing in relative interoperability: the smaller is β, the larger is j^*_{sw};

(c) The socially optimal coalition size is weakly increasing in the number of districts, n.

The impacts of the coalition economy of scale and relative interoperability parameters on the socially optimal coalition size (Proposition 5(a) and 5(b)) are similar to their impacts on the equilibrium coalition size (Proposition 3(a) and 3(b)). In Proposition 5(c), as the total number of districts increases from n to $n+1$, the existing separating lines for regions of different coalition sizes $\{1, 2..., n\}$ all shift upward and a new separating line (i.e., $n+1$) is added below. As a result, for a given market condition (i.e., a given α and β), the coalition size could remain unchanged or increase depending on whether the coalition size region shifted.

Next, we compare the equilibrium coalition structures to the socially optimal coalition structures.



Notes: This figure is based on parameter values of $n = 10$ and $\kappa = 0.35$.

Figure 3 Deviation of Equilibrium Coalition Structures from Socially Optimal Coalition Structures

Figure 3 illustrates how the socially optimal coalition structure regions in Figure 2 differ from the equilibrium coalition structure regions in Figure 1. In Figure 3, the symbols $<, =$, and $>$ represent regions where the equilibrium coalition structure is smaller than, the same as, or greater than the socially optimal coalition structure in size, respectively.

As shown in Figure 3, the equilibrium coalition structures are aligned with the social optimum in some regions, but are at odds with the social optimum in other regions. Specifically, individual districts prefer smaller size coalition structures than does the social planner under most combinations of α and β. In Figure 3 there is only a small region where the equilibrium coalition structure is larger than the socially optimal coalition structure. It is also worth noting that in the partial coalition region where the equilibrium and socially optimal coalition structures coincide, the coalitions under each regime may differ in size. That is, there is variation in size within the same structure. This is because the partial coalition region is defined as when $j^*_{eqm} \in \{2, ..., n-1\}$ for the equilibrium coalition and $j^*_{sw} \in \{2, ..., n-1\}$ for the socially optimal coalition.

As applied to our Direct Project example, the deviation shown in Figure 3 demonstrates the differences in levels of participation in the Direct Project – that involves resource levels determined by the ONC for those HIOs that choose to participate – between levels of participation that would be chosen by the HIOs individually and those that HHS would prefer.

7. Incentives to Induce the Socially Optimal Coalition

In this section we design an incentive mechanism to induce the socially optimal coalition structure, and in the case of a partial coalition to induce the socially optimal coalition size. Here, we solely focus on offering incentives (subsidy or tax) to participating districts, and not consider penalties for non-participating districts.[10] In our EUDCC example, this involves the EU Commission providing incentives to participating Member States to motivate the socially optimal level of Member

[10] In our setting there are several reasons for not considering a penalty to outside districts. First, the social planer may lack the jurisdictional authority to enforce a penalty on non-participating districts. Second, the contracts/agreements that are agreed to by participating districts are less restrictive, less invasive, and more easily enforceable compared to those imposed on non-participating districts. Third, aversive incentive solutions such as penalties are typically met with resistance from the districts' perspective and thus are deemed undesirable from the social planner's perspective.

State participation in the EUDCC program. And the EU Commission cannot impose penalties on Member States that choose not to participate in the EUDCC program.

7.1. Incentive Mechanism Design

In this subsection we model a social planner who provide incentives to districts for coalition participation. To our two-stage setup we add a Stage 0 where a social planner decides the incentive for districts inside the coalition to maximize social welfare. Stage 1 and Stage 2 remain the same as in our equilibrium model where in Stage 1 districts decide whether to participate in the coalition and in Stage 2 the coalition coordinator decides participating districts' resource levels.

In Stage 0, the social planner announces an incentive $\phi_j \in R$ (i.e., ϕ_j can be a subsidy or tax) given to each district inside the coalition when the size of the coalition is $j \in \{2, ..., n\}$.

Providing incentives to districts within a coalition often incurs an opportunity cost for the social planner, since the necessary funds must be obtained through interest-bearing loans or the diversion of resources from alternative government initiatives. We capture this cost as a fraction of the incentive, denoted by γ. Consequently, the social planner's objective is to maximize the total surplus across all districts, taking into account the incentives received by the districts within the coalition, while also factoring in the cost of providing such incentives.

$$\max_j \{SW(j) = CW(j) + [n-j]\, S_{out}(j) - [1+\gamma]\, j\phi_j\}$$

As a result, for each inside district, the total surplus is

$$S_{in}(j) = B_{in}(j) - \frac{TC(j)}{j} + \phi_j.$$

The total surplus for the outside districts $S_{out}(j)$ remains the same as in the analysis of optimal resource levels.

As before, in Stage 1 each district chooses whether to participate in the coalition based on its surplus, $S_{in}(j)$ and $S_{out}(j)$. In Stage 2, each outside district chooses its own resource level to maximize its surplus $S_{out}(j)$. Simultaneously, the coalition coordinator chooses the resource level for each inside district to maximize coalition welfare $CW(j) = jS_{in}(j) = jB_{in}(j) - TC(j) + j\phi_j$.

Because the incentive ϕ_j is both fixed and additive to the inside district surplus, it does not affect the choice of resource level for an individual district, and therefore the optimal resource levels for inside and outside districts are the same as in the optimal district resource analysis in Section 4. The socially optimal grand coalition (i.e., $j^*_{sw} = n$) is an equilibrium if and only if:

$$S_{in}(j = n) \geq S_{out}(j = n - 1). \tag{23}$$

The socially optimal partial coalition (i.e., $j^*_{sw} \in \{3, ..., n-1\}$) is an equilibrium if and only if:

$$S_{in}(j = j^*_{sw}) \geq S_{out}(j = j^*_{sw} - 1), \text{ and} \tag{24}$$

$$S_{out}(j = j^*_{sw}) \geq S_{in}(j = j^*_{sw} + 1). \tag{25}$$

The socially optimal minimal coalition (i.e., $j^*_{sw} = 2$) is an equilibrium if and only if:

$$S_{in}(j = 2) \geq S_{singleton}, \text{ and} \tag{26}$$

$$S_{out}(j = 2) \geq S_{in}(j = 3). \tag{27}$$

The socially optimal singleton is an equilibrium if and only if:

$$S_{singleton} \geq S_{in}(j = 2). \tag{28}$$

Conditions (23) through (28) are analogous to those in (10) through (15) when determining the equilibrium coalition size.

We again use our coalition economy of scale parameter, α, to determine thresholds for transitions between coalition structures. Employing the subscript im on α to denote thresholds for our incentive mechanism, setting inequality (28) to equality yields threshold:

$$\alpha_{im1}(\beta, \phi_2) = \frac{1}{2\kappa\left[2\beta\left[1-\kappa\right]+\kappa-2\right]-8p\phi_2+2}.$$

Setting inequalities (25) or inequality (27) to equality for $j^*_{sw} = j \in \{3, ..., n\}$ yield a group of thresholds:

$$\alpha_{im\{j-1\}}(\beta, \phi_j) = \frac{j\kappa\left[-2\beta\left[\left[j-3\right]\kappa+1\right]+\left[j-4\right]\kappa+2\right]-4\left[1-\kappa\right]\kappa+1}{j\left[\left[1-\kappa\right]^2-4p\phi_j\right]}.$$

The thresholds $\alpha_{im1}(\beta, \phi_2)$ and $\alpha_{im\{j-1\}}(\beta, \phi_j)$ represent solutions for our coalition economy of scale as incentive compatibility (IC) conditions for the equilibrium coalition size being the socially optimal coalition size as a function of our model parameters and our socially optimal incentive.

Setting inequality (18) or inequality (17) to equality from Section 6 when $j^*_{sw} = j \in \{3, ..., n\}$ yields the threshold IC condition for the socially optimal coalition being the partial or grand coalitions:

$$\alpha_{sw\{j-1\}}(\beta) = \frac{\kappa\left[2\left[1-\beta\right]+\kappa\left[2\beta\left[n+5\right]-2\left[2\beta-1\right]j-7\right]\right]}{\left[1-\kappa\right]\left[\kappa\left[2\beta\left[n-1\right]-1\right]+1\right]}.$$

Using these thresholds as IC conditions, we derive a sufficient incentive mechanism to ensure that the socially optimal coalition size is the equilibrium as a function of our model parameters. Denoting the incentive for a coalition of size j as ϕ_j, we solve for ϕ_j in the case where we have the minimal coalition (ϕ_2) and where we have a partial coalition by equating the corresponding IC conditions for the equilibrium and socially optimal coalition structures:

$$\alpha_{im1}(\beta, \phi_2) = \alpha_{sw1}(\beta, \phi_2) \implies \phi_2(\beta, n, \kappa, \gamma) = \frac{\beta\left[2\beta-1\right]\left[1-\kappa\right]\kappa^2\left[n-2\right]}{2p\left[1+\gamma+2\beta\kappa\left[n-2\right]\right]},$$

where $\alpha_{sw1}(\beta, \phi_2)$ is defined by $SW_{singleton} = SW(j=2)$. Other incentives ϕ_j for $j \in \{3, ..., n\}$ is defined recursively as follows:

$$\alpha_{im\{j-1\}}(\beta, \phi_j) = \alpha_{sw\{j-1\}}(\beta, \phi_{j-1}, \phi_j) \text{ for } j \in \{3, ..., n\}$$

$$\implies \phi_j(\beta, n, \kappa, \gamma) = F_1(\beta, n, \kappa, \gamma) * \phi_{j-1} + F_2(\beta, n, \kappa, \gamma) \text{ for } j \in \{3, ..., n\}$$

The expressions for functions $F_1(\beta, n, \kappa, \gamma)$ and $F_2(\beta, n, \kappa, \gamma)$ can be found in Appendix A.

Note that the socially optimal incentives $\phi_2(\beta, n, \kappa, \gamma)$ and $\phi_j(\beta, n, \kappa, \gamma)$ are functions of our parameters of interest: relative interoperability β, number of districts n, degree of spillover κ, and the opportunity cost γ. We shorten the socially optimal incentives as $\phi_2(\cdot)$ and $\phi_j(\cdot)$ for the remainder of our analyses.

7.2. Socially Optimal Incentives When the Opportunity Cost is Negligible

Due to the intricate nature of the socially optimal incentive mechanism design in a multi-district system and the additional layer of complexity introduced by the presence of opportunity cost, it is infeasible to obtain closed-form explicit analytical solutions. Consequently, we begin our analytical investigation by focusing on the special case where opportunity cost is negligible (i.e., when $\gamma = 0$). This allows us to gain insights under simplified conditions. To gain a more comprehensive understanding, in the next subsection we employ numerical analysis to explore the properties of the optimal incentives when the opportunity cost is not zero.

In the following proposition, we investigate the properties of the socially optimal incentives when the opportunity cost is negligible.

PROPOSITION 6 **(Socially Optimal Incentives When Opportunity Cost is Negligible)**. *When opportunity cost $\gamma = 0$, the optimal incentives for the socially optimal coalitions have the following properties:*

(a) Inside districts in the minimal coalition are subsidized,

i.e., $\phi_2(\cdot) > 0$.

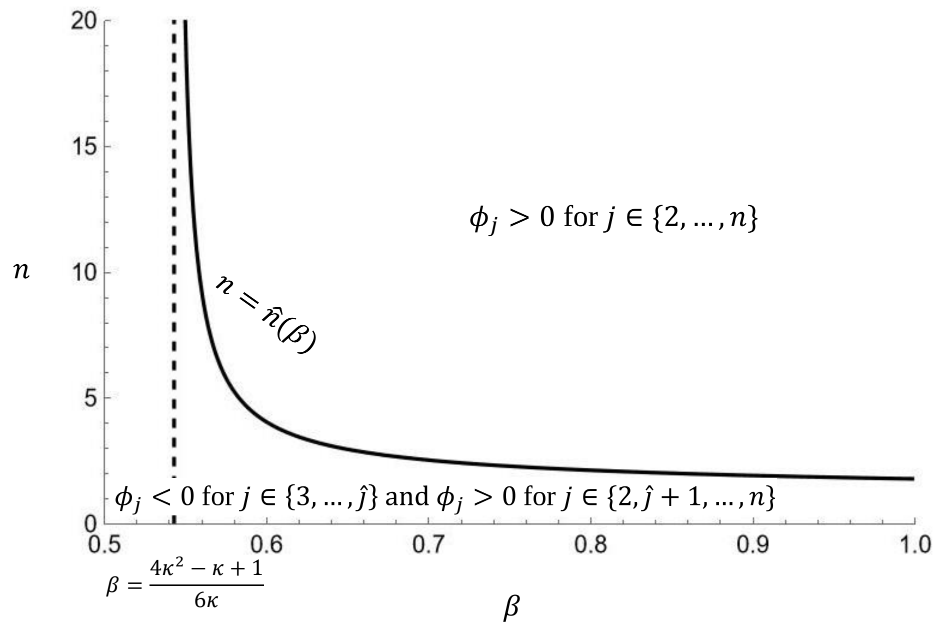
(b) If relative interoperability is high and there are a large number of districts, then inside districts in a partial or grand coalition are subsidized,

i.e., if $\beta \geq \frac{4\kappa^2-\kappa+1}{6\kappa}$ and $n \geq \hat{n}(\beta)$, then $\phi_j(\cdot) \geq 0$ for $j \in \{3, ..., n\}$.

(c) If relative interoperability is low, or if relative interoperability is high and there are a smaller number of districts, then inside districts in a small partial coalition are taxed and inside districts in a large partial or grand coalition are subsidized,

i.e., if $\beta < \frac{4\kappa^2-\kappa+1}{6\kappa}$, or if $\beta > \frac{4\kappa^2-\kappa+1}{6\kappa}$ and $n < \hat{n}(\beta)$, then $\phi_j(\cdot) < 0$ for $j \in \{3, ..., \hat{j}\}$ and $\phi_j(\cdot) > 0$ for $j \in \{\hat{j}, ..., n\}$.

The expressions for thresholds $\hat{n}(\beta)$ and $\hat{j}$ can be found in Appendix A.



Notes: This figure is based on parameter value of $\kappa = 0.35$.

Figure 4 Socially Optimal Incentives When Opportunity Cost is Negligible

From Proposition 6 both the incentive and the relative interoperability are coalition-size specific. The above Proposition is directly related to Figure 4, whereby the sign of $\phi_j(\cdot)$ indicates whether the equilibrium coalition size is smaller or larger than the socially optimal coalition size. If $\phi_j(\cdot) > 0$, then districts receive a subsidy to participate the coalition, and if $\phi_j(\cdot) < 0$, then districts in the coalition are taxed. Referring back to Figure 3, the regions where the equilibrium coalition structure is smaller than the socially optimal coalition structure call for a subsidy and vice-versa where the relative coalition structure sizes are reversed. However, there are situations when the partial coalition structure is both the equilibrium and socially optimal form where the coalition sizes are different, and in these cases a non-zero incentive is needed to align the equilibrium coalition size with the socially optimal one. Although our incentive is sufficient to align the equilibrium coalition size with the socially optimal coalition size under all our parameter ranges, it is not necessary when the equilibrium and socially optimal coalition sizes are the same, rather it is only necessary and sufficient when they are different.

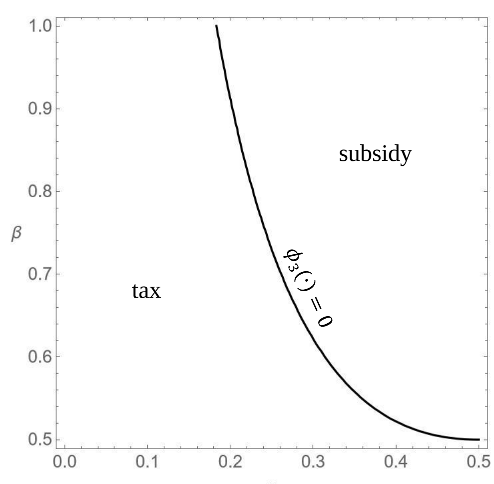
7.3. Socially Optimal Incentives in the Presence of Opportunity Cost

In this subsection, we investigate the properties of socially optimal incentives in the presence of opportunity cost (i.e., when $\gamma > 0$). As discussed in Section 7.1, socially optimal incentives to inside districts are recursive solutions. In the base case, the incentive to inside districts in a minimal coalition, ϕ_2, has an explicit solution and we derive analytical results for its properties. However, for other incentives beyond the minimum coalition ϕ_j, $j \in \{3, ..., n\}$, we must rely on numerical analysis to investigate their properties due to the interdependencies among the socially optimal incentives. In the following we present the analytical results concerning the properties of ϕ_2 and the numerical observations concerning the properties of ϕ_3 when opportunity cost is non-negligible.

Socially Optimal Incentives in the Presence of Opportunity Cost (Properties of ϕ_2 and ϕ_3): *When opportunity cost is non-negligible, the socially optimal incentives ϕ_2 and ϕ_3 have the following properties:*

(a) *The socially optimal incentive ϕ_2 always takes the form of a subsidy, i.e., $\phi_2 > 0$; the socially optimal subsidy ϕ_2 decreases in opportunity cost γ.*

(b) *The socially optimal incentive ϕ_3 can take the form of either a subsidy or tax, i.e., $\phi_3 > 0$ if β exceeds a threshold and $\phi_3 < 0$ otherwise; the socially optimal subsidy ϕ_3 decreases in opportunity cost γ; the socially optimal tax $|\phi_3|$ also decreases in opportunity cost γ.*



Notes: This figure is based on parameter values of $n = 10$ and $\gamma = 10\%$.

Figure 5 Socially Optimal Incentive ϕ_3 in the Presence of Opportunity Cost

We find that the structure of the optimal policy (subsidy or tax) for the social planner in the presence of opportunity cost is similar to that in the absence of opportunity cost (as summarized in Proposition 6). Specifically, inside districts for the minimal coalition should always be subsidized. Beyond the minimal coalition, as depicted in Figure 5, if β exceeds a certain threshold, the optimal policy for the social planner is to provide a subsidy to districts within the coalition, whereas if β is below this threshold, the optimal policy is to impose a tax on the districts within the coalition. Furthermore, we find that the socially optimal incentives, whether in the form of subsidies or taxes, decrease as the opportunity cost γ increases. As the opportunity cost γ increases, the social planner should reduce the amount of the provided incentives (either subsidy or tax) due to the reduced efficiency of this policy instrument.

7.4. Discussion about the Implementation of Socially Optimal Incentive

Our socially optimal incentive is not designed to achieve budget balance. Achieving budget balance is not a critical objective when the social planner faces substantial natural disasters or pandemics such as COVID-19. Allowing cross-Member State travel and trade while maintaining population health is a primary objective of the EU Commission, and the EUDCC program is essential in achieving it. In the US, health information exchange between HIOs has proven critical in treatment of ICU patients and in the distribution of vaccines, illustrating the importance of the Direct Project. However, when the socially optimal incentive $\phi_j(\cdot)$ is effectively a tax, $\phi_j(\cdot) < 0$, then overall social welfare is improved by the incentive both through achieving the socially optimal coalition structure/size and through the proceeds from the tax. When the incentive is a subsidy, the social planner has to weigh the gain in coalition welfare, CW, with the aggregate cost of the subsidy $\phi_j(\cdot) > 0$. In both the EUDCC program and the Direct Project the social planner (EU Commission and HHS, respectively) has made substantial investments of public funds.

Reviewing the timing of implementation of our socially optimal incentive, the stages unfold as follows:

Stage 0: The social planner announces the incentives ϕ_2 and ϕ_j where $j \in \{3, ..., n\}$, based on all parameters other than the economy of scale parameter α.

Stage 1: In response to the announced incentives, individual districts choose whether to participate in the coalition noncooperatively based on all parameters including the coalition economy of scale parameter α. The resulting equilibrium coalition size is socially optimal.

Stage 2: The coalition coordinator chooses optimal resource levels cooperatively for inside districts and outside districts choose their own optimal resource levels. The resources levels are not directly affected by the additive incentive but are indirectly affected by changes in the coalition structure or size.

8. Conclusions

We study a relatively general setting with districts choosing whether to participate in a critical IT infrastructure coalition noncooperatively where decisions about resource levels within the coalition are made cooperatively and centrally. In our setting there are resource spillovers between districts, there is lesser interoperability for districts outside the coalition, and there are different economies of scale and scope inside the coalition. As two illustrative examples, we use the EUDCC program, an EU initiative to implement digital COVID certificates (e.g., vaccine passports) to support movement across Member States, and we use the Direct Project, a US HHS initiative to make information exchange across HIOs fully interoperable.

Even with homogeneous districts we find that, in addition to no coalition or the grand coalition, a minimal, or partial coalition can be the equilibrium coalition structure. Similarly, all four coalition structures can be socially optimal. Although there are cases where the equilibrium coalition structure and even size correspond to the social optimum, there are other cases where they do not correspond. In these cases it is most likely that the socially optimal coalition is larger, and we present a straightforward incentive mechanism that aligns the equilibrium and socially optimal coalition structure and size.

We also find that benefits for inside districts and the surplus for outside districts are higher with a greater coalition economy of scale and with higher relative interoperability of resources from outside districts. However, the coalition size in equilibrium and at the social optimal is larger with a greater coalition economy of scale and smaller with higher relative interoperability of resources from outside districts.

Our results are important reminders to policymakers that critical IT infrastructure programs such as the Direct Project should not expect that all potential participants will choose to participate, and, more significantly, that it may not be socially optimal for all potential participants to participate – that is, the grand coalition may not be socially optimal. Interestingly, in the EUDCC program not only did all 27 Member States participate in the program, but equivalence was accepted in 40 non-EU nations including most European Economic Area countries, and others (e.g., Iceland) that directly participated in the EUDCC program. Thus, although the EUDCC program was designed for and targeted towards the participation of Member States, there was additional social benefit from the program.

Our modeling approach has some limitations common to most analytical models. We chose specific functional forms for our benefits and surplus functions where our key parameters such as spillovers, relative interoperability, and coalition economy of scale enter linearly, and our aggregation is additive. Our results are likely to generalize to a wide variety of settings and mathematical forms, but technically we only establish our results for those forms. A feature of our main model that can be considered a limitation is homogeneous districts, and through our extension we find that with limited heterogeneity, such as districts with two different levels of resource costs, our equilibrium results and transitions between coalition structures as our underlying parameters change are qualitatively the same – hence we believe most of our results would hold with heterogeneous districts. For example, in a franchise network with positive participation and investment externalities (i.e., spillovers) Nault and Dexter (1994) found that universal participation across heterogeneous territories (i.e., districts) was not necessarily profit maximizing for a franchisor nor franchisees because greater participation diluted the investment incentive.

In our model the coalition coordinator chooses and enforces investments for inside districts maximizing coalition welfare. When there is no coalition coordinator, the coalition needs a mechanism that fills these functions. The coalition could contract to an external party giving them decision and enforcement rights – in our examples the social planner (EU Commission and HHS) allocates these rights to their chosen coalition coordinator (eHealth network and ONC, respectively). Alternatively, the coalition could act as a decentralized autonomous organization (DAO) using algorithms to determine inside district investment and smart contracts to enforce investments, and communication between districts would be necessary to achieve this. The coalition in our model is in part a form of DAO that is characterized by a finite number of potentially autonomous units, and the decentralized autonomy extends to deciding whether to join the coalition. In our examples (EUDCC program and the Direct Project) we found that some elements of resource investment implementation are decentralized and some of the underlying standards are centralized – consistent with a DAO. However, given our model is directed towards public policy, it differs from DAOs in that levels of resource investment are determined cooperatively and centrally rather than being decentralized.

Generically our study examines benefits of interoperability and spillovers in a positive spillover setting with multiple (more than two) districts where a coalition coordinator dictates investment and benefits are shared through spillovers. In addition to treating the coalition as a DAO as discussed above, future research opportunities include developing mechanisms for multiple districts to directly share benefits through transfers or through cooperative game solutions that incorporate the critical feature of interoperability as well as incorporating decentralized investment decisions in coalition formation.

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Appendices for: Join Up or Stay Away? Coalition Formation for Critical IT Infrastructure

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Appendix A: Proof of Lemmas, Corollaries and Propositions in the Main Model

Proof of Proposition 1

(a) $\frac{\partial g_{in}(j)}{\partial j} = \frac{-[1-2\kappa]}{2j^2 p\alpha} < 0$. (b) g_{out} in equation (7) does not depend on j. (c) The result follows as $g_{in}(j)$ decreases in the coalition size j and g_{out} is independent of the coalition size. Q.E.D.

Proof of Corollary 1

Taking the partial derivatives of $\check{j}$ with respect to α and κ reveals that $\frac{\partial \check{j}}{\partial \alpha} = \frac{[3-2\kappa]\kappa-1}{[\alpha-\kappa[1+\alpha]]^2} < 0$ and $\frac{\partial \check{j}}{\partial \kappa} = \frac{1-\alpha}{[\alpha-\kappa[1+\alpha]]^2} > 0$. Q.E.D.

Proof of Corollary 2

Taking the partial derivatives of $B_{in}(j)$ in equation (5) and $S_{out}(j)$ in equation (8) with respect to α and β reveals that $\frac{\partial B_{in}(j)}{\partial \alpha} = -\frac{[[j-2]\kappa+1]^2}{2\alpha^2 jp} < 0$, $\frac{\partial B_{in}(j)}{\partial \beta} = \frac{[1-\kappa]\kappa[n-j]}{2p} > 0$, $\frac{\partial S_{out}(j)}{\partial \alpha} = -\frac{\beta\kappa[[j-2]\kappa+1]}{2\alpha^2 p} < 0$, and $\frac{\partial S_{out}(j)}{\partial \beta} = \frac{\kappa[[j-2]\kappa+\alpha[1-\kappa][n-j-1]+1]}{2\alpha p} > 0$. Q.E.D.

Proof of Lemma 1

Solving $S_{singleton} = S_{in}(j=2)$ yields: $\alpha_{eqm1}(\beta) = \frac{1}{2-4\kappa+4\beta\kappa+2\kappa^2-4\beta\kappa^2}$. Solving $S_{out}(j=2) = S_{in}(j=3)$ yields: $\alpha_{eqm2}(\beta) = \frac{1+\kappa[2-6\beta+\kappa]}{3[1-\kappa]^2}$. Solving $S_{in}(j=n) = S_{out}(j=n-1)$ yields: $\alpha_{eqm3}(\beta) = \frac{[1+[n-2]\kappa]^2-2n\beta\kappa[1+[n-3]\kappa]}{n[1-\kappa]^2}$.

Comparing the three threshold values we find that: The sign of $\alpha_{eqm1}(\beta) - \alpha_{eqm2}(\beta)$ depends on $[2-4\beta]\kappa^3 + [4\beta[6\beta-5]+2]\kappa^2 + [8\beta-5]\kappa + 1$, which is positive given that $\kappa \in [0, 0.5]$ and $\beta \in (1/2, 1]$. As a result, $\alpha_{eqm1}(\beta) > \alpha_{eqm2}(\beta)$. The sign of $\alpha_{eqm2}(\beta) - \alpha_{eqm3}(\beta)$ depends on $-4\kappa + \kappa^2\left[[6\beta-3]n+4\right]+1$, which is also positive given that $\kappa \in [0, 0.5]$ and $\beta \in (1/2, 1]$. As a result, $\alpha_{eqm2}(\beta) > \alpha_{eqm3}(\beta)$. Q.E.D.

Proof of Proposition 2

The singleton is the equilibrium if and only if $S_{singleton} \geq S_{in}(j=2)$. Solving this inequality yields: $\alpha > \alpha_{eqm1}(\beta)$. The minimum coalition is the equilibrium if and only if $S_{in}(j=2) \geq S_{singleton}$ and $S_{out}(j=2) \geq S_{in}(j=3)$. Solving both inequalities simultaneously yields: $\alpha_{eqm2}(\beta) < \alpha < \alpha_{eqm1}(\beta)$. The grand coalition is the equilibrium if and only if $S_{in}(j=n) \geq S_{out}(j=n-1)$. Solving this inequality yields: $\alpha < \alpha_{eqm3}$. As the conditions for all four possible equilibria are mutually exclusive and collectively exhaustive, we know that the partial coalition is the equilibrium if $\alpha_{eqm3}(\beta) < \alpha < \alpha_{eqm2}(\beta)$.

A partial coalition is the equilibrium if and only if $S_{in}(j=j^*_{eqm}) \geq S_{out}(j=j^*_{eqm}-1)$ and $S_{out}(j=j^*_{eqm}) \geq S_{in}(j=j^*_{eqm}+1)$ for $j^*_{eqm} \in \{3,...,n-1\}$. We can show that $S_{in}(j=j^*_{eqm}) - S_{out}(j=j^*_{eqm}-1)$ decreases in j for $j^*_{eqm} \in \{3,...,n-1\}$. Thus $S_{in}(j=j^*_{eqm}) \geq S_{out}(j=j^*_{eqm}-1)$ if and only if $j \leq \overline{j}$, where upper bound $\overline{j}$ is: $\overline{j} = \frac{\sqrt{\left[\alpha[1-\kappa]^2-2\kappa[3\beta\kappa-\beta-2\kappa+1]\right]^2+4[2\beta-1]\kappa^2[1-2\kappa]^2}-\alpha[1-\kappa]^2+2\kappa[3\beta\kappa-\beta-2\kappa+1]}{2[2\beta-1]\kappa^2}$.

Similarly, we can show that $S_{out}(j=j^*_{eqm}) - S_{in}(j=j^*_{eqm}+1)$ increases in j for $j^*_{eqm} \in \{3,...,n-1\}$. Thus $S_{out}(j=j^*_{eqm}) \geq S_{in}(j=j^*_{eqm}+1)$ if and only if $j \geq \underline{j}$, where lower bound $\underline{j}$ is: $\underline{j} = \frac{\sqrt{\alpha^2[1-\kappa]^4-4\alpha\kappa[1-\kappa]^2[3\beta\kappa-\beta-2\kappa+1]+4\beta\kappa^2\left[[9\beta-4]\kappa^2-6\beta\kappa+\beta+2\kappa\right]}-\alpha[1-\kappa]^2+2[1-\beta]\kappa[1-\kappa]}{2[2\beta-1]\kappa^2}$.

We choose $j_{eqm} = \frac{\overline{j}+\underline{j}}{2}$ as a continuous approximation for the equilibrium coalition size in the partial coalition region, where $j_{eqm} = \frac{\sqrt{\left[\alpha[1-\kappa]^2-2\kappa[3\beta\kappa-\beta-2\kappa+1]\right]^2+4[2\beta-1]\kappa^2[1-2\kappa]^2}-\alpha[1-\kappa]^2+\kappa[4\beta\kappa-2\beta-3\kappa+2]}{2[2\beta-1]\kappa^2}$. We can also show that $\overline{j} - \underline{j} = 1$. Thus $j^*_{eqm} = \lfloor j_{eqm} \rfloor$ is a uniquely defined integer. Q.E.D.

Proof of Proposition 3

(a) According to Proposition 2, as α decreases, the equilibrium coalition structure changes from singleton to minimum (i.e., $j^*_{eqm} = 2$), to partial (i.e., $j^*_{eqm} = \lfloor j_{eqm} \rfloor$) and eventually to the grand coalition(i.e., $j_{eqm} = n$). Furthermore, in the partial coalition region, taking partial derivative of j_{eqm} with respect to α yields $\frac{\partial j_{eqm}}{\partial \alpha} < 0$. Therefore, the equilibrium coalition size is decreasing in α: the smaller is α, the larger is j^*_{eqm}.

(b) According to Proposition 2, as β decreases, the equilibrium coalition structure changes from singleton to minimum (i.e., $j^*_{eqm} = 2$), to partial (i.e., $j^*_{eqm} = \lfloor j_{eqm} \rfloor$) and eventually to the grand coalition (i.e., $j_{eqm} = n$). Furthermore, in the partial coalition region, taking the partial derivative of j_{eqm} with respect to β yields $\frac{\partial j_{eqm}}{\partial \beta} < 0$. Therefore, the equilibrium coalition size is decreasing in β: the smaller is β, the larger is j^*_{eqm}.

(c) As n increases from n to $n+1$, we find that the equilibrium coalition size remain the same for the singleton or minimum coalition structures because α_{eqm1} and α_{eqm2} are independent of n. Taking the partial derivative of j_{eqm} with respect to the n yields $\frac{\partial j_{eqm}}{\partial n} = 0$. This indicates the coalition size is independent from n in the partial coalition region. However, as the number of districts increase from n to $n+1$, α_{eqm3} decreases. Consequently, the original grand coalition region is divided into two regions: n (now part of the partial coalition region) and $n+1$ (the new grand coalition region). Therefore, the equilibrium coalition size is weakly increasing in n. Q.E.D.

Proof of Lemma 2

Because $\frac{\partial^2 SW(j)}{\partial j^2} = -\frac{[2\beta-1]\kappa^2}{2\alpha p} < 0$ given $\frac{1}{2} < \beta < 1$, the social welfare $SW(j)$ is a concave in j. Also, because $CW(j)$ is a quadratic function of j and S_{out} is linear in j, as a result the social welfare function $SW(j) = \{CW(j) + [n-j]\, S_{out}(j)\}$ is both concave and quadratic in j. Therefore, the conditions as listed in (16) to (21) for the corresponding socially optimal coalition structures are both sufficient and necessary.

Solving $SW_{singleton} = SW(j=2)$ yields: $\alpha_{sw1}(\beta) = \frac{1+2[n-2]\beta\kappa}{2[1-\kappa][1+[2[n-1]\beta-1]\kappa]}$. Solving $SW(j=2) = SW(j=3)$ yields: $\alpha_{sw2}(\beta) = \frac{\kappa[2+\kappa+2\beta[[n-3]\kappa-1]]}{[1-\kappa][1+[2[n-1]\beta-1]\kappa]}$. Solving $SW(j=n) = SW(j=n-1)$ yields: $\alpha_{sw3}(\beta) = \frac{\kappa[[2n-5]\kappa-2\beta[1+[n-3]\kappa]+2]}{[1-\kappa][1+[2[n-1]\beta-1]\kappa]}$.

Comparing the three threshold values we find that: The sign of $\alpha_{sw1}(\beta) - \alpha_{sw2}(\beta)$ depends on $2\kappa\,[[6\beta-1]\,\kappa-2] + 2\beta\kappa\,[1-2\kappa]\,n+1$, which is positive given that $\kappa \in [0, 0.5]$ and $\beta \in (1/2, 1]$. As a result, $\alpha_{sw1}(\beta) > \alpha_{sw2}(\beta)$. The sign of $\alpha_{sw2}(\beta) - \alpha_{sw3}(\beta)$ depends on $2\,[2\beta-1]\,\kappa^2\,[n-3]$, which is also positive given that $\kappa \in [0, 0.5]$ and $\beta \in (1/2, 1]$. As a result, $\alpha_{sw2}(\beta) > \alpha_{sw3}(\beta)$. Q.E.D.

Proof of Proposition 4

Because $\frac{\partial SW(j)}{\partial \alpha} < 0$, $SW(j)$ is decreasing in α. As a result, condition (16) is equivalent to $\alpha < \alpha_{sw3}(\beta)$, conditions (17) and (18) are equivalent to $\alpha_{sw3}(\beta) < \alpha < \alpha_{sw2}(\beta)$, condition (21) is equivalent to $\alpha > \alpha_{sw1}(\beta)$. Otherwise, when $\alpha_{sw2}(\beta) < \alpha < \alpha_{sw1}(\beta)$, the partial coalition is socially optimal.

Solving the first order condition $\frac{\partial SW(j)}{\partial j} = 0$ yields $j_{sw} = \frac{2\kappa[1-2\kappa+\beta[\kappa[n+2]-1]]-\alpha[1-\kappa][1+\kappa[2\beta[n-1]-1]]}{2[2\beta-1]\kappa^2}$. Here, j_{sw} is a continuous approximation of the socially optimal coalition size. When partial coalition is the socially optimal coalition structure, the socially optimal coalition size j^*_{sw} is an integer which takes the value of $\lceil j_{sw} \rceil$ or $\lfloor j_{sw} \rfloor$ depending on which yields a higher social welfare. Q.E.D.

Proof of Proposition 5

(a) According to Proposition 4, as α decreases, the socially optimal coalition structure changes from singleton to minimum (i.e., $j^*_{sw} = 2$), to partial (i.e., $j^*_{sw} = max\{\lfloor j_{sw} \rfloor, \lceil j_{sw} \rceil\}$) and eventually to the grand coalition(i.e., $j_{sw} = n$). Furthermore, in the partial coalition region, taking the partial derivative of j_{sw} with respect to α yields $\frac{\partial j_{sw}}{\partial \alpha} < 0$. Therefore, the equilibrium coalition size is decreasing in α: the smaller is α, the larger is j^*_{sw}.

(b) We can show that all separating lines $\alpha_{sw1}(\beta), \alpha_{sw2}(\beta)$, and $\alpha_{sw3}(\beta)$ decrease in β. We also know from Lemma 2 that the three threshold values are strictly ordered (i.e., $\alpha_{sw1}(\beta) > \alpha_{sw2}(\beta) > \alpha_{sw3}(\beta)$). Thus as β decreases, the socially optimal coalition structure changes from singleton to minimum (i.e., $j^*_{sw} = 2$), to partial (i.e., $j^*_{sw} = max\{\lfloor j_{sw} \rfloor, \lceil j_{sw} \rceil\}$) and eventually to the grand coalition (i.e., $j_{sw} = n$). Furthermore, in the partial coalition region, taking the partial derivative of j_{sw} with respect to β yields $\frac{\partial j_{sw}}{\partial \beta} < 0$. Therefore, the equilibrium coalition size is decreasing in β: the smaller is β, the larger is j^*_{sw}.

(c) As n increases, $\alpha_{sw1}(\beta)$ and $\alpha_{sw2}(\beta)$ both increase in n. Thus, for a given α and β in the singleton or minimal coalition region, the coalition size could remain unchanged or increase depending on whether the coalition size region shifted or not. In the partial coalition region, taking the partial derivative of j_{sw} with respect to the n yields $\frac{\partial j_{sw}}{\partial n} > 0$. In the grand coalition region, the grand coalition size also increases as there are more districts in total. Therefore, the socially optimal coalition size is increasing in n. Q.E.D.

Proof of Proposition 6

The expressions for the socially optimal incentives are given below:

$$\phi_2 = \frac{\beta\,[2\beta-1]\,[1-\kappa]\,\kappa^2\,[n-2]}{2p\,[1+\gamma+2\beta\kappa\,[n-2]]}$$

$$\phi_j(\beta,n,\kappa,\gamma) = F_1(\beta,n,\kappa,\gamma) * \phi_{j-1} + F_2(\beta,n,\kappa,\gamma) \text{ for } j \in \{3,...,n\}, \text{ where}$$

$$F_1(\beta,n,\kappa,\gamma) = \frac{4\gamma\,[j-1]\,p\left[\kappa\left[2\beta\,[j-3]\,j\kappa + 2\,[\beta-1]\,j - \kappa\,[j-2]^2 + 4\right] - 1\right]}{4j\kappa^2 p\left[2\beta\gamma\,[j-3]\,j + [4\beta-2]\,j - \gamma\,[j-2]^2 - 2\beta\,[n+3] + 5\right] + 8j\kappa p\,[\beta+\gamma\,[[\beta-1]\,j+2]-1] - 4\gamma jp}$$

$$F_2(\beta,n,\kappa,\gamma) = \frac{\kappa^4\,[-j^2 + 4\beta\,[[j-2]\,j+2] + 4\beta^2\,[j-3]\,j\,[n-1] - 2\beta\,[[j-3]\,j+4]\,n + j + 4]}{4j\kappa^2 p\left[2\beta\gamma\,[j-3]\,j + [4\beta-2]\,j - \gamma\,[j-2]^2 - 2\beta\,[n+3] + 5\right] + 8j\kappa p\,[\beta+\gamma\,[[\beta-1]\,j+2]-1] - 4\gamma jp}$$
$$+ \frac{2\kappa^3\,[-[[2\beta-1]\,j^2\,[\beta\,[n-1]+1]] + [2\beta-1]\,j\,[4\beta\,[n-1]+1] + 8\beta\,[n-1] - 6]}{4j\kappa^2 p\left[2\beta\gamma\,[j-3]\,j + [4\beta-2]\,j - \gamma\,[j-2]^2 - 2\beta\,[n+3] + 5\right] + 8j\kappa p\,[\beta+\gamma\,[[\beta-1]\,j+2]-1] - 4\gamma jp}$$

$$+\frac{\kappa^2\left[-j^2-4\beta^2 j\left[n-1\right]2\beta\left[j\left[j+n-2\right]-5n+5\right]+j+13\right]+\kappa\left[2\beta\left[n-1\right]-6\right]+1}{4j\kappa^2 p\left[2\beta\gamma\left[j-3\right]j+\left[4\beta-2\right]j-\gamma\left[j-2\right]^2-2\beta\left[n+3\right]+5\right]+8j\kappa p\left[\beta+\gamma\left[\left[\beta-1\right]j+2\right]-1\right]-4\gamma jp}$$

When $\gamma=0$, $\beta>\frac{1}{2}$ and $\kappa\leq\frac{1}{2}$, $\phi_2(\cdot)>0$ follows immediately. For $\phi_j(\cdot)$, $j\in\{3,...,n\}$, we investigate the general form of $\phi_j(\cdot)$ to see if the incentive requires tax (i.e., $\phi_j(\cdot)<0$) or subsidy (i.e., $\phi_j(\cdot)>0$). The sign of $\phi_j(\cdot)$ is linear in n: $slope*n+intercept$, where $intercept=\kappa\left[-4\beta^2 j\kappa\left[\left[j-3\right]\kappa+1\right]+2\beta\left[2\kappa-1\right]\left[\left[\left[j-2\right]j+2\right]\kappa-1\right]+j\kappa\left[-\kappa j+j+\kappa-1\right]+4\left[\kappa-2\right]\kappa+5\right]-1$ and $slope=2\beta\kappa\left[\kappa\left[\left[2\beta-1\right]\left[\left[j-3\right]\kappa j+j\right]-4\kappa+4\right]-1\right]$. We can show that $intercept<0$ is always true. For $slope$, if $\beta>\frac{4\kappa^2-\kappa+1}{6\kappa}$, then $slope>0$ for all $j\in\{3,...,n\}$. If $\beta<\frac{4\kappa^2-\kappa+1}{6\kappa}$, then $slope>0$ for $j\in\{\tilde{j},...,n\}$ and $slope<0$ for $j\in\{3,...,\tilde{j}\}$, where $\tilde{j}=\frac{\beta[6\kappa-2]+\sqrt{[2\beta-1]\left[2\beta[1-3\kappa]^2+[1-\kappa][3-7\kappa]\right]}-3\kappa+1}{2[2\beta-1]\kappa}$. When $slope<0$, then $\phi_j(\cdot)<0$. When $slope>0$, we can derive $\tilde{n}_j=\frac{\kappa\left[4\beta^2 j\kappa[[j-3]\kappa+1]-2\beta[2\kappa-1][[[j-2]j+2]\kappa-1]+\kappa[[j-1]j[\kappa-1]-4\kappa+8]-5\right]+1}{2\beta\kappa[\kappa[[2\beta-1][[j-3]\kappa j+j]-4\kappa+4]-1]}$ by solving $\phi_j(\cdot)=0$. If $n>\tilde{n}_j$, then $\phi_j(\cdot)>0$. If $n<\tilde{n}_j$, then $\phi_j(\cdot)<0$.

Summarizing the above results, we define $\hat{n}(\beta)=\tilde{n}_3=\frac{\kappa\left[2[1-10\beta]\kappa^2+2[\beta+1][6\beta+1]\kappa-2\beta-5\right]+1}{2\beta\kappa[6\beta\kappa-4\kappa^2+\kappa-1]}$, and $\hat{j}=\left\lfloor\frac{[2\beta-1]\kappa^3[6\beta[n-1]+1]-[2\beta-1]\kappa^2[2\beta[n-1]+1]+\sqrt{[1-2\beta]^2\kappa^4[\kappa+2\beta[3\kappa-1][n-1]-1]^2+4[2\beta-1][1-2\kappa]^2\kappa^2[\kappa+2\beta\kappa[n-1]-1][\kappa[2\beta[n-1]-1]+1]}}{2[2\beta-1]\kappa^2[\kappa+2\beta\kappa[n-1]-1]}\right\rfloor$ by solving $\tilde{n}_j=n$ and then choose the floor of the solution. If $\beta>\frac{4\kappa^2-\kappa+1}{6\kappa}$ and $n>\hat{n}(\beta)$, then $\phi_j(\cdot)>0$ for all $j\in\{3,...,n\}$. If $\beta<\frac{4\kappa^2-\kappa+1}{6\kappa}$, or if $\beta>\frac{4\kappa^2-\kappa+1}{6\kappa}$ and $n<\hat{n}(\beta)$, then $\phi_j(\cdot)<0$ for $j\in\{3,...,\hat{j}\}$ and $\phi_j(\cdot)>0$ for $j\in\{\hat{j},...,n\}$. Q.E.D.

Appendix B: Extension – Impact of Heterogeneous Districts

In this appendix, as an extension to our main model, we consider when districts differ in their resource investment costs and examine the impact of such heterogeneity across districts. In many settings districts have varying capabilities in the provisioning of IT infrastructure resources, implementing applications and integration with "backend" systems (e.g., EUDCC program), managing information exchange (e.g., the Direct Project), analyzing data, defending against cyberattack, etc.; therefore, districts are naturally heterogeneous. Such heterogeneity in IT provisioning capability can be captured by differences in cost per resource unit among different districts. Here we investigate the impact of heterogeneous districts with this heterogeneity.

We consider two types of districts that differ in their cost per resource unit where we denote the different costs as p_H and p_L, and where $p_H>p_L$. There are n_H high-cost districts and n_L low-cost districts, with total districts being $n=n_H+n_L$. Let $j_H\in\{0,...,n_H\}$ and $j_L\in\{0,...,n_L\}$ denote the number of high-cost (H) districts and low-cost (L) districts that are inside the coalition, respectively.

We use $B_{Hi}(j_H, j_L)$ to denote the benefits of high-cost district $i \in \{1, ..., n_H\}$ for a coalition of size $j_H + j_L$. Following the same composition of benefits as in the homogeneous districts case in equation (1), for high-cost districts we have

$$B_{Hi}(j_H, j_L) = [1-\kappa]\, g_{Hi} + \kappa \left[\sum_{q=1, q\neq i}^{j_H} g_{Hq} + \sum_{q=1}^{j_L} g_{Lq} \right] + \kappa\beta \left[\sum_{q=j_H+1}^{n_H} g_{Hq} + \sum_{q=j_L+1}^{n_L} g_{Lq} \right],$$

for inside H district $i \in \{1, ..., j_H\}$.

$$B_{Hi}(j_H, j_L) = [1-\kappa]\, g_{Hi} + \kappa\beta \left[\sum_{q=1}^{j_H} g_{Hq} + \sum_{q=1}^{j_L} g_{Lq} + \sum_{q=j_H+1, q\neq i}^{n_H} g_{Hq} + \sum_{q=j_L+1}^{n_L} g_{Lq} \right],$$

for outside H district $i \in \{j_H + 1, ..., n_H\}$.

The benefits of low-cost district $i \in \{1, ..., n_L\}$ follow a similar mathematical form and are given by

$$B_{Li}(j_H, j_L) = [1-\kappa]\, g_{Li} + \kappa \left[\sum_{q=1}^{j_H} g_{Hq} + \sum_{q=1, q\neq i}^{j_L} g_{Lq} \right] + \kappa\beta \left[\sum_{q=j_H+1}^{n_H} g_{Hq} + \sum_{q=j_L+1}^{n_L} g_{Lq} \right],$$

for inside L district $i \in \{1, ..., j_L\}$, and

$$B_{Li}(j_H, j_L) = [1-\kappa]\, g_{Li} + \kappa\beta \left[\sum_{q=1}^{j_H} g_{Hq} + \sum_{q=1}^{j_L} g_{Lq} + \sum_{q=j_H+1}^{n_H} g_{Hq} + \sum_{q=j_L+1, q\neq i}^{n_L} g_{Lq} \right],$$

for outside L district $i \in \{j_L + 1, ..., n_L\}$.

We use C_{Hi} and C_{Li} to denote the resource cost of each outside H district and each outside L district, respectively. And we use $TC(j_H, j_L)$ to denote the total cost of resources in a coalition. For any district outside the coalition, costs are:

$$C_{Hi} = p_H g_{Hi}^2 \text{ for outside H districts } i \in \{j_H + 1, ..., n_H\}$$

$$C_{Li} = p_L g_{Li}^2 \text{ for outside L districts } i \in \{j_L + 1, ..., n_L\}.$$

The total cost of resources in the coalition is:

$$TC(j_H, j_L) = \frac{\alpha\, [j_H p_H + j_L p_L]}{j_H + j_L} \left[\sum_{i=1}^{j_H} g_{Hi} + \sum_{i=1}^{j_L} g_{Li} \right]^2,$$

where $j_H \in \{0, ..., n_H\}$, $j_L \in \{0, ..., n_L\}$, and $j_H + j_L \in \{2, ..., n_H + n_L\}$. Here, α captures the benefits of the coalition economy of scale and $\frac{j_H p_H + j_L p_L}{j_H + j_L}$ represents the average cost per resource unit for the coalition.

The surplus for each district inside the coalition is given by:

$$S_{Hi}(j_H,j_L)=B_{Hi}(j_H,j_L)-\frac{TC_H(j_H,j_L)}{j_H}$$
$$=[1-\kappa]\,g_{Hi}+\kappa\left[\sum_{q=1,q\neq i}^{j_H} g_{Hq}+\sum_{q=1}^{j_L} g_{Lq}\right]+\kappa\beta\left[\sum_{q=j_H+1}^{n_H} g_{Hq}+\sum_{q=j_L+1}^{n_L} g_{Lq}\right]-\frac{\alpha p_H}{j_H+j_L}\left[\sum_{i=1}^{j_H} g_{Hi}+\sum_{i=1}^{j_L} g_{Li}\right]^2,$$

for inside H district $i\in\{1,...,j_H\}$, and

$$S_{Li}(j_H,j_L)=B_{Li}(j_H,j_L)-\frac{TC_L(j_H,j_L)}{j_L}$$
$$=[1-\kappa]\,g_{Li}+\kappa\left[\sum_{q=1}^{j_H} g_{Hq}+\sum_{q=1,q\neq i}^{j_L} g_{Lq}\right]+\kappa\beta\left[\sum_{q=j_H+1}^{n_H} g_{Hq}+\sum_{q=j_L+1}^{n_L} g_{Lq}\right]-\frac{\alpha p_L}{j_H+j_L}\left[\sum_{i=1}^{j_H} g_{Hi}+\sum_{i=1}^{j_L} g_{Li}\right]^2,$$

for inside L district $i\in\{1,...,j_L\}$,

where $TC_H(j_H,j_L)$ and $TC_L(j_H,j_L)$ represents the portions of the total cost $TC(j_H,j_L)$ that can be attributed to H districts and L districts in the coalition, respectively. For the same cost-type of district (H or L), we take costs as equally shared. The surplus for each district outside the coalition is given by:

$$S_{Hi}(j_H,j_L)=B_{Hi}(j_H,j_L)-C_{H_i}$$
$$=[1-\kappa]\,g_{Hi}+\kappa\beta\left[\sum_{q=1}^{j_H} g_{Hq}+\sum_{q=1}^{j_L} g_{Lq}+\sum_{q=j_H+1,q\neq i}^{n_H} g_{Hq}+\sum_{q=j_L+1}^{n_L} g_{Lq}\right]-p_H g_{Hi}^2$$

for outside H district $i\in\{j_H+1,...,n_H\}$, and

$$S_{Li}(j_H,j_L)=B_{Li}(j_H,j_L)-C_{L_i}$$
$$=[1-\kappa]\,g_{Li}+\kappa\beta\left[\sum_{q=1}^{j_H} g_{Hq}+\sum_{q=1}^{j_L} g_{Lq}+\sum_{q=j_H+1}^{n_H} g_{Hq}+\sum_{q=j_L+1,q\neq i}^{n_L} g_{Lq}\right]-p_L g_{Li}^2$$

for outside L district $i\in\{j_L+1,...,n_L\}$.

B.1. Resource Levels with Heterogeneous Districts

The coalition coordinator maximizes coalition welfare by choosing resource levels for districts inside the coalition. We use $CW(j_H,j_L)$ to denote the coalition welfare for a coalition of size j_H+j_L, which is the total benefits from infrastructure resources of all inside districts minus the total cost of resources to the coalition:

$$CW(j_H,j_L)=\left[\sum_{i=1}^{j_H} B_{Hi}(j_H,j_L)+\sum_{i=1}^{j_L} B_{Li}(j_H,j_L)\right]-TC(j_H,j_L)$$
$$=[1-\kappa]\left[\sum_{i=1}^{j_H} g_{Hi}+\sum_{i=1}^{j_L} g_{Li}\right]+\kappa\,[j_H+j_L-1]\left[\sum_{i=1}^{j_H} g_{Hi}+\sum_{i=1}^{j_L} g_{Li}\right]$$
$$+\kappa\beta\,[j_H+j_L]\left[\sum_{i=j_H+1}^{n_H} g_{Hi}+\sum_{i=j_L+1}^{n_L} g_{Li}\right]-\frac{\alpha\,[j_H p_H+j_L p_L]}{j_H+j_L}\left[\sum_{i=1}^{j_H} g_{Hi}+\sum_{i=1}^{j_L} g_{Li}\right]^2.$$

The decision problem for the coalition coordinator with coalition size $j_H + j_L$ is:

$$\max_{g_{Hi}, i\in\{1,\dots,j_H\}; g_{Li}, i\in\{1,\dots,j_L\}} CW(j_H,j_L) = \left[\sum_{i=1}^{j_H} B_{Hi}(j_H,j_L) + \sum_{i=1}^{j_L} B_{Li}(j_H,j_L)\right] - TC(j_H,j_L)$$

$$\text{Subject to: } 0 \le g_{Hi} \le \bar{g} \text{ for inside H district } i \in \{1,...,j_H\}, \tag{B.1}$$

$$0 \le g_{Li} \le \bar{g} \text{ for inside L district } i \in \{1,...,j_L\},$$

$$S_{Hi}(j_H,j_L) = S_{Li}(j_H,j_L).$$

The constraint $S_{Hi}(j_H,j_L) = S_{Li}(j_H,j_L)$ ensures H districts and L districts are treated equally within the coalition, and results in a unique solution to formulation (B.1). The rationale behind treating high-cost and low-cost districts equally within the coalition is to ensure fairness and equity in the distribution of benefits and costs among the districts.

When solving the coalition coordinator's maximization problem in formulation (B.1), without the last constraint (the equal-treatment constraint), we have infinite solutions for optimal resource investments g_H and g_L. Specifically, the coalition welfare is maximized as long as g_H and g_L satisfy equation $j_H g_H + j_L g_L = \frac{[[j_H+j_L][1+\kappa[j_H+j_L-2]]]}{[2\alpha[j_H p_H + j_L p_L]]}$. The equal-treatment constraint helps impart structure on the solution. With this last constraint in, we can derive a unique solution for optimal resource investments for inside districts:

$$\begin{aligned} g^*_{H1} &= ... = g^*_{Hj_H} = g_{inH} \\ &= \frac{[1+\kappa[j_H+j_L-2]]\left[j_L p_L[1-\kappa[j_H+j_L+2]] + j_H p_H[2-4\kappa+\kappa j_L] + j_L p_H[1-2\kappa+\kappa j_L]\right]}{4\alpha[1-2\kappa][j_H p_H + j_L p_L]^2}, \end{aligned}$$

$$\begin{aligned} g^*_{L1} &= ... = g^*_{Lj_L} = g_{inL} \\ &= \frac{[1+\kappa[j_H+j_L-2]]\left[j_H p_H[1-\kappa[j_H+j_L+2]] + j_L p_L[2-4\kappa+\kappa j_H] + j_H p_L[1-2\kappa+\kappa j_H]\right]}{4\alpha[1-2\kappa][j_H p_H + j_L p_L]^2}. \end{aligned}$$

Correspondingly, $S^*_{H1}(j_H,j_L) = ... = S^*_{Hj_H}(j_H,j_L) = S^*_{L1}(j_H,j_L)... = S^*_{Lj_L}(j_H,j_L) = S_{in}(j_H,j_L)$. In other words, as per the equal-treatment constraint all districts in the coalition have the same surplus level, denoted by $S_{in}(j_H,j_L)$.

If a district chooses not to join the coalition and to build its IT infrastructure outside the coalition, then it chooses a resource level to maximize surplus for its own district. For high-cost outside districts we have

$$\max_{g_{Hi}} S_{Hi}(j_H,j_L) = B_{Hi}(j_H,j_L) - C_{Hi}$$

$$\text{Subject to: } 0 \le g_{Hi} \le \bar{g} \text{ for outside H district } i \in \{j_H+1,...,n_H\},$$

and similarly for low-cost outside districts.

Solving these outside districts' decision problems, we find that the optimal resource levels are

$$g^*_{j_H+1} = ... = g^*_{n_H} = g_{outH} = \frac{1-\kappa}{2p_H}, \text{ for outside H district } j_H+1, ..., n_H \text{ and}$$

$$g^*_{j_L+1} = ... = g^*_{n_L} = g_{outL} = \frac{1-\kappa}{2p_L}, \text{ for outside L district } j_L+1, ..., n_L,$$

which is identical to the mathematical form of optimal resource levels for outside districts when districts are homogeneous in equation (7). Correspondingly, $S^*_{j_H+1}(j_H, j_L) = ... = S^*_{n_H}(j_H, j_L) = S_{outH}(j_H, j_L)$ and all outside H districts have the same surplus level. Similarly, $S^*_{j_L+1}(j_H, j_L) = ... = S^*_{n_L}(j_H, j_L) = S_{outL}(j_H, j_L)$ and all outside L districts have the same surplus level. For the special case of singleton, where no coalition is formed, all H districts have the same surplus, denoted by $S_{singleH}$, and all L districts have the same surplus, denoted by $S_{singleL}$.

B.2. Equilibrium Analysis with Heterogeneous Districts

We now analyze the equilibrium coalition structures with heterogeneous districts. We analyze all possible coalition structures: grand coalition, partial coalition, minimal coalition and singleton. Moreover, with heterogeneous districts, partial and minimal coalition structures can be further categorized into pure high-cost, pure low-cost and hybrid coalition structures (i.e., a mixture of H and L districts inside the coalition).

Similar to the analysis in Section 5 of the main text, we present the necessary and sufficient conditions for different equilibrium coalition structures. As indicated in the main text, these conditions ensure that given other districts' coalition participation choices, a district inside the coalition does not have the incentive to leave the coalition and a district outside the coalition does not have the incentive to join the coalition. Noting that there are additional conditions to accommodate the different combinations of high- and low-cost districts that could make up a minimal or partial coalition. These necessary and sufficient conditions capture incentive compatibility such that no district inside the coalition has an incentive to leave the coalition, and no district outside the coalition has an incentive to join the coalition.

The grand coalition is an equilibrium, where all districts are in the coalition with $j^*_H = n_H$ and $j^*_L = n_L$, if and only if:

$$S_{in}(j_H = n_H, j_L = n_L) \geq S_{outH}(j_H = n_H - 1, j_L = n_L),$$

$$S_{in}(j_H = n_H, j_L = n_L) \geq S_{outL}(j_H = n_H, j_L = n_L - 1).$$

A partial coalition with size $j^*_{eqmH} + j^*_{eqmL}$, where $j^*_{eqmH} \in \{0, ..., j_H\}$, $j^*_{eqmL} \in \{0, ..., j_L\}$ and $\left[j^*_{eqmL} + j^*_{eqmH}\right] \in \{3, ..., n_H + n_L - 1\}$, can be further categorized into three possible coalition structures. The pure high-cost partial coalition, with only H districts in the coalition, is an equilibrium if and only if:

$$S_{in}(j_H = j^*_{eqmH}, j_L = 0) \geq S_{outH}(j_H = j^*_{eqmH} - 1, j_L = 0),$$

$$S_{outH}(j_H = j^*_{eqmH}, j_L = 0) \geq S_{in}(j_H = j^*_{eqmH} + 1, j_L = 0),$$

$$S_{outL}(j_H = j^*_{eqmH}, j_L = 0) \geq S_{in}(j_H = j^*_{eqmH}, j_L = 1).$$

The pure low-cost partial coalition, with only L districts in the coalition, is an equilibrium if and only if:

$$S_{in}(j_H = 0, j_L = j^*_{eqmL}) \geq S_{outL}(j_H = 0, j_L = j^*_{eqmL} - 1),$$

$$S_{outH}(j_H = 0, j_L = j^*_{eqmL}) \geq S_{in}(j_H = 1, j_L = j^*_{eqmL}),$$

$$S_{outL}(j_H = 0, j_L = j^*_{eqmL}) \geq S_{in}(j_H = 0, j_L = j^*_{eqmL} + 1).$$

The hybrid partial coalition is an equilibrium if and only if:

$$S_{in}(j_H = j^*_{eqmH}, j_L = j^*_{eqmL}) \geq S_{outH}(j_H = j^*_{eqmH} - 1, j_L = j^*_{eqmL}),$$

$$S_{in}(j_H = j^*_{eqmH}, j_L = j^*_{eqmL}) \geq S_{outL}(j_H = j^*_{eqmH}, j_L = j^*_{eqmL} - 1),$$

$$S_{outH}(j_H = j^*_{eqmH}, j_L = j^*_{eqmL}) \geq S_{in}(j_H = j^*_{eqmH} + 1, j_L = j^*_{eqmL}),$$

$$S_{outL}(j_H = j^*_{eqmH}, j_L = j^*_{eqmL}) \geq S_{in}(j_H = j^*_{eqmH}, j_L = j^*_{eqmL} + 1).$$

Similar to the partial coalition, a minimal coalition can also be further categorized into three possible coalition structures. The pure high-cost minimal coalition, with only two high-cost districts in the coalition: $j^*_H = 2$ and $j^*_L = 0$, is an equilibrium if and only if:

$$S_{in}(j_H = 2, j_L = 0) \geq S_{singleH},$$

$$S_{outH}(j_H = 2, j_L = 0) \geq S_{in}(j_H = 3, j_L = 0),$$

$$S_{outL}(j_H = 2, j_L = 0) \geq S_{in}(j_H = 2, j_L = 1).$$

The pure low-cost minimal coalition, with only two low-cost districts in the coalition: $j^*_H = 0$ and $j^*_L = 2$, is an equilibrium if and only if:

$$S_{in}(j_H = 0, j_L = 2) \geq S_{singleL},$$

$$S_{outH}(j_H=0,j_L=2)\geq S_{in}(j_H=1,j_L=2),$$

$$S_{outL}(j_H=0,j_L=2)\geq S_{in}(j_H=0,j_L=3).$$

The hybrid minimal coalition, with one low-cost district and one high-cost district in the coalition: $j_H^*=1$ and $j_L^*=1$, is an equilibrium if and only if:

$$S_{in}(j_H=1,j_L=1)\geq S_{singleH},$$

$$S_{in}(j_H=1,j_L=1)\geq S_{singleL},$$

$$S_{outH}(j_H=1,j_L=1)\geq S_{in}(j_H=2,j_L=1),$$

$$S_{outL}(j_H=1,j_L=1)\geq S_{in}(j_H=1,j_L=2).$$

Singleton, where no district chooses to join the coalition, is an equilibrium if and only if:

$$S_{singleH}\geq S_{in}(j_H=2,j_L=0),$$

$$S_{singleL}\geq S_{in}(j_H=0,j_L=2),$$

$$S_{singleH}\geq S_{in}(j_H=1,j_L=1),$$

$$S_{singleL}\geq S_{in}(j_H=1,j_L=1).$$

Our next proposition shows that the relative incentives of the different district cost-types to join the coalition depends on the coalition economy of scale such that a greater coalition economy of scale, a lower α, favors low-cost districts and vice-versa for high-cost districts.

Proposition B.1 **(Heterogeneous Districts' Incentive to Join the Coalition)**. *L districts have a higher incentive than H districts to join the coalition when α is relatively low, i.e., $S_{in}(j_H,j_L)-S_{outL}(j_H,j_L-1)>S_{in}(j_H,j_L)-S_{outH}(j_H-1,j_L)$, which can be reduced to $S_{outL}(j_H,j_L-1)<S_{outH}(j_H-1,j_L)$, when $\alpha<\frac{2\kappa\beta p_H p_L[j_H+j_L-1][\kappa[j_H+j_L-3]+1]}{[1-\kappa]^2[j_H p_H+[j_L-1]p_L][[j_H-1]p_H+j_L p_L]}$; Otherwise, H districts have a higher incentive to join the coalition.*

Proof: Proof of Proposition B.1 can be found in Appendix B.4.

With a greater coalition economy of scale (a lower α), all districts have a higher incentive to join the coalition. Specifically, districts' surpluses from joining the coalition and staying outside the coalition are both higher because the resource level for inside districts is higher and the higher resource level benefits both inside and outside districts through spillovers. However, a lower α has differential impacts on H and

L districts: a lower α increases L districts' surplus from staying outside the coalition relatively less when compared to the surplus increment for H districts. Consequently, as α decreases, L districts' incentive to join the coalition increases more than that of H districts.

Next, we solve for the equilibrium coalition size that leads to a set of equilibrium coalition structures. Again, similar to the analysis in Section 5, we develop a set of threshold values for the coalition economy of scale, $\alpha_{eqm4}(\beta)$ to $\alpha_{eqm9}(\beta)$, that determine which coalition structures are possible equilibria and when they occur. Proposition B.2 summarizes the different equilibrium coalition structures. The definitions of threshold values $\alpha_{eqm4}(\beta)$ to $\alpha_{eqm9}(\beta)$ can be found in the proof of Proposition B.2.

PROPOSITION B.2 **(Equilibrium Coalition Structures with Heterogeneous Districts)**. *All four coalition structures are possible equilibrium:*

(a) When $\alpha < \alpha_{eqm4}(\beta)$, the equilibrium is the grand coalition with $j^*_{eqmH} = n_H$ and $j^*_{eqmL} = n_L$;

(b) When $\alpha_{eqm5}(\beta) < \alpha < \alpha_{eqm6}(\beta)$, the equilibrium is a partial coalition with three possible combinations of districts: pure high-cost partial coalition ($j^*_{eqmH} = j_{eqm}, j^*_{eqmL} = 0$), pure low-cost partial coalition ($j^*_{eqmH} = 0, j^*_{eqmL} = j_{eqm}$), or hybrid partial coalition ($j^*_{eqmH} = j_{eqmH}, j^*_{eqmL} = j_{eqm} - j_{eqmH}$), where $j_{eqm} \in \{3, ..., n_H + n_L - 1\}$ and $j_{eqmH} \in \{1, ..., j_{eqm} - 1\}$;

(c) When $\alpha_{eqm7}(\beta) < \alpha < \alpha_{eqm8}(\beta)$, the equilibrium is the minimal coalition with three possible combinations of districts: pure high-cost minimal coalition ($j^*_{eqmH} = 2, j^*_{eqmL} = 0$), pure low-cost minimal coalition ($j^*_{eqmH} = 0, j^*_{eqmL} = 2$), or hybrid minimal coalition ($j^*_{eqmH} = 1, j^*_{eqmL} = 1$);

(d) When $\alpha > \alpha_{eqm9}(\beta)$, the equilibrium is a singleton.

Proof: Proof of Proposition B.2 can be found in Appendix B.4.

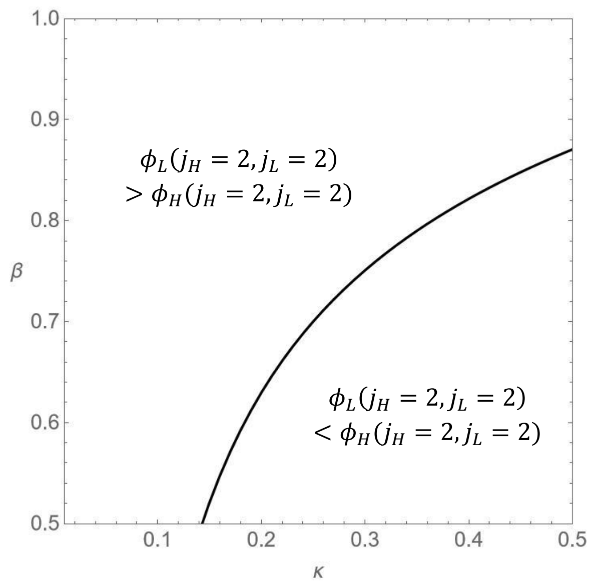
The equilibrium results presented in Proposition B.2 with heterogenous districts are similar to the results with homogeneous districts. As the coalition economy of scale increases, a coalition structure with a greater number of districts is the equilibrium. In other words, as α becomes smaller, the equilibrium coalition structure changes from singleton, to minimal, to partial, and eventually to the grand coalition. Additionally, for minimal and partial coalition structures, all three combinations of districts – pure high-cost, pure low-cost and hybrid coalitions are possible equilibrium outcomes.

What is critical about Proposition B.2 is that with heterogeneous districts defined by differences in their costs each coalition structure can be an equilibrium, qualitatively replicating the result in Proposition 2.

Moreover, the pattern of transitions between coalition structures as the coalition economy of scale changes is the same regardless of whether districts are homogeneous or heterogeneous.

B.3. Numerical Incentive Analysis with Heterogeneous Districts

Because there are so many coalition forms with overlapping regions within the parameter space, analytical analyses are not feasible. In this subsection, we instead resort to numerical analyses. Specifically, we consider a scenario where there are two high-cost and two low-cost districts, i.e., $n_H = n_L = 2$. In this setting, there exist seven possible coalition forms including: singleton, pure high-cost minimal coalition ($j_H = 2, j_L = 0$), pure low-cost minimal coalition ($j_H = 0, j_L = 2$), hybrid minimal coalition ($j_H = j_L = 1$), high-cost majority partial coalition ($j_H = 2, j_L = 1$), low-cost majority partial coalition ($j_H = 1, j_L = 2$), and grand coalition ($j_H = 2, j_L = 2$). In order to compare the incentives provided to the two types of districts, we consider the incentives needed to induce the socially optimal grand coalition ($j_H = 2, j_L = 2$). Following a similar procedure as described in Section 7, we find that if $\beta > \frac{[\kappa[5\kappa+2]-1]p_H+[\kappa[13\kappa+10]+1]p_L}{6\kappa[\kappa+1][p_H+p_L]}$, then the social planner should provide more incentives to low-cost districts, i.e., $\phi_L(j_H = 2, j_L = 2) > \phi_H(j_H = 2, j_L = 2)$; otherwise, the social planner should provide more incentives to high-cost districts, i.e., $\phi_H(j_H = 2, j_L = 2) > \phi_L(j_H = 2, j_L = 2)$. Figure B1 illustrate these results of incentive analysis for the two high-cost and two low-cost districts scenario.



Notes: This figure is based on parameter value of $p_H = 2p_L$.

Figure B1. Incentive Analysis for Heterogenous Districts

Based on our numerical analysis when there are more than two high-cost and two low-cost districts, we observe that the incentives provided by the social planner should depend on the relative interoperability β. Consistent with the solution to the two high-cost and two low-cost districts scenario above, when β is low, the social planner should allocate more incentives to the high-cost districts (i.e., $\phi_H > \phi_L$) to encourage their participation in the grand coalition. Conversely, when β is high, the social planner should allocate more incentives to the low-cost districts (i.e., $\phi_L > \phi_H$) to ensure their participation in the grand coalition. The counter-intuitive finding that the social planner should allocate more incentives to low-cost districts when interoperability is high can be explained by the social planner's preference for keeping low-cost districts in the coalition. When interoperability is high, the high-cost districts are more likely to join the grand coalition, even with lower incentives, because they can enjoy the spillover benefits and pooling the cost with the low-cost districts. On the other hand, the low-cost districts may be less willing to join the grand coalition without significant incentives, because they can operate more efficiently on their own. Therefore, by allocating more incentives to the low-cost districts in this scenario, the social planner can ensure that they participate in the grand coalition, leading to a more socially optimal outcome.

B.4. Proofs of Propositions B.1 and B.2

Proof of Proposition B.1

Substituting g_{inH} and g_{inL} into $S_{in}(j_H, j_L)$, g_{outH} into $S_{outH}(j_H, j_L)$, and g_{outL} into $S_{outL}(j_H, j_L)$, we can compare H districts' incentives to join the coalition, i.e., $S_{in}(j_H, j_L) - S_{outH}(j_H - 1, j_L)$, with L districts' incentives to join the coalition, i.e., $S_{in}(j_H, j_L) - S_{outL}(j_H, j_L - 1)$. Given the constraint $S_{Hi}(j_H, j_L) = S_{Li}(j_H, j_L) = S_{in}(j_H, j_L)$ that ensures H districts and L districts are treated equally within the coalition, the incentive comparison between H and L districts can be reduced to the comparison between $S_{outH}(j_H - 1, j_L)$ and $S_{outL}(j_H, j_L - 1)$. When $\alpha < \frac{2\kappa\beta p_H p_L [j_H + j_L - 1][\kappa[j_H + j_L - 3] + 1]}{[1-\kappa]^2 [j_H p_H + [j_L - 1] p_L][[j_H - 1] p_H + j_L p_L]}$, we find that $S_{outL}(j_H, j_L - 1) < S_{outH}(j_H - 1, j_L)$, indicating that L districts have a higher incentive to join the coalition. Otherwise, $S_{outL}(j_H, j_L - 1) > S_{outH}(j_H - 1, j_L)$, indicating that H districts have a higher incentive. Q.E.D.

Proof of Proposition B.2

Solving the necessary and sufficient conditions for grand coalition to be an equilibrium, yields $\alpha < \alpha_{eqm4}(\beta) = min\{\alpha_{eqm4H}, \alpha_{eqm4L}\}$, where α_{eqm4H} solves $S_{in}(j_H = n_H, j_L = n_L) = S_{outH}(j_H = n_H - 1, j_L = n_L)$ and α_{eqm4L} solves $S_{in}(j_H = n_H, j_L = n_L) = S_{outL}(j_H = n_H, j_L = n_L - 1)$.

Solving the necessary and sufficient conditions for singleton to be an equilibrium, yields $\alpha > \alpha_{eqm9}(\beta) = max\{\alpha_{eqm9H1}, \alpha_{eqm9H2}, \alpha_{eqm9L1}, \alpha_{eqm9L2}\}$, where α_{eqm9H1} solves $S_{singleH} = S_{in}(j_H = 2, j_L = 0)$, α_{eqm9H2} solves $S_{singleH} = S_{in}(j_H = 1, j_L = 1)$, α_{eqm9L1} solves $S_{singleL} = S_{in}(j_H = 0, j_L = 2)$, and α_{eqm9L2} solves $S_{singleL} = S_{in}(j_H = 1, j_L = 1)$.

Solving the necessary and sufficient conditions for minimal coalition to be an equilibrium, yields $\alpha_{eqm7}(\beta) < \alpha < \alpha_{eqm8}(\beta)$. Here, threshold $\alpha_{eqm7}(\beta) = min\{\alpha_{eqm7PH}, \alpha_{eqm7PL}, \alpha_{eqm7H}\}$, where $\alpha_{eqm7PH} = max\{\alpha_{eqm7PH1}, \alpha_{eqm7PH2}\}$, $\alpha_{eqm7PL} = max\{\alpha_{eqm7PL1}, \alpha_{eqm7PL2}\}$, and $\alpha_{eqm7H} = max\{\alpha_{eqm7HH}, \alpha_{eqm7HL}\}$, corresponding to the lower bounds of α for pure high-cost, pure low-cost and hybrid minimal coalition, respectively; threshold $\alpha_{eqm8}(\beta) = max\{\alpha_{eqm8PH}, \alpha_{eqm8PL}, \alpha_{eqm8H}\}$, where α_{eqm8PH} solves $S_{in}(j_H = 2, j_L = 0) = S_{singleH}$, α_{eqm8PL} solves $S_{in}(j_H = 0, j_L = 2) = S_{singleL}$, and $\alpha_{eqm8H} = min\{\alpha_{eqm8HH}, \alpha_{eqm8HL}\}$, corresponding to the upper bounds of α for pure high-cost, pure low-cost and hybrid minimal coalition, respectively. Furthermore, $\alpha_{eqm7PH1}$ solves $S_{outH}(j_H = 2, j_L = 0) = S_{in}(j_H = 3, j_L = 0)$, $\alpha_{eqm7PH2}$ solves $S_{outL}(j_H = 2, j_L = 0) = S_{in}(j_H = 2, j_L = 1)$, $\alpha_{eqm7PL1}$ solves $S_{outH}(j_H = 0, j_L = 2) = S_{in}(j_H = 1, j_L = 2)$, $\alpha_{eqm7PL2}$ solves $S_{outL}(j_H = 0, j_L = 2) = S_{in}(j_H = 0, j_L = 3)$, α_{eqm7HH} solves $S_{outH}(j_H = 1, j_L = 1) = S_{in}(j_H = 2, j_L = 1)$, α_{eqm7HL} solves $S_{outL}(j_H = 1, j_L = 1) = S_{in}(j_H = 1, j_L = 2)$, α_{eqm8HH} solves $S_{in}(j_H = 1, j_L = 1) = S_{singleH}$, and α_{eqm8HL} solves $S_{in}(j_H = 1, j_L = 1) = S_{singleL}$.

Solving the necessary and sufficient conditions for partial coalition to be an equilibrium, yields $\alpha_{eqm5}(\beta) < \alpha < \alpha_{eqm6}(\beta)$. Here, threshold $\alpha_{eqm5}(\beta) = max\{\alpha_{eqm5H}, \alpha_{eqm5L}\}$, where α_{eqm5H} solves $S_{outH}(j_H = n_H - 1, j_L = n_L) = S_{in}(j_H = n_H, j_L = n_L)$ and α_{eqm5L} solves $S_{outL}(j_H = n_H, j_L = n_L - 1) = S_{in}(j_H = n_H, j_L = n_L)$. Threshold $\alpha_{eqm6}(\beta) = max\{\alpha_{eqm6PH}, \alpha_{eqm6PL}, \alpha_{eqm6HH}, \alpha_{eqm6HL}\}$, where α_{eqm6PH} solves $S_{in}(j_H = 3, j_L = 0) = S_{outH}(j_H = 2, j_L = 0)$ and α_{eqm6PL} solves $S_{in}(j_H = 0, j_L = 3) = S_{outL}(j_H = 0, j_L = 2)$, corresponding to the upper bounds of α for pure high-cost and pure low-cost partial coalition, respectively; $\alpha_{eqm6HH} = min\{\alpha_{eqm6HH1}, \alpha_{eqm6HH2}\}$ and $\alpha_{eqm6HL} = min\{\alpha_{eqm6HL1}, \alpha_{eqm6HL2}\}$, corresponding to the upper bounds of α for hybrid partial coalition. Furthermore, $\alpha_{eqm6HH1}$ solves $S_{in}(j_H = 2, j_L = 1) = S_{outH}(j_H = 1, j_L = 1)$, $\alpha_{eqm6HH2}$ solves $S_{in}(j_H = 2, j_L = 1) = S_{outL}(j_H = 2, j_L = 0)$, $\alpha_{eqm6HL1}$ solves $S_{in}(j_H = 1, j_L = 2) = S_{outH}(j_H = 0, j_L = 2)$, and $\alpha_{eqm6HL2}$ solves $S_{in}(j_H = 1, j_L = 2) = S_{outL}(j_H = 1, j_L = 1)$. Q.E.D.

Appendix C: Extension – Impact of Low Interoperability

In this appendix, as another extension to our main model, we consider when interoperability outside the coalition is less than 50% (i.e., $\beta \leq 1/2$) and examine the impact of such low range of relative interoperability.

Specifically, we examine our analysis of the social optimum when interoperability of districts outside the coalition relative to districts inside the coalition is lower than 1/2, that is, when our Assumption 1 is relaxed to $0 \leq \beta \leq 1$.

When $\beta \leq 1/2$, the social welfare function $SW(j) = CW(j) + [n-j]\, S_{out}(j)$ is no longer concave in j because $\partial SW^2(j)/\partial j^2 \geq 0$ when $\beta \leq 1/2$. Since $SW(j)$ is no longer concave in j, the socially optimal coalition structure is either singleton, minimal, or grand coalition. That is, the partial coalition cannot be socially optimal. Comparing the three possible socially optimal coalition structures, we derive the necessary and sufficient conditions for the different socially optimal coalition structures when $\beta \leq 1/2$ as follows: The grand coalition is socially optimal, i.e., $j^*_{sw} = n$, if and only if:

$$SW(j=n) \geq SW(j=2) \text{ and } SW(j=n) \geq SW_{single}.$$

The minimal coalition is socially optimal, i.e., $j^*_{sw} = 2$, if and only if:

$$SW(j=2) \geq SW(j=n) \text{ and } SW(j=2) \geq SW_{single}.$$

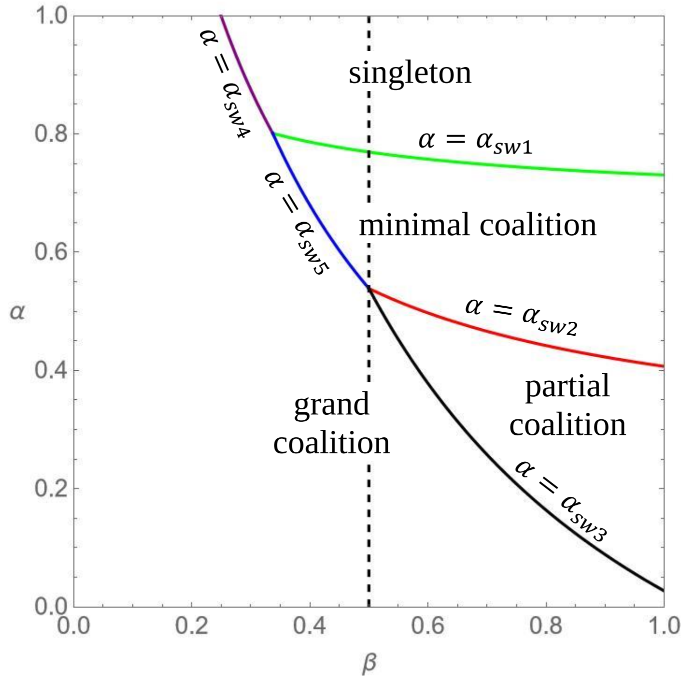
Singleton is socially optimal if and only if:

$$SW_{single} \geq SW(j=n) \text{ and } SW_{single} \geq SW(j=2).$$

Setting inequality $SW_{single} \geq SW(j=2)$ to equality, we obtain the same threshold value of α_{sw1} as in Section 6. Setting inequality $SW_{single} \geq SW(j=n)$ to equality, we obtain a new threshold value of $\alpha_{sw4} = \frac{[\kappa[n-2]+1]^2}{[1-\kappa]n[\kappa[2\beta[n-1]-1]+1]}$. Setting inequality $SW(j=n) \geq SW(j=2)$ to equality, we obtain another new threshold value of $\alpha_{sw5} = \frac{\kappa[-2\beta+\kappa[n-2]+2]}{[1-\kappa][\kappa[2\beta[n-1]-1]+1]}$.

The grand coalition is socially optimal, if and only if $\alpha < \alpha_{sw4}$ and $\alpha < \alpha_{sw5}$. The minimal coalition is socially optimal, if and only if $\alpha_{sw5} < \alpha < \alpha_{sw1}$. Lastly, singleton is socially optimal, if and only if $\alpha > \alpha_{sw4}$ and $\alpha > \alpha_{sw1}$.

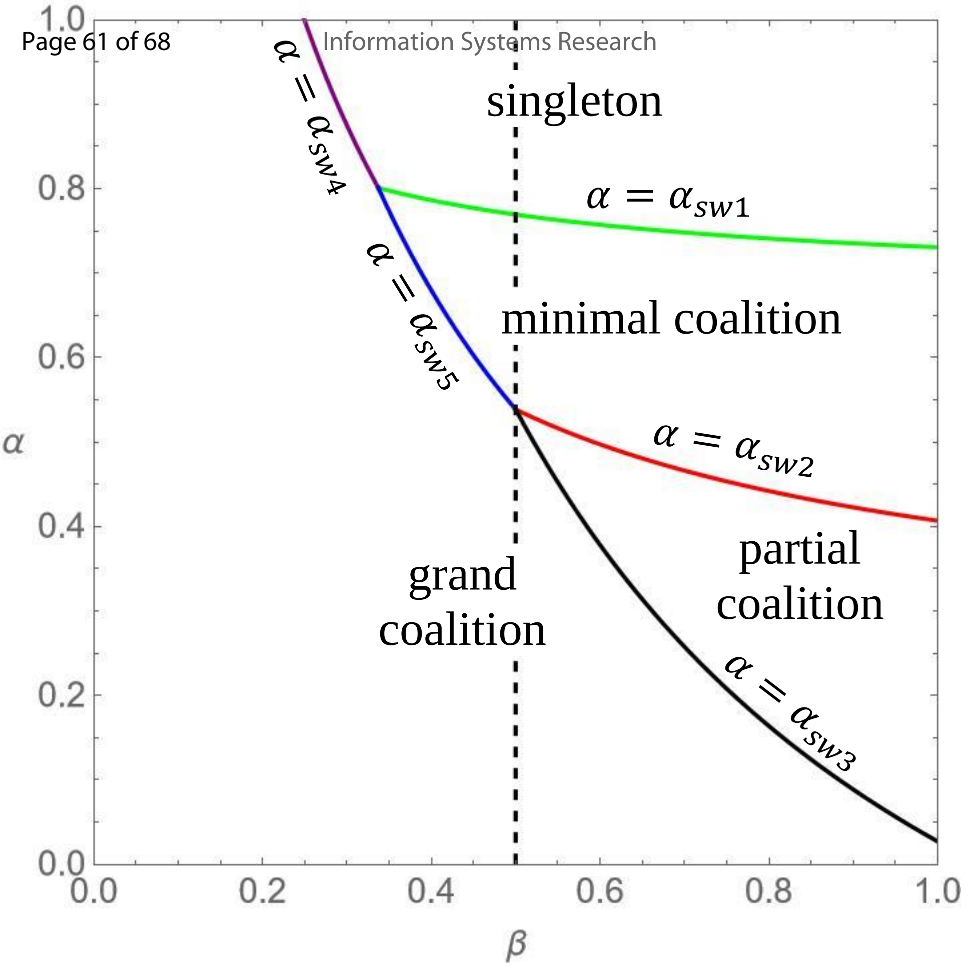
Next we combine our results under $\beta \leq 1/2$ analyzed here with results when $\beta > 1/2$ (our Assumption 1) as analyzed in Proposition 4 in Section 6. We can show that thresholds α_{sw1}, α_{sw4} and α_{sw5} are equal when $\beta = \frac{2\kappa[\kappa[n-2]+2]-1}{2\kappa n}$; thresholds α_{sw2}, α_{sw3} and α_{sw5} are equal when $\beta = 1/2$. As a result, these threshold values partition the parameter space (based on coalition economy of scale parameter $0 < \alpha < 1$ and relative interoperability parameter $0 \leq \beta \leq 1$) into four regions (corresponding to the four socially optimal coalition structures), which is illustrated in Figure B2.

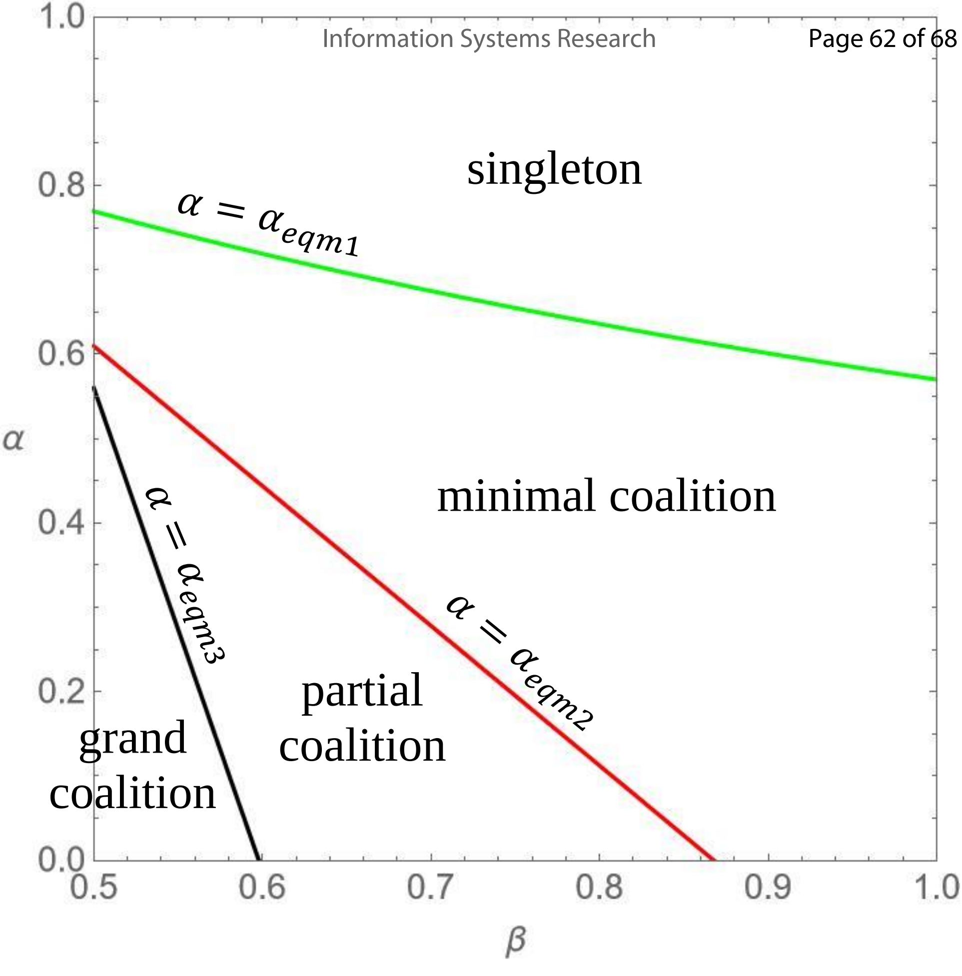


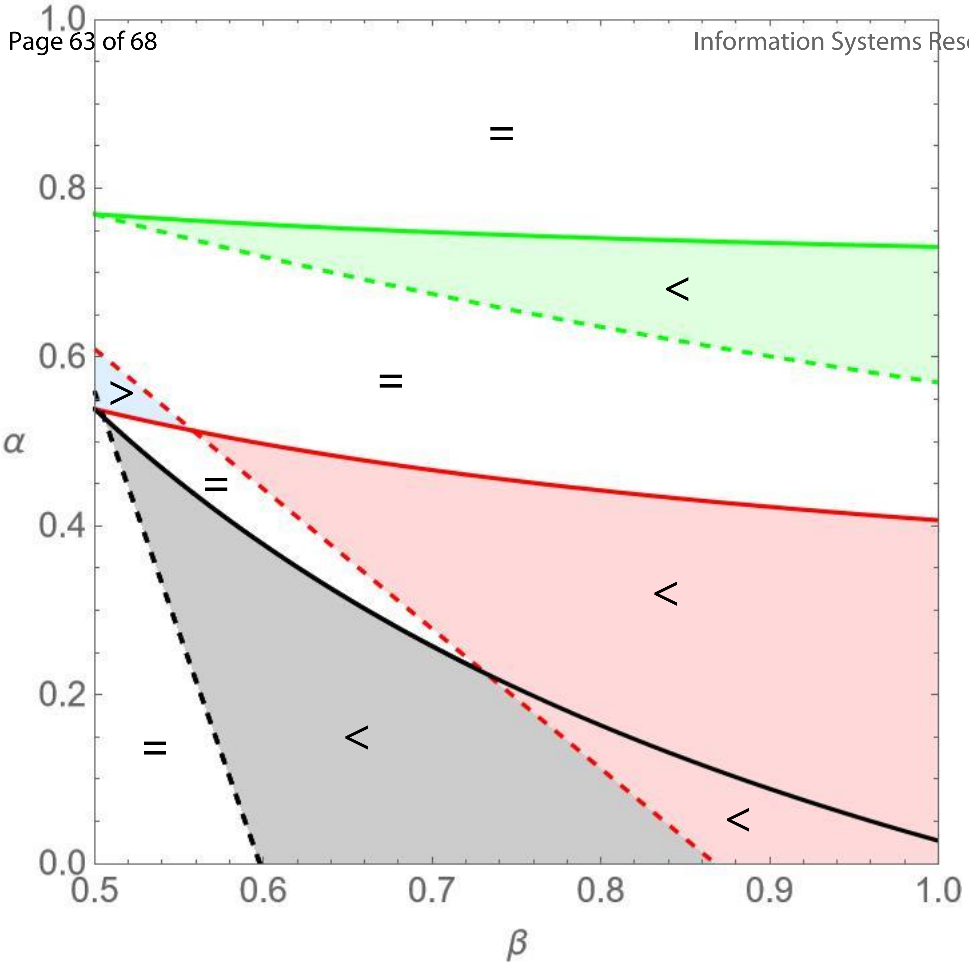
Notes: This figure is based on parameter values of $n = 10$ and $\kappa = 0.35$.

Figure B2. Socially Optimal Coalition Structures for $0 \leq \beta \leq 1$

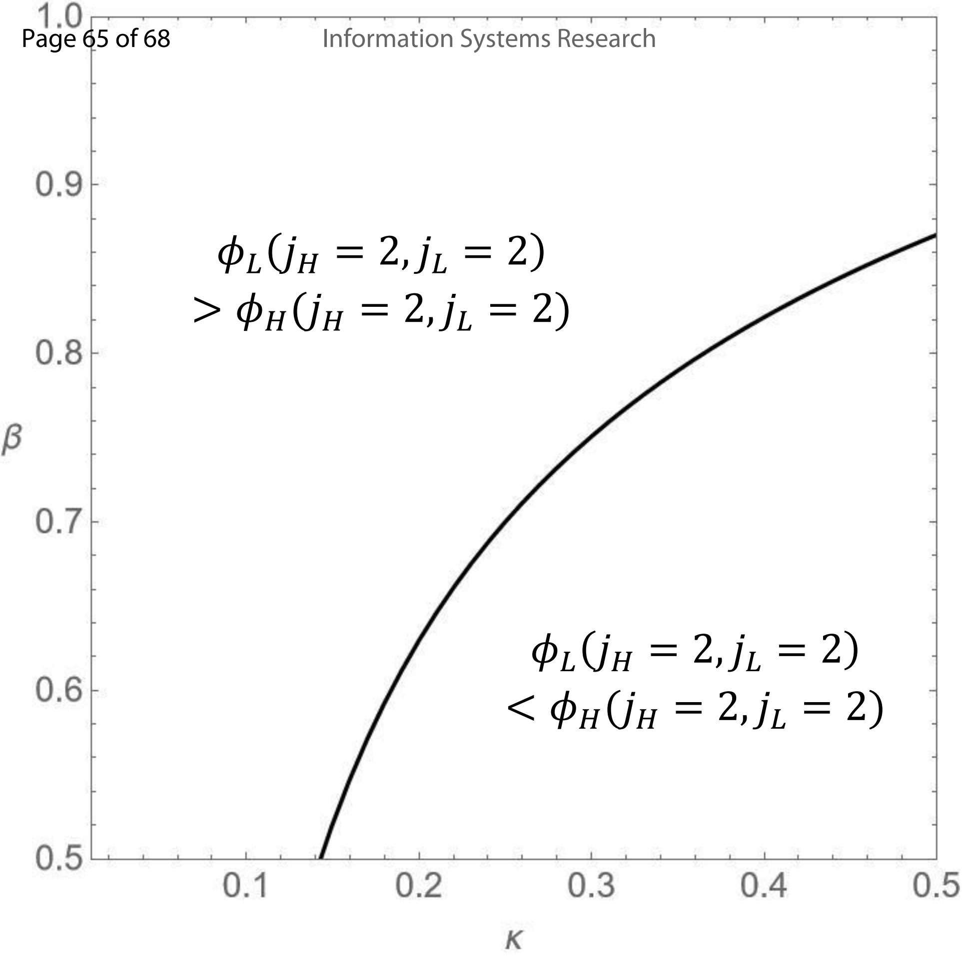
In Figure B2, the socially optimal coalition structure changes from no coalition (singleton) to minimal coalition to partial coalition and eventually to the grand coalition when moving from the upper right corner to the lower left corner because a greater coalition economy of scale (lower α) and lesser relative interoperability (lower β) both favor larger size coalition structures. This result for the socially optimal coalition structure under $0 \leq \beta \leq 1$ (illustrated in Figure B2) is qualitatively the same as that under $1/2 < \beta \leq 1$ (illustrated in Figure 2 in the main text).

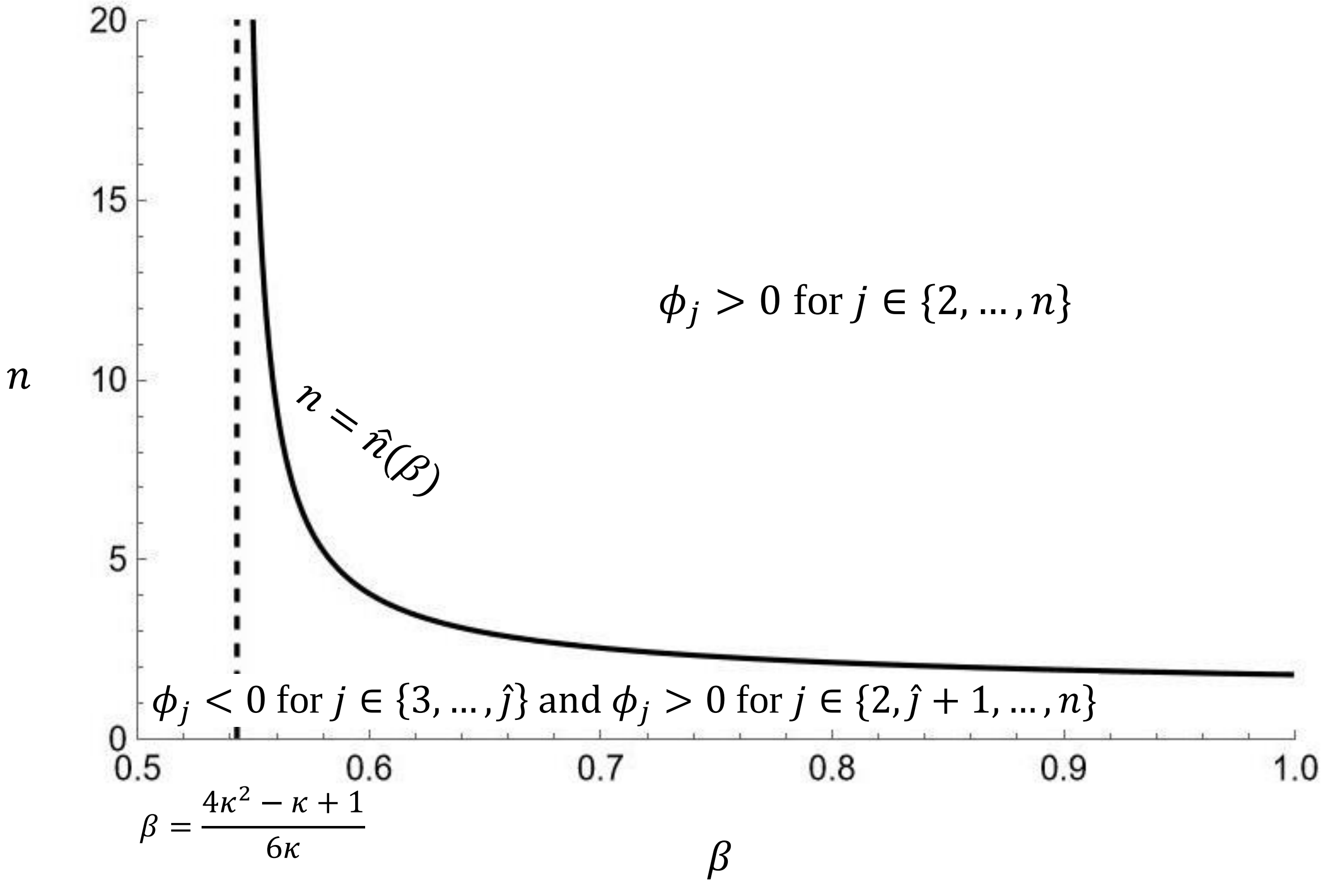

singleton
minimal coalition
grand coalition
partial coalition
$\alpha = \alpha_{sw1}$
$\alpha = \alpha_{sw2}$
$\alpha = \alpha_{sw3}$
$\alpha = \alpha_{sw4}$
$\alpha = \alpha_{sw5}$
α
β
0.0
0.2
0.4
0.6
0.8
1.0

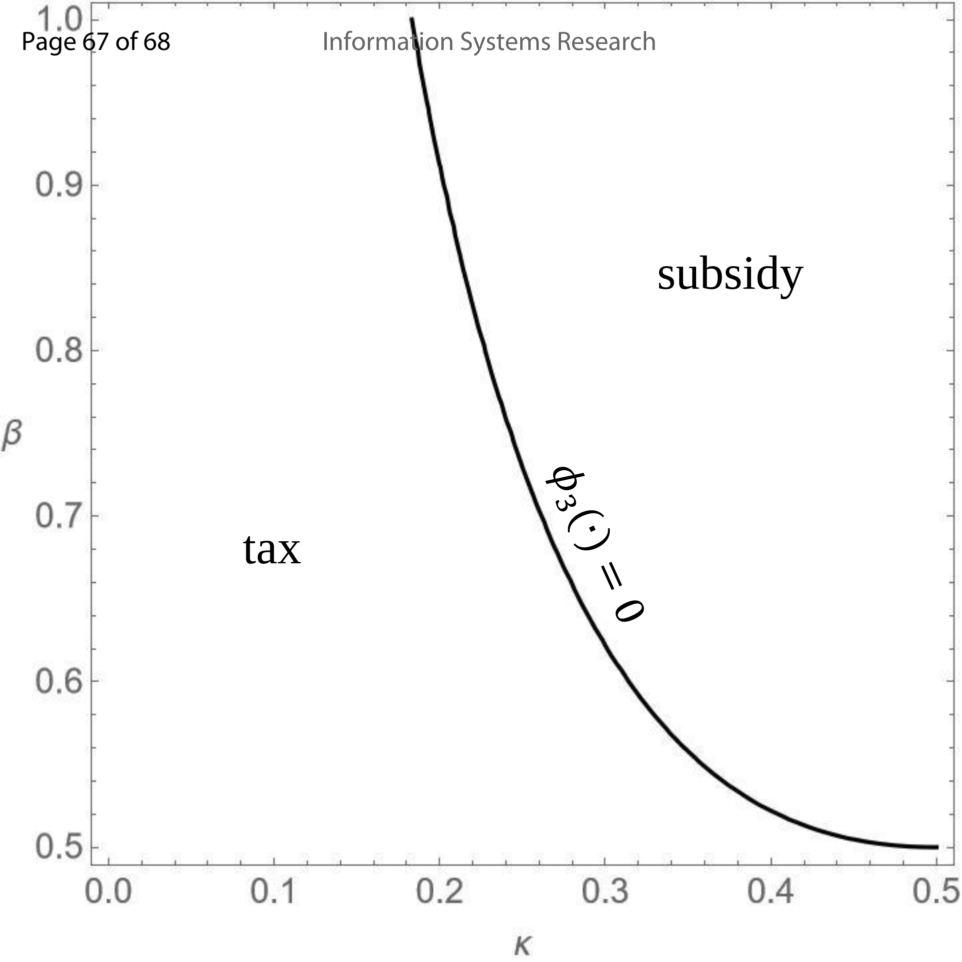

1.0
0.8
0.6
0.4
0.2
0.0
0.5
0.6
0.7
0.8
0.9
1.0
α
β
singleton
$\alpha = \alpha_{eqm1}$
minimal coalition
$\alpha = \alpha_{eqm2}$
$\alpha = \alpha_{eqm3}$
partial coalition
grand coalition

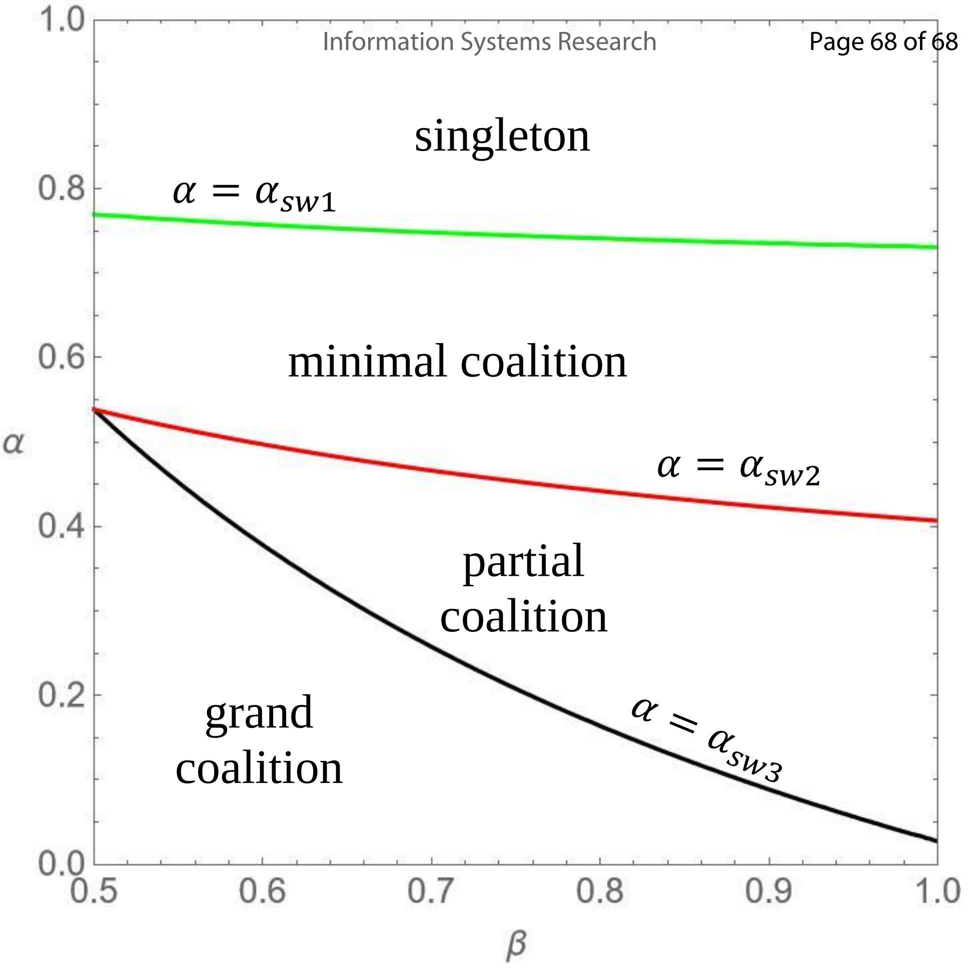


- Solid lines ——— : Separating lines for regions of socially optimal coalition structures
- Dashed lines - - - : Separating lines for regions of equilibrium coalition structures
- Symbol **<** : regions where the equilibrium coalition structure is *smaller than* the socially optimal coalition structure
- Symbol **=** : regions where the equilibrium coalition structure is *the same as* the socially optimal coalition structure
- Symbol **>** : regions where the equilibrium coalition structure is *greater than* the socially optimal coalition structure


1.0
0.9
0.8
0.7
0.6
0.5
β
0.1
0.2
0.3
0.4
0.5
κ
$\phi_L(j_H = 2, j_L = 2)$
$> \phi_H(j_H = 2, j_L = 2)$
$\phi_L(j_H = 2, j_L = 2)$
$< \phi_H(j_H = 2, j_L = 2)$


20
15
10
5
0
0.5
0.6
0.7
0.8
0.9
1.0
n
β
$\phi_j > 0$ for $j \in \{2, \ldots, n\}$
$n = \hat{n}(\beta)$
$\phi_j < 0$ for $j \in \{3, \ldots, \hat{j}\}$ and $\phi_j > 0$ for $j \in \{2, \hat{j}+1, \ldots, n\}$
$\beta = \frac{4\kappa^2 - \kappa + 1}{6\kappa}$


subsidy
tax
$\phi_3(\cdot) = 0$
β
κ
1.0
0.9
0.8
0.7
0.6
0.5
0.0
0.1
0.2
0.3
0.4
0.5


singleton
$\alpha = \alpha_{sw1}$
minimal coalition
$\alpha = \alpha_{sw2}$
partial coalition
grand coalition
$\alpha = \alpha_{sw3}$
1.0
0.8
0.6
0.4
0.2
0.0
α
0.5
0.6
0.7
0.8
0.9
1.0
β