

# Using Subsidies, Fines, and Restitution with Budget Balance to Combat Digital Piracy

Meysam Fereidouni

Beedie School of Business, Simon Fraser University, Burnaby, British Columbia V5A 1S6, Canada, meysam\_fereidouni@sfu.ca

Barrie R. Nault

Haskayne School of Business, University of Calgary, Calgary, Alberta T2N 1N4, Canada, nault@ucalgary.ca

Despite utilizing technical prevention methods and enacting copyright protection legislation, digital piracy has remained a persistent problem. We examine policy remedies to digital piracy whereby the policy-maker has to balance its budget between fines on detected pirates, subsidies for legal purchases, and restitution to the firm. In our model, users choose whether to subscribe, copy, or not use the good; a firm decides on subscription fee and quality; and a policy-maker determines subsidies, fines, and restitution. We find that the firm's subscription fee is always increasing in subsidies, fines, and restitution under a budget balance constraint. The impact of these policy instruments on the firm's investment in quality depends on how user marginal utility is influenced by the quality of the good and how the policy-maker redistributes fines back to society, if at all. Using two specific functional forms with additive and multiplicative utility, we explain how these factors come into play. With additive utility fines increase the firm's investment in quality. However, with multiplicative utility the firm's investment in quality decreases with fines if fines are used alone or alongside subsidies. In contrast to prior research, findings of our general framework illustrate that imposing fines on detected pirates "can" be socially optimal when accounting for the policy-maker's budget balance. Our specific functional forms also serve as two instances where imposing fines is always welfare maximizing. Finally, we find that although the policy-maker's optimal intervention improves social welfare and mitigates digital piracy, it leads to a reduction in consumer surplus.

*Key words:* digital piracy, pricing, copyright enforcement, restitution, budget balance, welfare

*History:* November 23, 2024

---

## 1. Introduction

In addition to their large development but negligible marginal costs, information goods exhibit other unique characteristics such as being piracy-prone. Digital piracy provides consumers with an opportunity to substitute purchases from legal distribution channels with illegal copies from unlicensed Internet sources. Although digital piracy may increase a digital firm's revenue by increasing the popularity of digital goods, its severe impact on industry and society is undeniable. Annually, digital piracy results in 70,000 job losses and costs more than \$12 billion in the United States (Eisend 2019). In 2018, the Business Software Alliance reported that almost 40% of software installed on

personal computers was unlicensed ([Business Software Alliance 2018](#)). Likewise, the International Federation of the Phonographic Industry estimated that 40% of consumers use illegal channels to listen to music ([International Federation of the Phonographic Industry 2018](#)). In addition to policy-makers' interventions, many organizations, such as the Recording Industry Association of America and Alliance for Creativity and Entertainment, have exerted efforts to fight digital piracy. Yet despite their efforts, digital piracy is widespread in a myriad of industries including movies, music, and software ([Lu et al. 2020](#)).

If digital piracy only reduces total sales by decreasing generated revenue (and not through production incentives), then it can be viewed as a welfare transfer from the firm to pirates, which does not reduce social welfare. In fact, under such a condition, piracy may even improve social welfare by reducing the deadweight loss from pirates whose willingness-to-pay is below the market price. In addition, prior studies such as [Lu et al. \(2020\)](#) argue that digital piracy can benefit firms producing digital goods by increasing word-of-mouth. Although this is true in some cases, the profitability of digital piracy for businesses is limited to particular goods and specific times. According to [Lu et al. \(2020\)](#), piracy that appears before a movie's theatrical premiere can reduce box office revenue by 11% and the effect of word-of-mouth from piracy that occurs after a movie's premiere can increase box office revenue by 3%. [Smith and Telang \(2016\)](#) show that piracy during the theatrical window significantly reduces sales in home-video release channels so that the total sales drop by 2-3%. Therefore, we believe that emphasizing the word-of-mouth aspect of digital piracy can be misleading as it creates a school of thought that firms' and policy-makers' efforts to fight digital piracy are counter-productive.

Although there are a few exceptions (e.g., [Oberholzer-Gee and Strumpf 2007](#), [Smith and Telang 2009](#)), most of the literature also acknowledges that digital piracy reduces the production of digital goods. Moreover, [Danaher et al. \(2014a\)](#) argue that policy-makers have a solid theoretical rationale to take socially negative impacts of digital piracy into account because they discourage firms from developing digital goods.

Firms and policy-makers can use antipiracy interventions to mitigate the negative impacts of digital piracy on firms' sales and on social welfare. Prior studies have shown the crucial role of firms in fighting digital piracy (e.g., [Chen and Png 2003](#), [Guo et al. 2019](#), [Dey et al. 2019](#)). However, with digital peer-to-peer (P2P) sharing developments like Napster, digital piracy has grown dramatically and file-sharing technology has evolved quickly. Subsequent applications such as BitTorrent operate over decentralized network architectures, facilitating the exchange of any type of digital good. These architectures make it extremely difficult for the movie and music industries to compete against BitTorrent because there is no single shutdown point. This has prompted firms to fight digital piracy by improving the quality of their digital goods or offering complementary services. To

incentivize firms to produce high quality digital goods, governments around the globe have enacted laws to encourage Internet service providers (ISPs) to diminish accessibility of illegal channels of digital goods using technologies such as deep packet inspection (Mo et al. 2017). For example, the Digital Millennium Copyright Act in the U.S. and the Copyright Modernization Act in Canada can require ISPs to provide identification of users accused of violation. However, ISPs often have no incentive to monitor piracy because illegal P2P file-sharing sites account for more than 20% of total Internet traffic (Mo et al. 2017). Moreover, P2P file-sharing sites also allow pirates to reroute their Internet traffic to bypass monitoring with services such as virtual private networks. As a result, pirates are detected with a probability which depends on the market behavior of ISPs, potential pirates, and policy-makers.

Over the last two decades, policy-makers have taken initial steps toward reinforcing the enforcement of intellectual property rights. Empirical evidence shows that the total sales of digital goods increases when policy-makers utilize copyright law, such as France's Creation and Internet Law (or HADOPI), and impose fines or other sanctions on pirates (e.g., Danaher et al. 2014b, Adermon and Liang 2014). This evidence supports the crucial role of policy-makers in fighting digital piracy. Despite a significant number of academic studies, a theoretical understanding of how both the policy-maker's instruments (e.g., imposing fines and offering subsidies) and the firm's decisions can affect digital piracy is not complete.

We use a general model to study the implications of policy-makers' interventions on digital piracy, as well as implications for the firm's profit, consumer surplus, and social welfare. In our formulation, users that differ in their valuation for the digital good choose among subscribing, copying, and not using the good. Although pirates copy the good at no cost, they might be detected. In the case of being detected, pirates are subject to monetary fines. To maximize social welfare, the policy-maker imposes fines on detected pirates, also known as a legal attack, and supports legal purchases by offering subsidies to those users that obtain the good through legitimate channels. The policy-maker can also pay the firm a fraction of its revenue from imposing fines as restitution for losses. It is worth noting that it is exceptionally challenging for policy-makers to subsidize legal purchases because of limited budget and resources. Therefore, in our formulation, the policy-maker solves the social welfare problem with the limitation that the budget must be balanced. The budget balance constraint implies that what the firm obtains from restitution plus what the policy-maker spends on subsidies for legal purchases cannot exceed society's revenue from imposing fines on detected pirates. Thus, two novel features of our formulation are partial restitution to the firm and a balanced budget.

Budget balance is crucial in evaluating the impact of policy instruments on social welfare. Without accounting for budget balance, accurately assessing the effects of policy instruments is challenging. Prior studies (e.g., Chen and Png 2003, Lahiri and Dey 2013) on digital piracy have

shown that social welfare decreases when fines are imposed on detected pirates. The underlying assumption in these studies is that the revenue from fines is simply retained by the policy-maker, without being redistributed into the economy. Moreover, other studies (e.g., [Chen and Png 2003](#)) have introduced subsidies as a means of improving welfare. However, these studies often operate under the assumption that the policy-maker has unlimited financial resources to fund such subsidies. Our findings, however, suggest that when a budget balance constraint is introduced, these conclusions may change significantly.

With our general model we find that although fines may initially harm social welfare, the policy-maker can mitigate or even reverse this effect by using the fine revenue to subsidize legal purchases or compensate the firm through restitution – the latter being the novel instrument we bring to the analysis. In both cases, the reallocation of fine revenue has the potential to increase social welfare to a level that offsets, and potentially exceeds, the initial negative impact of the fines. This dynamic suggests that the welfare consequences of fines depend crucially on how the resulting revenues are utilized. When paired with effective redistribution, fines may enhance social welfare, contrary to what the literature on digital piracy has traditionally concluded. Our analysis using both additive and multiplicative utility functions demonstrates that the policy maker always offsets the negative impact of fines on social welfare by using the fine revenue either as subsidies to encourage legal purchases or as restitution to the firm.

Based on user choices, we partition our analysis into following regions: (i) the *piracy region* where the policy-maker and the firm choose to allow copying; (ii) the *no piracy region with threat of copying*, or the limit pricing region, where the firm deters copying; and (iii) the *no piracy region without threat of copying* where the policy-maker deters copying. In the *piracy region*, our analysis yields a series of results. We first find that the subscription fee is increasing in the policy-maker's subsidies, fines, and restitution. Second, we show that firm's investment in quality varies non-monotonically with subsidies, fines, and restitution. Our findings also indicate that subsidies, fines, and restitution may encourage users to copy the digital good. This unintended consequence can be explained by policy instruments' impact on quality. In the *no piracy region with threat of copying*, we find that the firm's investment in quality decreases in fines if subscribers' valuation for quality is sufficiently higher than that of pirates. We also show that the *no piracy region without threat of copying* effectively represents a monopoly market where the firm sets decisions independent of the policy-maker's instruments. It is not surprising that by exerting its monopoly power, the firm sets the lowest level of quality in this region.

The result of our general framework implies that the form of user utility functions can explain contradictory results in the literature concerning the impact of fines on quality. Some prior studies (e.g, [Cho and Ahn 2010](#), [Jain 2008](#)) show that when users have the same preference for quality,

imposing higher fines increases the firm's investment in quality. In contrast, [Lahiri and Dey \(2013\)](#) find that when users differ in their valuation for quality, the firm's investment in quality decreases with imposed fines on pirates. In other words, [Lahiri and Dey \(2013\)](#) claim that increased enforcement decreases the firm's investment in quality, implying that decreased digital piracy decreases quality of the digital good. Our findings, however, indicate that this intuition does not always hold even when users differ in their valuation for quality. Overall, we find that the impact of fines on the firm's investment in quality depends on two factors. Firstly, it depends on how user marginal utility is influenced by the quality of the good, if at all. Secondly, it depends on how the policy-maker redistributes collected fines, if choosing to do so. Using two specific functional forms representing additive and multiplicative utility, we explain how these factors come into play. With additive utility, the firm's investment in quality increases with fines, regardless of whether the policy-maker redistributes revenues from fines to society (through subsidies or restitution). In contrast, with multiplicative utility, the impact of fines on the firm's investment in quality depends on how collected fines are redistributed to society. If the policy-maker uses fines as restitution (subsidies), then the firm's investment in quality increases (decreases) with fines.

Evidence suggests that policy-makers around the globe have enacted legislation allowing them to impose fines on detected pirates. However, prior studies (e.g., [Novos and Waldman 1984](#), [Chen and Png 2003](#), [Lahiri and Dey 2013](#)) have shown that social welfare decreases with fines. To justify the policy-maker's use of fines, [Lahiri and Dey \(2013\)](#) introduced secondary measures such as legal surplus, and show that imposing fines can improve such measures. In contrast to the literature, our welfare analysis shows that with a budget balance constraint the policy-maker can improve social welfare using fines. Imposing fines might directly decrease social welfare, but the policy-maker can offset this effect by redistributing fine revenues back to society, either through restitution or subsidies. The budget balance constraint allows us to study a more refined formulation in which the policy-maker's intervention improves social welfare and reduces digital piracy.

Using our specific functional forms, we show that with additive and multiplicative utility imposing fines on detected pirates is social welfare maximizing. With additive utility the policy-maker either deters digital piracy by imposing relatively large fines or tolerates some level of piracy by imposing lower fines and using collected fines as restitution or subsidies. With multiplicative utility, the policy-maker either deters digital piracy by imposing relatively large fines or tolerates some level of piracy by imposing lower fines and using collected fines as subsidies. Thus, in contrast to the multiplicative form, with the additive utility the policy-maker may find restitution social welfare maximizing.

Our analysis proceeds as follows. We first examine two literature streams related to our work. Next we introduce our notation and detail the assumptions we use in our models. This is succeeded

by our general model formulation and analysis that yields our main results. Then we present our analysis using two different specific functional forms – additive utility followed by multiplicative utility – that provide additional insights. We finish by recapping our results and suggest future extensions.

## 2. Literature

The first stream of literature about piracy of digital goods investigates the economic impact of digital piracy. Several studies examine the pricing of digital goods in the presence of piracy (e.g., [Sundararajan 2004](#), [Chellappa and Shivendu 2005](#), [Herings et al. 2018](#)). Although the prevailing wisdom is that digital piracy harms the firm, prior research indicates that under some conditions digital piracy may increase the firm’s profit. For example, [Bhattacharjee et al. \(2006\)](#) show that the firm can make additional profits using new licensing structures even if a proportion of users pirate. Other critical reasons for supporting digital piracy are network effects and competition. [Jain \(2008\)](#) shows that in a duopoly, both firms supporting piracy can intensify price competition. However, when positive network effects are strong and digital piracy expands the market size, a monopoly may tolerate or even support digital piracy ([Conner 1995](#)). Similarly, a negative network effect can motivate the firm to tolerate digital piracy when unpatched systems on the network result in negative security externalities on subscribers’ willingness to pay. [August and Tunca \(2008\)](#) show that to lessen security harms that pirates impose on subscribers, the firm may find it profitable to support pirates by allowing them to apply security patches. In addition to its possible impact on firm profits, digital piracy can incentivize the firm to improve quality by intensifying competition between legitimate and pirated versions which in turn increases social welfare ([Lahiri and Dey 2013](#)). Nevertheless, as digital piracy simultaneously affects dynamic efficiency (supporting innovation) and static efficiency (promoting diffusion), it is likely to reduce social welfare in the long run ([Belleflamme and Peitz 2010](#), [Bae and Choi 2006](#)).

The second literature stream examines strategies for fighting digital piracy. Such strategies include sampling ([Chellappa and Shivendu 2005](#)), bundling ([Bhattacharjee et al. 2009](#), [Gopal and Gupta 2010](#)), versioning ([Baird et al. 2016](#), [Wu and Chen 2008](#)), software patching ([August and Tunca 2008](#)), content delivery technology ([Johar et al. 2012](#)), and implementing copyright enforcement policy ([Dey et al. 2019](#)). [Dey et al. \(2019\)](#) compare demand-side and supply-side enforcement in terms of innovation and social welfare, and find that, in the long run, the impact of supply-side enforcement on social welfare is more desirable. [Guo and Meng \(2015\)](#) address the impact of copyright enforcement on consumer search, the firm’s profit, and quality of digital goods. They show that an increase in copyright protection is associated with lower quality. Although several studies have examined the welfare implications of copyright enforcement, only a few have discussed

policy-makers' role in fighting digital piracy. [Chen and Png \(2003\)](#) consider a firm that decides on detection probability (copyright enforcement) and price. In their model, the policy-maker sets three instruments: subsidies on legal purchases, fines on detected pirates, and taxes on the copying medium. They show that offering subsidies on legal purchases is the most effective instrument to maximize social welfare.

Our work is also related to the literature on monetary sanctions such as fines and restitution. Theoretical studies address the importance of the probability of detecting potential violators of law and the magnitude of fines. Our formulation incorporates these features by modeling detection probability and fines simultaneously. We show that the impact of fines on social welfare depends on the probability of detection (e.g., [Polinsky and Shavell 1979](#), [Shavell 1987](#), [Polinsky and Shavell 2000](#)). On the other hand, unlike fines that are normally imposed to deter individuals from illegal activities, restitution is used to compensate crime victims for potential losses ([Martin et al. 2018](#)). Our formulation extends this stream of the literature by investigating the impact of fines and restitution on digital piracy.

We formulate a setting where the firm determines the subscription fee and quality; and where the policy-maker sets three policy instruments including imposing fines on detected pirates, restitution to the firm, and subsidies on legal purchases. We examine the role of such policy instruments on the user segmentation, the firm's decisions, and social welfare. Unlike [Chen and Png \(2003\)](#)'s conclusion, we show that subsidizing legal purchases is not always socially optimal, and that the policy-maker can boost social welfare by balancing its budget between fines on detected pirates and restitution to the firm.

### 3. Notation and Assumptions

We consider a market where a firm produces a digital good, known hereafter as a good. We make the general assumption that the utilized functions are twice continuously differentiable where necessary. To fight digital piracy and encourage more users to subscribe to the good, the firm can invest in improving quality of the good,  $0 \leq e \leq \bar{e}$ , hereafter known as quality. The firm can increase quality by improving its functionality, flexibility, and portability. The firm can also improve quality by providing complementary services such as enriching its digital library or using a better recommendation system to incentivize user participation. The firm offers only a single version of the good with a subscription fee,  $p \geq 0$ . The firm sets the profit-maximizing subscription fee and quality. A user has three options in such a market: legally subscribe to the good (subscriber), copy the good (pirate), or not use the good. Because some users may not use the good, the market may not be covered.

We also examine the role of a policy-maker in fighting digital piracy by considering the impact of three policy instruments. The first is a subsidy,  $s \geq 0$ , provided to subscribers. The second is a

monetary fine,  $f \geq 0$  used in the case of a pirate being detected. In our formulation, we assume that pirates are detected with a probability, which is shown with  $0 \leq \mu \leq 1$  and is taken as exogenous. Finally, the policy-maker may share a fraction,  $0 \leq \alpha \leq 1$ , of its revenue from imposing fines on detected pirates as restitution with the firm. Thus, the policy-maker can redistribute revenue from fines to subsidize subscribers or to support the firm through restitution. The policy-maker determines the social welfare maximizing subsidies, fines, and restitution.

We consider users to be heterogeneous in their valuation for the good and to choose the option that maximizes their utility, shown below in our assumptions.

**ASSUMPTION 1 (Heterogeneity and Observability).** *Users differ in their valuation for the good. An individual user's valuation for the good is not verifiable.*

With a little loss of generality, we assume that users are uniformly distributed based on their valuation for the good,  $v \sim U[0, 1]$ , so that the density is positive over the support,  $g(v) > 0 \quad \forall v \in [0, 1]$ ,  $G(0) = 0$ , and  $G(1) = 1$ .

Following prior research (e.g., Belleflamme and Peitz 2010, Lahiri and Dey 2013), we assume the cost of investment in quality,  $C(e)$ , is a quadratic function of  $e$ . Our results are qualitatively the same under more general convex costs.

**ASSUMPTION 2 (The Firm's Costs).** *The cost of investment in quality is  $C(e) = e^2/2$ .*

Users that pay the subscription fee become subscribers. Subscribers' valuation for the good and the firm's choice of quality can both increase their consumption utility,  $u^S(v, e)$ . Moreover, to motivate subscriber participation, the policy-maker offers subsidies. Thus, the utility of a subscriber is

$$U^S(v, e, p, s) = u^S(v, e) + s - p.$$

Users that obtain the good through illegal channels such as P2P sharing networks become pirates. Although pirates get the good at a negligible cost, they might be detected with probability  $\mu$ . In the case of being detected, pirates are subject to the policy-maker's fine. According to the U.S. Federal Copyright Act, "a single infringer of a single work is liable for a single amount between \$250 and \$10,000, no matter how many acts of infringement are involved in the action and regardless of whether the acts were separate, isolated, or occurred in a related series" (Copyright Law of the U.S. 2010). Here the underlying assumption is that even if pirates are detected they can still use the good. Therefore, the utility of a pirate is

$$U^C(v, e, f) = u^C(v, e) - \mu f.$$

Assumption 3 characterizes the behavior of subscribers and pirates' utility from consumption.

**ASSUMPTION 3 (User Utility).** (a) *Consumption utility is increasing in the firm's choice of quality (weakly increasing for pirates) at a decreasing rate.* (b) *Consumption utility is increasing in user valuation for the good.* (c) *The marginal consumption utility of subscribers and that of pirates are weakly increasing in the firm's choice of quality.*

Assumption 3 can be described by the partial derivatives

$$(a) \frac{\partial u^S(v, e)}{\partial e} > 0, \quad \frac{\partial u^C(v, e)}{\partial e} \geq 0, \quad \frac{\partial^2 u^S(v, e)}{\partial e^2} \leq 0, \quad \text{and} \quad \frac{\partial^2 u^C(v, e)}{\partial e^2} \leq 0,$$

$$(b) \frac{\partial u^S(v, e)}{\partial v} > 0 \quad \text{and} \quad \frac{\partial u^C(v, e)}{\partial v} > 0, \quad (c) \frac{\partial^2 u^S(v, e)}{\partial v \partial e} \geq 0 \quad \text{and} \quad \frac{\partial^2 u^C(v, e)}{\partial v \partial e} \geq 0.$$

Because pirates use illegal channels to access the good, they only capture a fraction of the positive effect of the firm's choice of quality. For example, although using a more sophisticated recommendation system benefits subscribers, it is unlikely to benefit pirates. Therefore, an increase in quality benefits subscribers more than pirates. That is, besides pirates' valuation for the good, the firm's choice of quality can only slightly increase, if at all, their consumption utility,  $u^C(v, e)$ . Assumption 3 also implies that utility is increasing in the firm's investment at a decreasing rate. We assume that such a decreasing rate is greater in magnitude for subscribers compared to pirates. Assumption 4 formalizes these characteristics of our model.

**ASSUMPTION 4 (Quality).** *The increase in consumption utility from an increase in quality is substantially greater for subscribers than pirates and the rate of increase for subscribers decreases faster than that of pirates.*

The effects of Assumption 4 can be described by

$$\frac{\partial u^S(v, e)}{\partial e} \gg \frac{\partial u^C(v, e)}{\partial e} \quad \text{and} \quad \left| \frac{\partial^2 u^S(v, e)}{\partial e^2} \right| > \left| \frac{\partial^2 u^C(v, e)}{\partial e^2} \right|,$$

where  $\ll$  signifies a substantial difference.

The second part of Assumption 4 is sufficient but not necessary for our results as it ensures that the firm's profit is concave in quality.

Following [Chellappa and Shivendu \(2005\)](#), we assume that subscribers have a higher marginal willingness to pay compared to pirates. Assumption 5 formalizes this characteristic of our model.

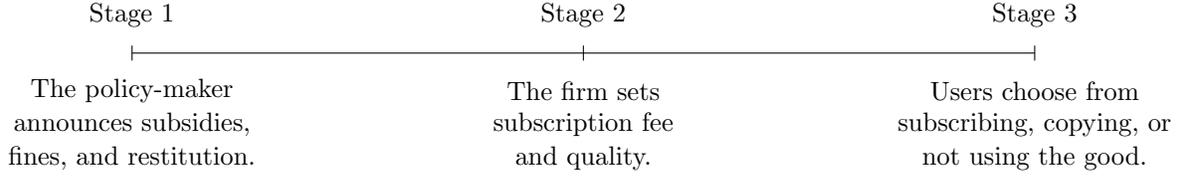
**ASSUMPTION 5 (User Valuation).** *As the user valuation for the good increases, subscribers obtain greater consumption utility than pirates.*

Using partial derivatives, Assumption 5 can be described by

$$0 < \frac{\partial u^S(v, e)}{\partial v} - \frac{\partial u^C(v, e)}{\partial v} \leq 1.$$

Assumption 5 ensures that users have a preference for subscribing at the margin, and the proportion of subscribers is sensitive to changes in the subscription fee.

**Timeline:** The timeline of events has three stages. In Stage 1 the policy-maker announces fines on detected pirates ( $f$ ), subsidies on legal purchases ( $s$ ), and restitution to the firm ( $\alpha$ ). In Stage 2, given the policy-maker's instruments, the firm sets a subscription fee ( $p$ ) and investment in quality ( $e$ ). In Stage 3, given the policy-maker's instruments and the firm's decisions, each user chooses to either subscribe, copy, or not use the good. The timeline of events is shown in Figure 1.



**Figure 1** Timeline of events.

## 4. Model Analysis

In our model we work through the three stages described above in reverse order.

### 4.1. Stage 3: User Behavior

Potential users divide into three segments: subscribers, pirates, and those that do not use the good. First, we examine the choice by subscribers. A subscriber purchases the good under individual-rationality (IR) and incentive-compatibility (IC) conditions.

When users only choose between subscribing and not using the good, we define an IR condition so that subscribers receive non-negative utility from subscribing,

$$u^S(v, e) + s - p \geq 0. \quad [IR.1] \quad (1)$$

With  $u^S(v, e)$  increasing in  $v$ , setting (1) to equality defines  $\bar{v}(e, p, s)$ , hereafter denoted by  $\bar{v}(\cdot)$ . This represents the utility of the user that is indifferent between subscribing and not using the good.

When users only choose between subscribing and copying, using Assumption 5 we define an IC condition so that subscribers weakly prefer subscribing,

$$u^S(v, e) + s - p \geq u^C(v, e) - \mu f. \quad [IC] \quad (2)$$

Setting (2) to equality defines  $\tilde{v}(e, p, s, f)$ , hereafter denoted by  $\tilde{v}(\cdot)$ , which is the utility of the user indifferent between subscribing and copying the good. For the indifferent subscriber, either the IR.1 condition in (1), the IC condition in (2), or both are binding. Therefore, when copying

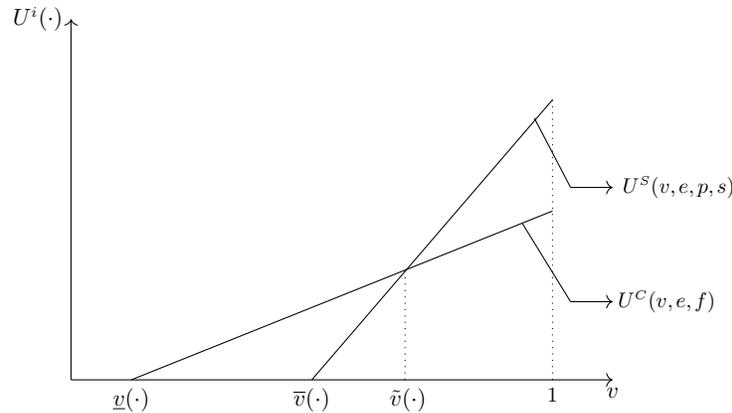
occurs, from Assumption 5 users whose utility for the good is  $v \in [\tilde{v}(\cdot), 1]$  are subscribers. On the other hand, when copying does not occur, subscribers are users whose valuation for the good is  $v \in [\bar{v}(\cdot), 1]$ .

When users only choose between copying the good and not using it, we define a second IR condition so that pirates receive non-negative value from copying

$$u^C(v, e) - \mu f \geq 0. \quad [IR.2] \quad (3)$$

With  $u^C(v, e)$  increasing in  $v$ , setting (3) to equality defines  $\underline{v}(e, f)$ , hereafter denoted by  $\underline{v}(\cdot)$ . This represents the utility of the user indifferent between copying and not using the good. Therefore, users with  $v \in [\underline{v}(\cdot), \tilde{v}(\cdot)]$  copy the good.

From Assumption 5, Figure 2 shows that the slope of subscribers' utility is greater than that of pirates; hence, after some point, here  $v = \tilde{v}(\cdot)$ , subscribing results in higher utility compared to copying. When the policy-maker's instruments and the firm's decisions satisfy  $\bar{v}(\cdot) > \underline{v}(\cdot)$ , copying occurs.



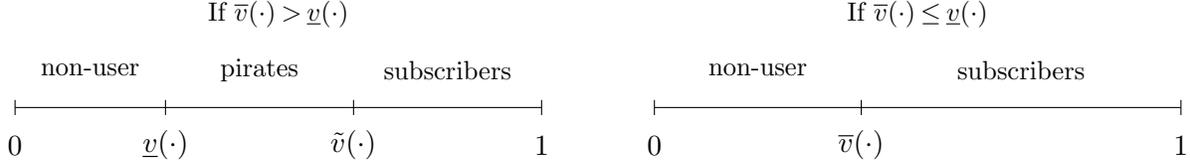
**Figure 2** Utility function of subscribers and pirates when copying occurs

It is worth noting that the policy-maker's instruments and the firm's choice of subscription fee and quality may remove piracy. If the policy-maker's choice of instruments and the firm's decisions increase subscribers' utility or decrease pirates' utility to a point where  $\bar{v}(\cdot) \leq \underline{v}(\cdot)$ , then no users choose to copy the good. Figure 3 represents user segmentation when piracy occurs,  $\bar{v}(\cdot) > \underline{v}(\cdot)$ , and when it does not,  $\bar{v}(\cdot) \leq \underline{v}(\cdot)$ .

#### 4.1.1. Subscribers

As shown in Figure 2, when copying occurs,  $\bar{v}(\cdot) > \underline{v}(\cdot)$ , subscribers are users with the highest valuation for the good. The proportion of subscribers,  $SU$ , depends on the magnitude of  $\tilde{v}(\cdot)$ . Therefore,

$$SU(e, p, s, f) = \int_{\tilde{v}}^1 dv = 1 - \tilde{v}(e, p, s, f).$$



**Figure 3** User Behavior

For economy of notation we substitute  $SU(\cdot)$  for  $SU(e, p, s, f)$ . In this case, from Assumption 3(b) and using the IC condition in (2), it is straightforward that as the firm increases the subscription fee, the proportion of subscribers decreases. Moreover, from Assumptions 3(a), 3(b), and 4, as the firm's choice of quality increases, subscribers obtain a greater utility compared to pirates. Thus, the proportion of subscribers increases with quality of the good. This gives  $\partial SU(\cdot)/\partial e > 0$  and  $\partial SU(\cdot)/\partial p < 0$ .

When copying does not occur,  $\bar{v}(\cdot) \leq \underline{v}(\cdot)$ , the proportion of subscribers is

$$SU_{np}(e, p, s) = \int_{\bar{v}}^1 dv = 1 - \bar{v}(e, p, s),$$

where the subscript  $np$  means no piracy. This proportion increases with the firm's investment in quality and decreases with the subscription fee. Therefore, substituting  $SU_{np}(\cdot)$  for  $SU_{np}(e, p, s)$  we have  $\partial SU_{np}(\cdot)/\partial e > 0$  and  $\partial SU_{np}(\cdot)/\partial p < 0$ .

#### 4.1.2. Pirates

As shown in Figure 2, users with moderate valuation for the good prefer copying to subscribing. The proportion of pirates,  $CU$ , depends on the magnitude of  $\underline{v}(\cdot)$  and  $\tilde{v}(\cdot)$  where users  $v \in [\underline{v}(\cdot), \tilde{v}(\cdot)]$  copy the good. Therefore,

$$CU(e, p, s, f) = \int_{\underline{v}}^{\tilde{v}} dv = \tilde{v}(e, p, s, f) - \underline{v}(e, f).$$

Substituting  $CU(\cdot)$  for  $CU(e, p, s, f)$ , from Assumptions 3(b) and 4, and by using the IC condition in (2) and the IR condition in (3), it is straightforward that as the firm increases the subscription fee, the proportion of pirates increases. However, an increase in quality has two opposing effects on the proportion of pirates. First, it reduces the proportion of pirates by increasing the proportion of subscribers. Second, it increases the proportion of pirates by reducing the proportion of users that do not use the good. From Assumption 4 the first impact dominates the second because the benefit of an increase in quality is substantially greater for subscribers. Thus, in aggregate, an increase in quality reduces the proportion of pirates. This gives  $\partial CU(\cdot)/\partial e \leq 0$  and  $\partial CU(\cdot)/\partial p \geq 0$ . It is worth noting that although user behavior depends on subsidies and fines when copying occurs, user behavior does not directly depend on restitution.

## 4.2. Stage 2: The Firm's Response

As discussed above, the firm's choice of subscription fee and quality affect user choice. In this section, we examine the firm's optimal decisions in the following regions: (i) the piracy region where copying occurs,  $\bar{v}(\cdot) > \underline{v}(\cdot)$ ; (ii) the no piracy region with threat of copying, or limit pricing,  $\bar{v}(\cdot) = \underline{v}(\cdot)$ ; and (iii) the no piracy region with no threat of copying,  $\bar{v}(\cdot) < \underline{v}(\cdot)$ .

### 4.2.1. Piracy Region

In the piracy region where the policy-maker's instruments and the firm's decisions satisfy  $\bar{v}(\cdot) > \underline{v}(\cdot)$  some users copy the good. As Figure 2 shows, in this region users with  $v \in [\tilde{v}(\cdot), 1]$  subscribe to the good, users with  $v \in [\underline{v}(\cdot), \tilde{v}(\cdot)]$  copy it, and users with  $v \in [0, \underline{v}(\cdot)]$  do not use it.

Because a proportion of users copy, the firm gains a proportion of the policy-maker's revenues,  $0 \leq \alpha \leq 1$ , from imposing fines on detected pirates as restitution. Therefore, the firm's profit is its revenue from subscribers and digital piracy restitution less the cost of improving quality. The firm solves

$$\max_{p,e} \left\{ \pi(e, p; s, f, \alpha) = pSU(\cdot) + \alpha\mu fCU(\cdot) - \frac{e^2}{2} \right\}, \quad (4)$$

where  $\pi(e, p; s, f, \alpha)$  represents the profit function of the firm. In (4) the first term represents the firm's revenue from subscribers, the second term represents the gained restitution, and the last term shows the cost of investment in improving quality.

Substituting  $\pi(\cdot)$  for  $\pi(e, p; s, f, \alpha)$ , the first-order conditions (FOCs) of (4) are

$$\begin{aligned} \frac{\partial \pi(\cdot)}{\partial p} &= SU(\cdot) + p \frac{\partial SU(\cdot)}{\partial p} + \alpha\mu f \frac{\partial CU(\cdot)}{\partial p} = \phi_1(e, p, s, f, \alpha) = 0, \\ \frac{\partial \pi(\cdot)}{\partial e} &= p \frac{\partial SU(\cdot)}{\partial e} + \alpha\mu f \frac{\partial CU(\cdot)}{\partial e} - e = \phi_2(e, p, s, f, \alpha) = 0. \end{aligned}$$

Substituting  $(\cdot)$  for  $(e, p, s, f, \alpha)$  in the implicit functions,  $\phi_1(\cdot)$  and  $\phi_2(\cdot)$  implicitly define the firm's optimal subscription fee and quality:  $p(e, s, f, \alpha)$ , and  $e(p, s, f, \alpha)$ , respectively. The concavity of the firm's profit is the sufficient condition for optimal subscription fee and quality. In Appendix EC.1, we show that the firm's profit function satisfies this sufficient condition. For simplicity of notation we use  $p(\cdot)$  and  $e(\cdot)$  in what follows.

We begin our analysis by examining the impact of each policy instrument on the firm's profit-maximizing subscription fee and quality. Each instrument affects the firm's choices of subscription fee and investment in quality directly and indirectly. Therefore, the total impacts of instrument  $y \in \{s, f, \alpha\}$  on the firm's subscription fee and investment in quality are obtained by solving the following equations simultaneously,

$$\frac{dp(\cdot)}{dy} = \frac{\partial p(\cdot)}{\partial y} + \frac{\partial p(\cdot)}{\partial e} \frac{de(\cdot)}{dy}, \quad (5)$$

$$\frac{de(\cdot)}{dy} = \frac{\partial e(\cdot)}{\partial y} + \frac{\partial e(\cdot)}{\partial p} \frac{dp(\cdot)}{dy}, \quad (6)$$

where the first term on the right-hand side is the direct effect through one of the firm's choice variables and the second product is the indirect effect through the other of the firm's choice variables for both (5) and(6).

Lemma 1 describes how the firm's optimal decisions are effected as each policy instrument varies. For clarity of presentation, all proofs of lemmas, theorems, and detailed technical expressions are presented in Appendix EC.2.

LEMMA 1. *In the piracy region, an increase in each policy instrument (a) increases the subscription fee; and (b) decreases the firm's investment in quality if the negative direct effect dominates the positive indirect effect through the subscription fee.*

Lemma 1(a) indicates that with an increase in each policy instrument the positive direct effect on the subscription fee always dominates the potential negative indirect effect from the firm's investment in quality.

Lemma 1(b) shows that policy instruments affect the firm's investment in quality non-monotonically. An increase in subsidies, fines, or restitution have two opposing effects on the firm's investment in quality. The first effect is weakly negative and captures the direct effect of each policy instrument on quality. The second effect is always positive and reflects the indirect effect of policy instruments on quality by increasing the subscription fee. Therefore, a given policy instrument diminishes the firm's investment in quality if the first effect outweighs the second.

In Section 5 where we develop our specific functional forms, we show that two factors affect the magnitude of direct and indirect effects of imposing fines on the firm's investment in quality. First is whether the policy-maker redistributes revenue from fines as restitution or as subsidies. Second is the choice of specific functional form. Thus, the form of utility functions can explain the contradictory results in studies that examine the impact of policy instruments on the quality of digital goods. For example, [Bae and Choi \(2006\)](#) and [Cho and Ahn \(2010\)](#) show that when users are homogeneous in their valuation for quality, imposing higher fines on pirates increases quality. On the other hand, [Lahiri and Dey \(2013\)](#) find that when users are heterogeneous and have multiplicative utility, imposing higher fines discourages the firm from investing in quality.

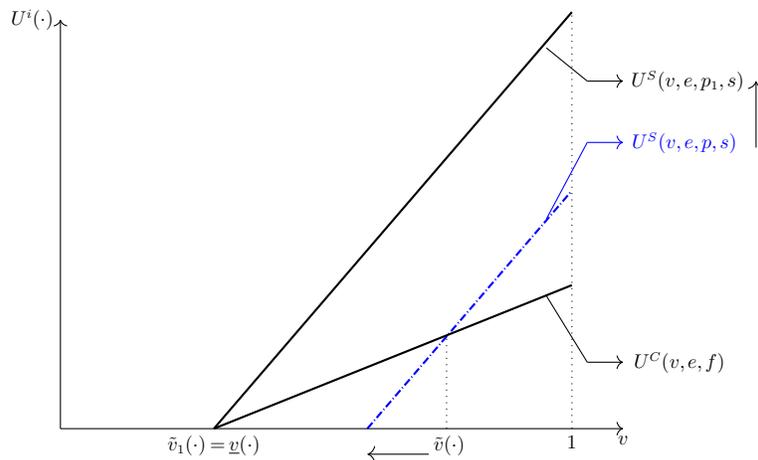
According to the first theorem in [Lahiri and Dey \(2013\)](#), when users' valuations for quality are different, increased enforcement (shown with  $\mu f$  in our formulation) decreases the firm's investment in quality, implying that decreased digital piracy decreases quality. However, our analysis reveals that this intuition does not always hold and under certain conditions imposing higher fines may increase the firm's investment in quality (even when users value quality differently). A sufficient condition for enforcement to increase the firm's investment in quality is

$$\frac{\partial u^S(\tilde{v}, e)}{\partial e} - \frac{\partial u^C(\tilde{v}, e)}{\partial e} > SU(\cdot) \left[ \frac{\partial^2 u^S(\tilde{v}, e)}{\partial \tilde{v} \partial e} - \frac{\partial^2 u^C(\tilde{v}, e)}{\partial \tilde{v} \partial e} \right]. \quad (7)$$

#### 4.2.2. No Piracy Region with Threat of Copying

In the no piracy region with threat of copying, although piracy does not occur, the potential for piracy causes the firm to consider itself in competition with the illegal channel. Under such conditions, in order to compete against the illegal channel and deter piracy, the firm reduces the subscription fee or alternatively increases quality.

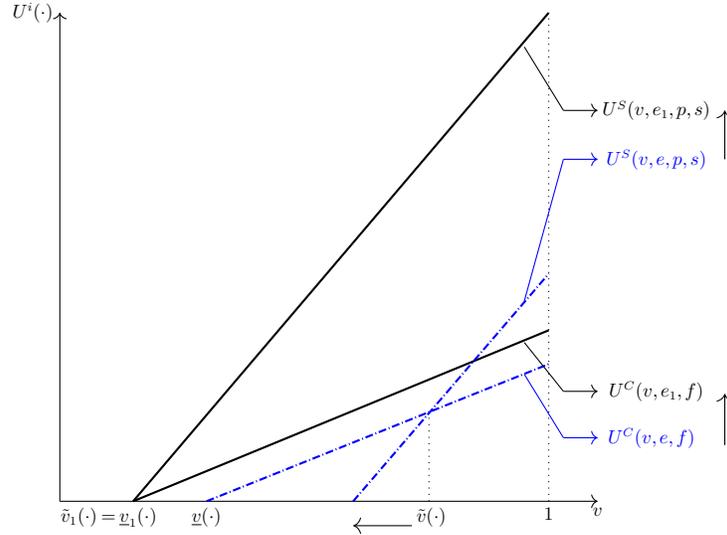
Thinking of  $\tilde{v}(\cdot)$  as the point at which utilities from subscribing and copying intersect, Figure 4 shows that users with  $v > \tilde{v}(\cdot)$  subscribe to the good and users with  $\underline{v}(\cdot) < v \leq \tilde{v}(\cdot)$  copy it. Figure 4 also illustrates that when the firm decreases the subscription fee,  $p$ , to compete against the illegal channel, the utility from subscribing increases (shifts upward). Therefore, the intersection point,  $\tilde{v}(\cdot)$ , is shifted to the left and the gap between  $\tilde{v}(\cdot)$  and  $\underline{v}(\cdot)$  is reduced. To deter piracy, the firm keeps reducing the subscription fee to  $p_1$ , so that utilities from subscribing and copying intersect at  $\tilde{v}_1(\cdot) = \underline{v}(\cdot)$ . Therefore, in this region users with  $v > \tilde{v}_1(\cdot)$  subscribe to the good and users with  $v \leq \tilde{v}_1(\cdot)$  do not use it.



**Figure 4** Utility functions of subscribers and pirates when subscription fee decreases

From Assumption 4, when the firm increases its investment in quality, the utility from subscribing increases faster than the utility of copying. Thinking of  $\tilde{v}(\cdot)$  as the point at which utilities from subscribing and copying intersect and  $\underline{v}(\cdot)$  as the point at which the utility from copying is zero, when the firm's investment in quality is  $e$ , Figure 5 shows that by increasing the firm's investment in quality both  $\tilde{v}(\cdot)$  and  $\underline{v}(\cdot)$  are shifted to the left. However, from Assumption 4,  $\tilde{v}(\cdot)$  is shifted more than  $\underline{v}(\cdot)$ . When the firm increases the quality (from  $e$  to  $e_1$ ) to deter digital piracy, utilities from subscribing and copying shift upward so that they intersect at  $\tilde{v}_1(\cdot) = \underline{v}_1(\cdot)$ . Under such conditions, users with  $v > \tilde{v}_1(\cdot)$  subscribe to the good and users with  $v \leq \tilde{v}_1(\cdot)$  do not use it.

In this case, because there is no copying the policy-maker's revenue from fines is zero. Further, from budget balance the subsidy and restitution are zero,  $s = \alpha = 0$ . Thus, we can characterize the



**Figure 5** Utility functions of subscribers and pirates when investment in quality increases

firm as choosing the subscription fee,  $p$ , and quality,  $e$ , to maximize profits, subject to there being no users that copy:

$$\max_{p,e} \left\{ \pi_{np}(e,p;f) = pSU_{np}(e,p) - \frac{e^2}{2} \right\}, \quad \text{subject to: } \bar{v}(e,p) - \underline{v}(e,f) = 0, \quad (8)$$

where the first term of the max function in (8) represents the firm's revenue from sales to subscribers and the second term is the firm's cost of investment in quality. The constraint shows the condition for the no piracy region with threat of copying. As there is no copying, the policy-maker raises no revenue from fines and compared to (4) there is no restitution term in (8).

To solve the constrained profit maximization problem in (8), we form the Lagrangian function and use KKT conditions,

$$L(e,p,f,\gamma) = pSU_{np}(e,p) - \frac{e^2}{2} - \gamma [\bar{v}(e,p) - \underline{v}(e,f)], \quad (9)$$

where  $L(e,p,f,\gamma)$  represents the Lagrangian function and  $\gamma \geq 0$  is the KKT multiplier. If  $(e,p)$  maximizes the constrained profit maximization problem (8), then it also satisfies KKT conditions. A closer look at these KKT conditions reveals that imposing higher fines in this region may decrease the firm's investment in quality indirectly through increasing the subscription fee. In particular, we find that in the no piracy region with threat of copying imposing higher fines increases the firm's investment in quality if

$$2 \frac{\partial u^C(\underline{v}(\cdot), e) / \partial e}{\partial u^C(\underline{v}(\cdot), e) / \partial \underline{v}} + \frac{\partial^2 u^S(\bar{v}(\cdot), e) / \partial \bar{v} \partial e}{\partial u^S(\bar{v}(\cdot), e) / \partial \bar{v}} > \frac{\partial u^S(\bar{v}(\cdot), e) / \partial e}{\partial u^S(\bar{v}(\cdot), e) / \partial \bar{v}}.$$

### 4.2.3. No Piracy Region without Threat of Copying

In the no piracy region without threat of copying, where  $\bar{v}(\cdot) < \underline{v}(\cdot)$ , independent of the firm's decisions, copying does not occur. In other words, in this region the policy-maker increases fines to a point where no users copy. Under such conditions, due to the budget balance constraint and because  $CU(\cdot) = 0$ , the policy-maker cannot use the other two instruments,  $s = \alpha = 0$ . As shown in Figure 4, as the policy-maker increases fines, the utility from copying reduces to the point where no users copy. Thus, the firm can exert its monopoly power. As a result, given the policy-maker's optimal fines, the firm maximizes its profit if users only choose between subscribing and not using the good, which is modeled as an IR condition in (1) when  $s = 0$ . Therefore, the proportion of subscribers does not depend on the firm's detection probability and the policy-maker's fines,  $SU_{np}(e, p) = 1 - \bar{v}(e, p)$ .

The firm's profit is its revenue from subscribers less the cost of investment in quality. The firm solves

$$\max_{p, e} \left\{ \pi_{np}(e, p) = pSU_{np}(e, p) - \frac{e^2}{2} \right\}, \quad (10)$$

where  $\pi_{np}(e, p)$  represents the firm's profit function in the no piracy region without threat of copying. In the no piracy region without threat of copying, because implicit functions that define the firm's optimal decisions do not depend on fines, the firm's subscription fee and investment in quality are unaffected by fines.

### 4.3. Stage 1: The Policy-Maker's Response

In this section, we study the consequences of our policy instruments focusing on user segmentation and social welfare.

#### 4.3.1. User Segmentation

We examine the impact of each policy instrument on user segmentation in: (i) the piracy region where copying occurs,  $\bar{v}(\cdot) > \underline{v}(\cdot)$ , and (ii) the no piracy region with threat of copying,  $\bar{v}(\cdot) = \underline{v}(\cdot)$ . *Piracy Region.* We begin our analysis in stage 1 by examining the impact of each policy instrument on user segmentation in the piracy region. Theorem 1 establishes the impact of each policy instrument on the aggregate proportion of subscribers and pirates, that is

$$TU(e, f) = \int_{\underline{v}(\cdot)}^1 dv = 1 - \underline{v}(e, f).$$

**THEOREM 1.** *In the piracy region, (a) if the firm's investment in quality increases with subsidies and restitution, then the aggregate proportion of subscribers and pirates increases with subsidies and restitution, respectively, and (b) if the firm's investment in quality decreases with fines, then the aggregate proportion of subscribers and pirates decreases with fines.*

From the proof of Theorem 1, presented in Appendix EC.2, the impact of an increase in subsidies and restitution on the aggregate proportion of subscribers and pirates is

$$\frac{dTU(\cdot)}{ds} = \frac{de(\cdot)}{ds} \frac{\partial TU(e, f)}{\partial e} = \frac{de(\cdot)}{ds} \frac{\partial u^C(\underline{v}, e)/\partial e}{\partial u^C(\underline{v}, e)/\partial \underline{v}}, \text{ and}$$

$$\frac{dTU(\cdot)}{d\alpha} = \frac{de(\cdot)}{d\alpha} \frac{\partial TU(e, f)}{\partial e} = \frac{de(\cdot)}{d\alpha} \frac{\partial u^C(\underline{v}, e)/\partial e}{\partial u^C(\underline{v}, e)/\partial \underline{v}},$$

respectively, which indicate that increasing subsidies and restitution increase the aggregate proportion of subscribers and pirates if they increase the firm's investment in quality.

Using the proof of Theorem 1 the impact of increasing fines on the aggregate proportion of subscribers and pirates is

$$\frac{dTU(\cdot)}{df} = \frac{\partial TU(e, f)}{\partial f} + \frac{de(\cdot)}{df} \frac{\partial TU(e, f)}{\partial e} = -\frac{\mu}{\partial u^C(\underline{v}, e)/\partial \underline{v}} + \frac{de(\cdot)}{df} \frac{\partial u^C(\underline{v}, e)/\partial e}{\partial u^C(\underline{v}, e)/\partial \underline{v}},$$

in which the first term is negative and the second term is non-monotonic. If the firm's investment in quality is decreasing in fines, then the second term is also negative and the aggregate proportion of subscribers and pirates is decreasing. In contrast, for the aggregate proportion of subscribers and pirates to be increasing in fines, the impact of fines on the firm's investment in quality must be positive and large in magnitude.

**COROLLARY 1.** *In the piracy region, subsidies, fines, and restitution are not necessarily effective in fighting digital piracy.*

Corollary 1 reveals that imposing higher fines on detected pirates does not always decrease the proportion of pirates. That is because increasing fines has opposing effects on the proportion of pirates including (i) the negative direct effect, (ii) the positive indirect effect by increasing the subscription fee, and (iii) the non-monotonic indirect effect by changing the firm's investment in quality. Depending on the magnitude of these effects, imposing higher fines on detected pirates may encourage users to copy. Through a similar combination of effects, restitution and subsidies may increase the proportion of pirates by encouraging marginal subscribers to copy the good.

Our first corollary also demonstrates that even if an increase in fines decreases the firm's investment in quality, it can still discourage users from copying the good through the negative direct effect. This is consistent [Lahiri and Dey \(2013\)](#) where they show that with multiplicative utility, imposing higher fines (increasing piracy costs) reduces quality and digital piracy. As we show later, we find that with additive utility, imposing higher fines increases the firm's investment in quality.

*No Piracy Region with Threat of Copying* We now examine user segmentation when the firm's choices of subscription fee and/or investment in quality deter piracy. Because the firm completely deters pirates,  $CU = 0$ , the policy-maker's fine revenue is zero and with budget balancing legal purchases are not subsidized,  $s = 0$ . Therefore, in this region, we drop  $s$  from the notation. According

to our analysis in section 4.2.2, in this region, the proportion of subscribers equals the proportion of users that obtain non-negative utility by subscribing to the good. Therefore, we can obtain the proportion of subscribers by setting the IR condition in (1) to equality, that is  $1 - \bar{v}(e, p)$ . Our next theorem establishes the impact of fines on the proportion of subscribers,

$$SU_{np}(e, p) = \int_{\bar{v}(\cdot)}^1 dv = 1 - \bar{v}(e, p).$$

**THEOREM 2.** *In the no piracy region with threat of copying, if the impact of fines on the firm's investment in quality is positive and sufficiently large, then the proportion of subscribers increases in fines.*

Theorem 2 shows that because increasing fines directly raises the subscription fee, it does not necessarily increase the proportion of subscribers. Therefore, if the firm's investment in quality increases in fines so that it reverses the negative impact through the subscription fee, then the proportion of subscribers increases with fines. In other words, if

$$\frac{de(f)}{df} > \mu \frac{\partial u^C(\underline{v}, e)}{\partial e},$$

then in this region the proportion of subscribers increases with fines.

#### 4.3.2. Social Welfare

Considering a budget balance constraint, we now examine how social welfare is affected by the policy-maker's intervention in: (i) the piracy region where copying occurs,  $\bar{v}(\cdot) > \underline{v}(\cdot)$  and (ii) the no piracy region with threat of copying,  $\bar{v}(\cdot) = \underline{v}(\cdot)$ .

*Piracy Region.* In the piracy region the policy-maker maximizes social welfare and must balance its budget between revenue from fines on detected pirates, subsidies for legal purchases, and restitution to the firm.

We define social welfare,  $SW(s, f, \alpha)$ , as the net benefit of all users, the firm's profit, and the policy-maker's net revenue. The subscription fee (the subsidy) enters the subscribers' surplus and the firm's profit (the policy-maker's revenue) but is netted out in the social welfare analysis as it is a transfer between subscribers and the firm (the policy-maker). Similarly, the fines on detected pirates can be viewed as a transfer between pirates and the policy-maker and do not affect social welfare directly. Thus, social welfare is the sum of the net benefit of all users less the firm's incurred costs of investment in quality. The budget balance constraint states that total subsidies for legal purchases,  $sSU(s, f, \alpha)$ , should be smaller than or equal to the policy-maker's revenue from fines on detected pirates,  $\mu fCU(s, f, \alpha)$ , less the firm's restitution,  $\alpha \mu fCU(s, f, \alpha)$ . The policy-maker maximizes

$$\max_{s, f, \alpha} \left\{ SW(s, f, \alpha) = \int_{\bar{v}(\cdot)}^1 u^S(v, e(\cdot)) dv + \int_{\underline{v}(\cdot)}^{\bar{v}(\cdot)} u^C(v, e(\cdot)) dv - \frac{e(\cdot)^2}{2} \right\}, \quad (11)$$

$$\text{subject to:} \quad sSU(s, f, \alpha) \leq [1 - \alpha] \mu fCU(s, f, \alpha). \quad (12)$$

Given that  $1 - \tilde{v}(\cdot) = SU(\cdot)$  and  $1 - \underline{v}(\cdot) = TU(\cdot)$ , by differentiating (11) with respect to subsidies, restitution, and fines, we have

$$\begin{aligned} \frac{dSW(s, f, \alpha)}{ds} &= \frac{de(\cdot)}{ds} \left[ \int_{\tilde{v}(\cdot)}^1 \frac{\partial u^S(v, e(\cdot))}{\partial e} dv + \int_{\underline{v}(\cdot)}^{\tilde{v}(\cdot)} \frac{\partial u^C(v, e(\cdot))}{\partial e} dv \right] + \frac{dTU(\cdot)}{ds} u^C(\underline{v}(\cdot), e(\cdot)) \\ &\quad + \frac{dSU(\cdot)}{ds} [u^S(\tilde{v}(\cdot), e(\cdot)) - u^C(\tilde{v}(\cdot), e(\cdot))] - \frac{de(\cdot)}{ds} e(\cdot), \end{aligned} \quad (13)$$

$$\begin{aligned} \frac{dSW(s, f, \alpha)}{d\alpha} &= \frac{de(\cdot)}{d\alpha} \left[ \int_{\tilde{v}(\cdot)}^1 \frac{\partial u^S(v, e(\cdot))}{\partial e} dv + \int_{\underline{v}(\cdot)}^{\tilde{v}(\cdot)} \frac{\partial u^C(v, e(\cdot))}{\partial e} dv \right] + \frac{dTU(\cdot)}{d\alpha} u^C(\underline{v}(\cdot), e(\cdot)) \\ &\quad + \frac{dSU(\cdot)}{d\alpha} [u^S(\tilde{v}(\cdot), e(\cdot)) - u^C(\tilde{v}(\cdot), e(\cdot))] - \frac{de(\cdot)}{d\alpha} e(\cdot), \end{aligned} \quad (14)$$

$$\begin{aligned} \frac{dSW(s, f, \alpha)}{df} &= \frac{de(\cdot)}{df} \left[ \int_{\tilde{v}(\cdot)}^1 \frac{\partial u^S(v, e(\cdot))}{\partial e} dv + \int_{\underline{v}(\cdot)}^{\tilde{v}(\cdot)} \frac{\partial u^C(v, e(\cdot))}{\partial e} dv \right] + \frac{dTU(\cdot)}{df} u^C(\underline{v}(\cdot), e(\cdot)) \\ &\quad + \frac{dSU(\cdot)}{df} [u^S(\tilde{v}(\cdot), e(\cdot)) - u^C(\tilde{v}(\cdot), e(\cdot))] - \frac{de(\cdot)}{df} e(\cdot). \end{aligned} \quad (15)$$

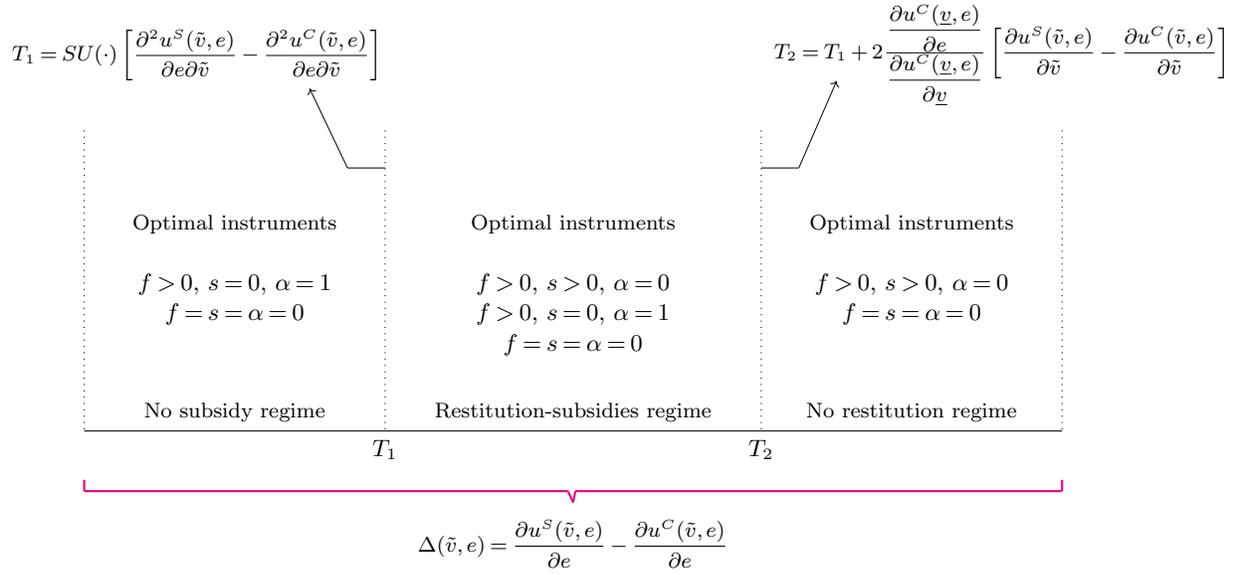
Based on the impact of subsidies and restitution on the firm's investment in quality, we partition our welfare analysis into three regimes. The first regime is called *no restitution regime* and refers to the conditions under which the policy-maker never uses restitution. In this regime, the firm's investment in quality increases with subsidies and decreases with restitution. The second regime is called the *no subsidy regime* in which the policy-maker never uses subsidies. In this regime, the firm's investment in quality decreases with subsidies and increases with restitution. Finally, the third regime is *restitution-subsidies regime* which depicts the condition under which the policy-maker may redistribute revenues from fines to subsidize legal purchases or to support the firm through restitution. In this regime, both subsidies and restitution influence the firm's investment in quality in the same direction.

According to Lemma 1, the difference between the impact of the firm's investment in quality on utility of subscribers and that of pirates,  $\Delta(\tilde{v}, e) = \partial u^S(\tilde{v}, e)/\partial e - \partial u^C(\tilde{v}, e)/\partial e$ , determines how subsidies and restitution affect the firm's investment in quality. As shown in Figure 6, following the value of  $\Delta(\tilde{v}, e)$  defines the three regimes. To delineate each regime, we define two thresholds developed in the proof of Theorem 3,  $0 < T_1 < T_2$ , where

$$T_1 = SU(\cdot) \left[ \frac{\partial^2 u^S(\tilde{v}, e)}{\partial e \partial \tilde{v}} - \frac{\partial^2 u^C(\tilde{v}, e)}{\partial e \partial \tilde{v}} \right] \quad \text{and} \quad T_2 = T_1 + 2 \frac{\frac{\partial u^C(\underline{v}, e)}{\partial e}}{\frac{\partial u^C(\underline{v}, e)}{\partial \underline{v}}} \left[ \frac{\partial u^S(\tilde{v}, e)}{\partial \tilde{v}} - \frac{\partial u^C(\tilde{v}, e)}{\partial \tilde{v}} \right].$$

If  $\Delta(\tilde{v}, e)$  is smaller than  $T_1$  (greater than  $T_2$ ), then the firm's investment in quality decreases with subsidies (restitution).

Accounting for the budget balance constraint, our next theorem shows that for all values of  $\Delta(\tilde{v}, e)$ , the policy-maker can improve social welfare by imposing fines on detected pirates.



**Figure 6** Optimal policy instruments

**THEOREM 3.** *In the piracy region, imposing higher fines on detected pirates can improve social welfare.*

Even though policy-makers have imposed fines on detected pirates as anti-piracy enforcement, prior studies (e.g., Novos and Waldman 1984, Chen and Png 2003, Lahiri and Dey 2013) have shown that social welfare decreases with fines. In contrast to this literature, Theorem 3 reveals that when the policy-maker can redistribute revenues from fines to society (either through restitution or subsidies) and faces a budget balance constraint, then imposing fines on detected pirates can be socially optimal.

*No restitution regime.* In the piracy region with the no restitution regime, the policy-maker can improve social welfare by imposing positive fines on detected pirates and by redistributing revenues from fines to subsidize legal purchases. Thus, referring to the no restitution regime in Figure 6, we have either (i)  $f > 0, s > 0, \alpha = 0$ , or (ii)  $f = s = \alpha = 0$ .

*No subsidy regime.* In the piracy region with the no subsidy regime, the policy-maker can improve social welfare by imposing positive fines on detected pirates and by redistributing revenues from fines as restitution. Referring to the no subsidy regime in Figure 6 we have either (i)  $f > 0, s = 0$ , and  $\alpha = 1$ , or (ii)  $f = s = \alpha = 0$ . These possible outcomes lead us to our next corollary.

**COROLLARY 2.** *Subsidizing legal purchases is not always social welfare maximizing.*

When  $\Delta(\tilde{v}, e) < T_1$ , piracy becomes an attractive option even for users with a lower valuation for the good. In response, the policy-maker must consider imposing substantial fines on detected pirates to effectively combat digital piracy. However, caution is needed, as imposing large fines and

using revenue from fines as subsidies incentivizes the firm to significantly raise subscription fee. This, in turn, diminishes social welfare by reducing subscriber surplus. Consequently, as  $\Delta(\tilde{v}, e)$  decreases, the effectiveness of subsidizing legal purchases in maximizing social welfare declines.

The welfare analysis in [Chen and Png \(2003\)](#) shows that social welfare increases with subsidies. However, Corollary 2 indicates that in the piracy region with the no subsidy regime, where  $\Delta(\tilde{v}, e) < T_1$ , subsidizing legal purchases decreases social welfare. These contradictory findings can be explained by the choice of additive utility functions investigated in [Chen and Png \(2003\)](#). With additive utility,  $T_1 = 0$  indicating that the firm's investment in quality always increases with subsidies. In other words, the no subsidy regime does not occur when utility functions are additive.

*Restitution-subsidies regime.* In the piracy region with the restitution-subsidies regime, the policy-maker can improve social welfare by imposing fines on detected pirates and by redistributing revenue from fines as restitution to the firm or as subsidies for legal purchases. We find that if the policy-maker imposes fines on detected pirates, then it can always improve social welfare by redistributing revenue from fines as restitution. Under such conditions, we have  $f > 0$ ,  $s = 0$ , and  $\alpha = 1$ . The policy-maker may also maximize social welfare by redistributing revenue from fines as subsidies. Under such conditions, the optimal intervention is  $f > 0$ ,  $s > 0$ , and  $\alpha = 0$ . If  $f = 0$ , then  $s = \alpha = 0$ .

*No Piracy Region with Threat of Copying.* In the no piracy region with threat of copying, because users do not copy the good, the policy-maker's revenue from fines is zero indicating that  $s = \alpha = 0$ . Thus, the budget balance constraint is relaxed and the policy-maker maximizes social welfare by only setting the level of fines. In this region, the subscription fee can be viewed as a transfer between subscribers and the firm. Thus, social welfare is the sum of the net benefit of all subscribers less the firm's cost of investment in quality,

$$\max_f \left\{ SW_{np}(f) = \int_{\bar{v}(\cdot)}^1 u^S(v, e(\cdot)) dv - \frac{e(\cdot)^2}{2} \right\}, \quad (16)$$

where  $SW_{np}(f)$  represents social welfare in the no piracy region with threat of copying.

By differentiating (16) with respect to fines, the first-order condition can be written as

$$\frac{dSW_{np}(f)}{df} = -\frac{d\bar{v}(\cdot)}{df} u^S(\bar{v}(\cdot), e(\cdot)) + \int_{\bar{v}(\cdot)}^1 \frac{du^S(v, e(\cdot))}{df} dv - e(\cdot) \frac{de(\cdot)}{df} = 0. \quad (17)$$

In (17) if  $de(\cdot)/df < 0$ , then from Theorem 2 the first term two terms are negative and the last term is positive. Thus, in contrast to [Lahiri and Dey \(2013\)](#), social welfare in this region is not strictly increasing in fines.

## 5. Specific Functional Forms

Although one advantage of our general framework is that our results apply to a wide range of settings, to provide more detailed insights, we develop two examples, one with additive utility and the other with multiplicative utility. With additive utility we solve the policy-maker's social welfare maximizing problem and characterize its optimal intervention. With multiplicative utility an analytical characterization is not tractable, so we illustrate the results numerically. Hence, our welfare analysis with multiplicative utility primarily shows that the policy-maker's use of subsidies and fines can be welfare-maximizing.

### 5.1. Additive Utility

With the additive form, the utility function of a subscriber is

$$U^S = v + \eta e + s - p,$$

where  $0 < \eta \leq 1$  scales the benefit of an increase in quality for a subscriber. The remaining variables have the same meaning as in the general form.

Compared to subscribers, we expect the perceived utility of pirates to be lower. According to the literature, patches, updates, and certain functionalities may be missing in pirated versions. Thus, there is a quality degradation between the legitimate and pirated versions. Moreover, independent of the quality of the pirated version, in certain cases during the copying process, pirates may face challenges that lower their valuation for the good. For example, when pirates obtain movies through torrent websites they face a longer downloading time (Sundararajan 2004). Accounting for quality degradation and copying challenges, the additive utility function of a pirate is

$$U^C = \beta[v + \eta e] - \mu f,$$

where  $0 < \beta < 1$  accounts for a pirate's lower valuation for the good compared to subscribers and quality degradation between legitimate and pirated versions. Because copying lowers pirates' utility from consumption, it also affects the benefit of an increase in quality for a pirate.

#### 5.1.1. Stage 3: User Behavior

With additive utility, a user subscribes under the following IR and IC conditions:

$$v \geq \bar{v} = p - \eta e - s \quad [IR] \quad \text{and} \quad v \geq \tilde{v} = \frac{p - \mu f - s}{1 - \beta} - \eta e \quad [IC].$$

A user copies the good if the following IR and IC conditions hold:

$$v \geq \underline{v} = \frac{\mu f}{\beta} - \eta e \quad [IR] \quad \text{and} \quad v \leq \tilde{v} = \frac{p - \mu f - s}{1 - \beta} - \eta e \quad [IC].$$

When copying does not occur, the budget balance constraint leads the policy-maker to set  $s = \alpha = 0$ . Thus, copying does not occur if  $\bar{v}(\cdot) \leq \underline{v}(\cdot)$ , or equally if  $\mu f \geq \beta p$ . On the other hand, copying occurs if  $\bar{v}(\cdot) > \underline{v}(\cdot)$ , or equally if  $\mu f < \beta p$ . The proportion of subscribers and that of pirates can be written as

$$SU = \begin{cases} 1 + \eta e - \frac{p - \mu f - s}{1 - \beta} & \text{if } \mu f < \beta p \\ 1 + \eta e - p & \text{if } \mu f \geq \beta p \end{cases} \quad \text{and} \quad CU = \begin{cases} \frac{\beta[p - s] - \mu f}{\beta[1 - \beta]} & \text{if } \mu f < \beta p \\ 0 & \text{if } \mu f \geq \beta p. \end{cases}$$

In our additive utility functions, when piracy occurs an increase in the firm's investment in quality increases the proportion of subscribers, however it does not affect the proportion of pirates,  $\partial SU / \partial e = \eta$  and  $\partial CU / \partial e = 0$ .

### 5.1.2. Stage 2: The Firm's Response

*Piracy Region.* Substituting for the proportion of subscribers and that of pirates in the piracy region where  $\mu f < \beta p$ , the firm's profit function can be written as

$$\max_{p,e} \left\{ \pi(p, e; s, \alpha, f) = p \left[ 1 + \eta e - \frac{p - s - \mu f}{1 - \beta} \right] + \alpha \mu f \left[ \frac{\beta[p - s] - \mu f}{\beta[1 - \beta]} \right] - \frac{e^2}{2} \right\}. \quad (18)$$

By solving the FOCs of (18) simultaneously, the firm's optimal decisions in the piracy region are

$$p^* = \frac{1 - \beta + s + [1 + \alpha]\mu f}{2 - \eta^2[1 - \beta]} \quad \text{and} \quad e^* = \frac{\eta[1 - \beta + s + [1 + \alpha]\mu f]}{2 - \eta^2[1 - \beta]}.$$

**PROPOSITION 1.** *With additive utility, imposing fines increases the firm's investment in quality, regardless of how the policy-maker redistributes revenue from fines.*

By differentiating the optimal subscription fee and the firm's investment in quality with respect to subsidies, restitution, and fines, we get

$$\begin{aligned} \frac{dp^*}{ds} &= \frac{1}{2 - \eta^2[1 - \beta]}, & \frac{dp^*}{d\alpha} &= \frac{\mu f}{2 - \eta^2[1 - \beta]} & \text{and} & \frac{dp^*}{df} &= \frac{\mu[1 + \alpha]}{2 - \eta^2[1 - \beta]}, \\ \frac{de^*}{ds} &= \frac{\eta}{2 - \eta^2[1 - \beta]}, & \frac{de^*}{d\alpha} &= \frac{\eta \mu f}{2 - \eta^2[1 - \beta]} & \text{and} & \frac{de^*}{df} &= \frac{\eta \mu [1 + \alpha]}{2 - \eta^2[1 - \beta]}, \end{aligned}$$

which are all positive indicating that the firm's choice of subscription fee and quality both increase with policy instruments.

Consistent with findings from our general framework in Lemma 1, our analysis with additive utility shows that the total effect of an increase in subsidies, fines, and restitution on the firm's subscription fee is positive. Moreover, with additive utility the negative direct effects of subsidies, restitution, and fines on the firm's investment in quality are dominated by the positive indirect effects (through increasing the subscription fee). The result being that increases in these instruments increase the firm's investment in quality.

The use of functional forms also allows us to define the condition that determines the piracy region based on policy instruments. Obtaining the firm's optimal decisions, the piracy region condition,  $\mu f < \beta p$ , can be expressed as

$$\mu f < \frac{\beta[1-\beta]}{2-\beta-\eta^2[1-\beta]}, \quad (19)$$

which indicates that in this region the policy-maker's fines must be lower than a threshold.

*No Piracy Region with Threat of Copying.* We next examine the firm's profit maximization problem in the no piracy region with threat of copying. Because in this region  $\underline{v} = \bar{v}(\cdot)$ , we can substitute  $p = \mu f / \beta$  for the subscription fee. Thus, the firm's profit in this region can be written as

$$\max_e \left\{ \pi(e; f) = \frac{\mu f}{\beta} \left[ 1 + \eta e - \frac{\mu f}{\beta} \right] - \frac{e^2}{2} \right\}. \quad (20)$$

From the FOC of (20), the firm's optimal investment in quality is  $e^* = \eta \mu f / \beta$ . Following the firm's optimal decisions, the condition that determines the no piracy region with threat of copying ( $\bar{v}(\cdot) = \underline{v}(\cdot)$ ) can be expressed as

$$\frac{\beta[1-\beta]}{2-\beta-\eta^2[1-\beta]} < \mu f < \frac{\beta}{2-\eta^2}. \quad (21)$$

*No Piracy Region without Threat of Copying.* In the no piracy region without threat of copying the firm maximizes

$$\max_{p,e} \left\{ \pi(p, e) = \frac{\mu f}{\beta} [1 + \eta e - p] - \frac{e^2}{2} \right\}. \quad (22)$$

From the FOCs of (22), the firm's optimal decisions are  $e^* = \eta p^* = \eta / [2 - \eta^2]$ . Given the firm's optimal decisions, the condition that determines this region,  $\mu f > \beta p$ , can be simplified as  $\mu f > \beta / [2 - \eta^2]$ , which indicates that to deter copying and its threat, the policy-maker must set large fines. Because this region effectively represents a conventional monopoly market, similar to our general framework we exclude this region from further analysis.

Integrating our findings in each region, the firm's optimal decisions are

$$p^* = \begin{cases} \frac{1-\beta+s+\mu f[1+\alpha]}{2-\eta^2[1-\beta]} & \text{if } \mu f < \frac{\beta[1-\beta]}{2-\beta-\eta^2[1-\beta]}, \\ \frac{\mu f}{\beta} & \text{if } \frac{\beta[1-\beta]}{2-\beta-\eta^2[1-\beta]} \leq \mu f \leq \frac{\beta}{2-\eta^2}, \\ \frac{1}{1-\eta^2} & \mu f > \frac{\beta}{2-\eta^2}, \end{cases}$$

and

$$e^* = \begin{cases} \frac{\eta[1-\beta+s+\mu f[1+\alpha]]}{2-\eta^2[1-\beta]} & \text{if } \mu f < \frac{\beta[1-\beta]}{2-\beta-\eta^2[1-\beta]}, \\ \frac{\eta \mu f}{\beta} & \text{if } \frac{\beta[1-\beta]}{2-\beta-\eta^2[1-\beta]} \leq \mu f \leq \frac{\beta}{2-\eta^2}, \\ \frac{\eta}{1-\eta^2} & \mu f > \frac{\beta}{2-\eta^2}. \end{cases}$$

### 5.1.3. Stage 1: The Policy-Maker's Response

**User Segmentation.** We now examine the impact of our policy instruments on user segmentation when utility functions are additive.

*Piracy Region.* We begin our analysis in the piracy region where (19) holds. Substituting for the firm's optimal decision in this region, the valuation of the user indifferent between subscribing and copying and that of the user indifferent between copying and not using are

$$\tilde{v}(s, \alpha, f) = 1 - \frac{1 - \beta + s - \mu f [1 - \alpha + \beta \eta^2 \alpha]}{[1 - \beta][2 - [1 - \beta]\eta^2]}, \quad (23)$$

$$\underline{v}(s, \alpha, f) = \frac{\mu f}{\beta} - \frac{\eta^2 [1 - \beta + s + \mu f [1 + \alpha]]}{2 - [1 - \beta]\eta^2}. \quad (24)$$

Next we examine the impact of each policy instrument on the aggregate proportion of subscribers and pirates,  $TU(s, \alpha, f) = 1 - \underline{v}(s, \alpha, f)$ , the proportion of pirates,  $CU(s, \alpha, f) = \tilde{v}(s, \alpha, f) - \underline{v}(s, \alpha, f)$ , and the proportion of subscribers  $SU(s, \alpha, f) = 1 - \tilde{v}(s, \alpha, f)$ .

By differentiating  $TU(s, \alpha, f) = TU(\cdot)$  with respect to subsidies, fines, and restitution, we get

$$\frac{dTU(\cdot)}{ds} = \frac{\eta^2}{2 - [1 - \beta]\eta^2} > 0, \quad \frac{dTU(\cdot)}{d\alpha} = \frac{\eta^2 \mu f}{2 - [1 - \beta]\eta^2} > 0 \quad \text{and} \quad \frac{dTU(\cdot)}{df} = \frac{-\mu [2 - [1 + \alpha\beta]\eta^2]}{\beta [2 - [1 - \beta]\eta^2]} < 0.$$

Consistent with Theorem 1, with additive utility an increase in subsidies always increases  $TU(\cdot)$  because the firm's investment in quality is always increasing in subsidies. Moreover, an increase in restitution increases the aggregate proportion of subscribers and pirates because with additive utility the firm's investment in quality is increasing in restitution. Finally, the aggregate proportion of subscribers and pirates increases with fines.

By differentiating  $CU(s, \alpha, f) = CU(\cdot)$  with respect to subsidies, fines, and restitution, we get

$$\begin{aligned} \frac{dCU(\cdot)}{ds} &= \frac{-[1 - [1 - \beta]\eta^2]}{[1 - \beta][2 - [1 - \beta]\eta^2]} < 0, & \frac{dCU(\cdot)}{d\alpha} &= \frac{\mu f}{[1 - \beta][2 - [1 - \beta]\eta^2]} > 0 \quad \text{and} \\ \frac{dCU(\cdot)}{df} &= \frac{-\mu [2 - \eta^2 [1 - \beta] - \beta [1 + \alpha]]}{[1 - \beta][2 - [1 - \beta]\eta^2]} < 0. \end{aligned}$$

The negative impact of subsidies and fines on the proportion of pirates indicates that with additive utility their direct negative effect dominates the positive indirect effect through increases in the subscription fee. On the other hand, an increase in restitution indirectly increases the proportion of pirates by increasing the subscription fee.

By differentiating  $SU(s, \alpha, f) = SU(\cdot)$  with respect to subsidies, fines, and restitution, we get

$$\begin{aligned} \frac{dSU(\cdot)}{ds} &= \frac{1}{[1 - \beta][2 - [1 - \beta]\eta^2]} > 0, & \frac{dSU(\cdot)}{d\alpha} &= \frac{-2\mu f [1 - [1 - \beta]\eta^2]}{[1 - \beta][2 - [1 - \beta]\eta^2]} < 0 \quad \text{and} \\ \frac{dSU(\cdot)}{df} &= \frac{\mu [1 - \alpha + \alpha [1 - \beta]\eta^2]}{[1 - \beta][2 - [1 - \beta]\eta^2]} > 0. \end{aligned}$$

The positive impact of subsidies and fines on the proportion of subscribers indicates that the positive effect of increasing subsidies and fines (directly and indirectly by increasing the firm's investment in quality) outweighs the negative indirect effect through increases in the subscription fee. On the other hand, the negative impact of restitution on the proportion of subscribers indicates that the negative indirect effect of increasing restitution by increasing the subscription fee outweighs the positive indirect effect by increasing the firm's investment in quality.

*No Piracy Region with Threat of Copying.* We next examine the impact of fines on the proportion of subscribers (also representing the aggregate proportion of subscribers and pirates) in the no piracy region with threat of copying where (21) holds. Substituting for the firm's optimal decision in this region, the valuation of the user indifferent between subscribing and not using the good is

$$\bar{v}_{lp}(f) = \frac{\mu f [1 - \eta^2]}{\beta}. \quad (25)$$

By differentiating  $SU(f) = TU(f) = 1 - \bar{v}_{lp}(f)$  with respect to fines, we get

$$\frac{dSU(f)}{df} = \frac{-\mu [1 - \eta^2]}{\beta} < 0,$$

indicating that in the no piracy region with threat of copying the proportion of subscribers decreases with fines. Following Theorem 2 in the no piracy region with threat of copying, an increase in fines increases the proportion of pirates if  $de(f)/df > \mu \partial u^C(\underline{v}, e)/\partial e$ . With additive utility, the left hand-side of this condition is  $\mu f \eta / \beta$  and its right hand-side is  $\beta \mu \eta$  indicating that the condition is violated.

**Social Welfare.** We now examine how social welfare is affected by the policy-maker's intervention facing a budget balance constraint in: (i) the piracy region where copying occurs,  $\bar{v}(\cdot) > \underline{v}(\cdot)$  and (ii) the no piracy region with threat of copying,  $\bar{v}(\cdot) = \underline{v}(\cdot)$ .

*Piracy Region.* Before discussing whether the policy-maker's intervention in the piracy region can improve social welfare, we first show that with additive utility the policy-maker's optimal intervention resides only in *restitution-subsidies regime*. As shown in Figure 6, the *no subsidy regime* and the *no restitution regime* are characterized by  $\Delta(\tilde{v}, e) > T_2$  and  $\Delta(\tilde{v}, e) < T_1$ , respectively. With additive utility, we have  $\Delta(\tilde{v}, e) = \eta [1 - \beta]$ ,  $T_1 = 0$ , and  $T_2 = 2\eta [1 - \beta]$  indicating that  $\Delta(\tilde{v}, e) > T_2$  and  $\Delta(\tilde{v}, e) < T_1$  are violated. Thus, with additive utility the *no subsidy regime* and the *no restitution regime* are infeasible.

For the moment, let us assume the policy-maker imposes positive fines on detected pirates. Under such conditions, the policy-maker has the following options: redistributing revenue from fines (i) as restitution to the firm ( $\alpha = 1$  and  $s = 0$ ) and (ii) as subsidies to support subscribers ( $\alpha = 0$  and  $s > 0$ ).

When the policy-maker redistributes revenue from fines as restitution to the firm ( $\alpha = 1$  and  $s = 0$ ), the valuation of the user indifferent between subscribing and copying the good in (23), and the valuation of the user indifferent between copying and not using the good in (24) can be written as

$$\tilde{v}_\alpha(f) = \frac{1 - \eta^2[1 - \beta + \mu f]}{2 - [1 - \beta]\eta^2} \quad \text{and} \quad \underline{v}_\alpha(f) = \frac{\mu f}{\beta} - \frac{\eta^2[1 - \beta + 2\mu f]}{2 - [1 - \beta]\eta^2},$$

respectively.

Substituting for  $\alpha = 1$  and  $s = 0$ , the policy-maker's problem described in (11) and (12) can be written as

$$\max_f \left\{ SW_\alpha(f) = \int_{\tilde{v}_\alpha(f)}^1 \left[ v + \eta \frac{\eta[1 - \beta + 2\mu f]}{2 - \eta^2[1 - \beta]} \right] dv + \int_{\underline{v}_\alpha(f)}^{\tilde{v}_\alpha(f)} \beta \left[ v + \eta \frac{\eta[1 - \beta + 2\mu f]}{2 - \eta^2[1 - \beta]} \right] dv - \frac{1}{2} \left[ \frac{\eta[1 - \beta + 2\mu f]}{2 - \eta^2[1 - \beta]} \right]^2 \right\}, \quad (26)$$

where  $SW_\alpha(f)$  represents social welfare when the policy-maker redistributes revenue from fines as restitution. From the FOC of (26) with respect to fines, the policy-maker's choice of fines is

$$f_\alpha = \frac{\beta\eta^2[1 + 3\beta]}{\mu[4[1 - \eta^2] + \eta^4[1 - 5\beta] + 8\beta\eta^2]}, \quad (27)$$

where  $f_\alpha$  represents the optimal level of fines when revenue from fines is redistributed as restitution.

Substituting the optimal level of fines in (27) into the condition identifying the piracy region defined in (19) and solving for  $\beta$  yields a threshold  $T_\alpha$ , where

$$T_\alpha = \frac{4\eta^4 - 7\eta^2 + 4}{2\eta^2[5 - 2\eta^2]} + \frac{1}{2} \sqrt{\frac{32\eta^6 - 71\eta^4 + 24\eta^2 + 16}{\eta^4[5 - 2\eta^2]^2}}, \quad (28)$$

such that  $0 \leq \beta < T_\alpha$  defines the piracy region in terms of  $\beta$  when revenue from fines are redistributed as restitution.

When the policy-maker redistributes revenue from fines as subsidies ( $\alpha = 0$  and  $s > 0$ ), the valuation of the user indifferent between subscribing and copying the good in (23), and the valuation of the user indifferent between copying and not using the good in (24) can be written as

$$\tilde{v}_s(s, f) = \frac{1 - \beta - s - \eta^2 + \beta\eta^2[2 - \beta] - \mu f}{[1 - \beta][2 - [1 - \beta]\eta^2]} \quad \text{and} \quad \underline{v}_s(s, f) = \frac{\mu f}{\beta} - \frac{\eta^2[1 - \beta + s + \mu f]}{2 - [1 - \beta]\eta^2},$$

respectively.

Substituting for  $\alpha = 0$ , the policy-maker's problem described in (11) and (12) can be written as

$$\max_f \left\{ SW_s(f, s) = \int_{\tilde{v}_s(s, f)}^1 \left[ v + \eta \frac{\eta[1 - \beta + s + \mu f]}{2 - \eta^2[1 - \beta]} \right] dv + \int_{\underline{v}_s(s, f)}^{\tilde{v}_s(s, f)} \beta \left[ v + \eta \frac{\eta[1 - \beta + s + \mu f]}{2 - \eta^2[1 - \beta]} \right] dv - \frac{1}{2} \left[ \frac{\eta[1 - \beta + s + \mu f]}{2 - \eta^2[1 - \beta]} \right]^2 \right\}, \quad (29)$$

$$\text{subject to: } s[1 - \tilde{v}_s(s, f)] dv \leq \mu f [\tilde{v}_s(s, f) - \underline{v}_s(s, f)], \quad (30)$$

where  $SW_s(f, s)$  represents social welfare when the policy-maker redistributes revenue from fines as subsidies.

At the optimum, the budget balance constraint, shown in (30), binds. Thus, we can obtain the optimal level of subsidies as function of fines, denoted by  $s(f)$ . Letting  $\omega = \eta^2[1 - \beta]$ , the optimal level of subsidies is

$$s(f) = \sqrt{\frac{1 - \beta}{4} \left[ 1 - \beta + 2\mu f [4 - \omega] - \frac{\mu^2 f^2 [8 - \omega [4 + \eta^2 \beta]]}{\beta} \right]} - \frac{1 - \beta + \mu f [2 - \omega]}{2}.$$

We next substitute  $s(f)$  in (29) and take the FOC with respect to fines, and then solve for the welfare maximizing level of fines, denoted by  $f_s$ . Substituting  $f_s$  into the piracy region condition (19) and solving for  $\beta$  yields a threshold  $T_s$ , where

$$T_s = 1 - \eta^2, \quad (31)$$

such that  $0 \leq \beta < T_s$  defines the piracy region in terms of  $\beta$  when revenue from fines is redistributed as subsidies.

*No Piracy Region With Threat of Copying.* In the no piracy region with threat of copying, the valuation of the user indifferent between subscribing and not using the good is shown in (25). Denoting social welfare in this region by  $SW_{lp}$  ( $lp$  for limit pricing), the policy-maker maximizes

$$\max_f \left\{ SW_{lp}(f) = \int_{\bar{v}_{lp}(f)}^1 \left[ v + \eta \frac{\eta \mu f}{\beta} \right] dv - \frac{1}{2} \left[ \frac{\eta \mu f}{\beta} \right]^2 \right\}. \quad (32)$$

From the FOC of (32), the policy-maker's choice of fine in the no piracy region with threat of copying is

$$f_{lp} = \frac{\beta \eta^2}{\mu [1 + \eta^2 [1 - \eta^4]]}, \quad (33)$$

where  $f_{lp}$  represents the optimal level of fines in the no piracy region with threat of copying.

Substituting the optimal level of fines in (33) into the condition identifying the no piracy region with threat of copying defined in (21) and solving for  $\beta$  yields a threshold  $T_{lp}$ , where

$$T_{lp} = 1 - \frac{\eta^2}{1 + \eta^4 [1 - \eta^2]}, \quad (34)$$

such that  $T_{lp} \leq \beta \leq 1$  defines the no piracy region with threat of copying in terms of  $\beta$ .

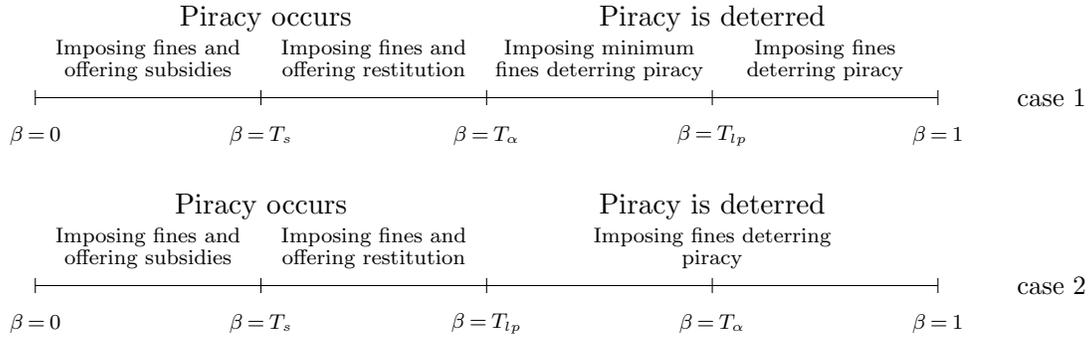
Integrating our findings in each region, Proposition 2 explores the policy-maker's optimal intervention in our additive form.

**PROPOSITION 2.** *With additive utility if the quality degradation between legitimate and pirated versions is*

- (a) large,  $0 < \beta \leq T_s$ , then the optimal instruments are fines and subsidies,  
 (b) moderate,  $T_s < \beta \leq \min\{T_\alpha, T_{lp}\}$ , then the optimal instruments are fines and restitution,  
 (c) small,  $\min\{T_\alpha, T_{lp}\} < \beta \leq 1$ , then the optimal instruments are large fines that deter digital piracy.

From the proof of Proposition 2, two possible orderings emerge for  $T_s$ ,  $T_\alpha$ , and  $T_{lp}$ . The first, referred to as case 1, is where  $T_s < T_\alpha < T_{lp}$ , and this holds when  $\sqrt{0.802} \leq \eta \leq 1$ . The second scenario, case 2, is characterized by  $T_s < T_{lp} < T_\alpha$ , applicable when  $0 < \eta < \sqrt{0.802}$ .

As shown in Figure 7, when the quality degradation between the legitimate and pirated versions of the good is large,  $0 < \beta < T_s$ , or moderate,  $T_s < \beta < \min\{T_\alpha, T_{lp}\}$ , digital piracy occurs. Under such conditions, the policy-maker imposes fines on detected pirates and redistributes revenue from fines to society either through subsidies or restitution. Finally, when the quality degradation between the legitimate and pirated versions of the good is small,  $\min\{T_\alpha, T_{lp}\} < \beta < 1$ , the digital piracy is deterred by setting large fines.



**Figure 7** Optimal instruments with additive utility functions

We next examine how consumer surplus and the firm's profit are affected across all possible outcomes. We begin with consumer surplus which in the piracy region is defined as the net payoff of subscribers and the net payoff of pirates. The first possibility is redistributing revenue from fines as subsidies as per Proposition 2(a). Let  $s(f)$  represent the optimal level of subsidies. Here consumer surplus can be written as

$$\begin{aligned}
 CS_s(f) = & \int_{\tilde{v}_s(f)}^1 \left[ v + \eta \frac{\eta[1 - \beta + s(f) + \mu f]}{2 - \eta^2[1 - \beta]} - \frac{1 - \beta + s(f) + \mu f}{2 - \eta^2[1 - \beta]} + s(f) \right] dv \\
 & + \int_{\underline{v}_s(f)}^{\tilde{v}_s(f)} \left[ \beta v + \beta \eta \frac{\eta[1 - \beta + s(f) + \mu f]}{2 - \eta^2[1 - \beta]} - \mu f \right] dv,
 \end{aligned} \tag{35}$$

where the first integral shows the net payoff subscribers and the second integral represents the net payoff of pirates.

The second possibility is redistributing revenue from fines as restitution as per Proposition 2(b). Under such conditions, consumer surplus is

$$\begin{aligned}
 CS_{\alpha}(f) = & \int_{\bar{v}_{\alpha}(f)}^1 \left[ v + \eta \frac{\eta[1-\beta+2\mu f]}{2-\eta^2[1-\beta]} - \frac{1-\beta+2\mu f}{2-\eta^2[1-\beta]} \right] dv \\
 & + \int_{\underline{v}_{\alpha}(f)}^{\bar{v}_{\alpha}(f)} \left[ \beta v + \beta \eta \frac{\eta[1-\beta+2\mu f]}{2-\eta^2[1-\beta]} - \mu f \right] dv,
 \end{aligned} \tag{36}$$

where the first integral shows the net payoff subscribers and the second integral represents the net payoff of pirates.

In the *no piracy region with threat of copying*, Proposition 2(c), consumer surplus is the net payoff of subscribers

$$CS_{tp}(f) = \int_{\bar{v}_{tp}(f)}^1 \left[ v + \eta \frac{\eta \mu f}{\beta} - \frac{\mu f}{\beta} \right] dv. \tag{37}$$

The following proposition outlines how consumer surplus and the firm's profit are affected when the optimal instruments are implemented.

**PROPOSITION 3.** *With additive utility the policy-maker's intervention decreases consumer surplus and increases the firm's profit.*

As shown in Proposition 2, we find that when the policy-maker can redistribute revenue from fines back to society through restitution or subsidies, imposing fines is welfare maximizing. In line with previous research, Proposition 3 shows that imposing fines on detected pirates leads to a reduction in consumer surplus. Nevertheless, our model's unique feature of allowing the policy-maker to redistribute revenue from fines as restitution or subsidies mitigates the negative impact of fines on consumer surplus. For instance, when revenue from fines is redistributed as subsidies, the adverse effects on consumer surplus are diminished as subscribers' net payment decreases and the firm's investment in quality increases.

## 5.2. Multiplicative Utility

With the multiplicative form, the utility functions of a subscriber and that of a pirate are

$$U^S = ve + s - p \quad \text{and} \quad U^C = \beta ve - \mu f,$$

respectively. Because  $0 < \beta < 1$  comparing the two above utility functions indicates that for a given user the perceived utility of copying is lower than subscribing.

### 5.2.1. Stage 3: User Behavior

With multiplicative utility, a user subscribes under the following IR and IC conditions

$$v \geq \bar{v} = \frac{p-s}{e} \quad [IR] \quad \text{and} \quad v \geq \tilde{v} = \frac{p-s-\mu f}{[1-\beta]e} \quad [IC].$$

A user copies the good if the following IR and IC conditions hold:

$$v \geq \underline{v} = \frac{\mu f}{\beta e} \quad [IR] \quad \text{and} \quad v \leq \tilde{v} = \frac{p-s-\mu f}{[1-\beta]e} \quad [IC].$$

Copying does not occur if  $\bar{v}(\cdot) \geq \underline{v}(\cdot)$  or equally if  $\mu f \geq \beta p$  because  $s = \alpha = 0$ . Therefore, if  $\mu f < \beta p$ , then some users copy the good and copying occurs. The proportion of subscribers and that of pirates can be written as

$$SU = \begin{cases} 1 - \frac{p-\mu f-s}{[1-\beta]e} & \text{if } \mu f < \beta p \\ 1 - \frac{p}{e} & \text{if } \mu f \geq \beta p \end{cases} \quad \text{and} \quad CU = \begin{cases} \frac{\beta[p-s]-\mu f}{\beta[1-\beta]e^2} & \text{if } \mu f < \beta p \\ 0 & \text{if } \mu f \geq \beta p. \end{cases}$$

When copying occurs, by differentiating  $SU$  and  $CU$  with respect to  $e$ , we get

$$\frac{\partial SU}{\partial e} = \frac{p-s-\mu f}{[1-\beta]e^2} > 0 \quad \text{and} \quad \frac{\partial CU}{\partial e} = -\frac{\beta[p-s]-\mu f}{\beta[1-\beta]e^2} < 0,$$

respectively. Thus, in our multiplicative utility functions, an increase in the firm's investment in quality increases the proportion of subscribers. However, unlike additive utility functions where  $\partial CU/\partial e = 0$ , in our multiplicative utility functions, an increase in the firm's investment in quality decreases the proportion of pirates.

### 5.2.2. Stage 2: The Firm's Response

*Piracy Region.* Substituting for the proportion of subscribers and that of pirates in the piracy region where  $\mu f < \beta p$ , the firm's profit function can be written as

$$\max_{p,e} \left\{ \pi(e,p;s,f,\alpha) = p \left[ 1 - \frac{p-\mu f-s}{[1-\beta]e} \right] + \alpha \mu f \left[ \frac{\beta[p-s]-\mu f}{\beta[1-\beta]e} \right] - \frac{e^2}{2} \right\}. \quad (38)$$

It makes no difference whether the firm chooses the optimal subscription fee and investment in quality simultaneously or sequentially. Solving the firm's profit maximization problem sequentially allows us to separate direct effects of each instrument from indirect effects. We assume that the firm sets the investment in quality before choosing the subscription fee. Under such conditions, the firm knows the quality at the time of pricing.

From the FOC of (38) with respect to  $p$ , for a given level of investment in quality, the optimal subscription fee is

$$p(e,s,f,\alpha) = \frac{[1-\beta]e + s + \mu f[1+\alpha]}{2}. \quad (39)$$

Substituting the above optimal subscription fee for  $p$  in  $\mu f < \beta p$ , the condition defining the piracy region can be written as

$$e > \frac{2\mu f - \beta\mu f[1 + \alpha] + \beta s}{\beta[1 - \beta]}. \quad (40)$$

We next substitute the optimal subscription fee in (39) back into (38) and differentiate the outcome, denoted by  $\pi(e; s, f, \alpha)$ , with respect to  $e$ . It is straightforward that as  $e \rightarrow \infty$ , then  $\partial\pi(e; s, f, \alpha)/\partial e < 0$ . Moreover, when the firm's investment in quality is equal to the threshold defined in (40), this partial derivative can be simplified as

$$\frac{\partial\pi(e; s, f, \alpha)}{\partial e} = \frac{\mu f[2 - \beta - \alpha\beta] + \beta s}{\beta[1 - \beta]} > 0$$

which is positive. Thus, there must exist at least one  $e$  satisfying  $\partial\pi(e; s, f, \alpha)/\partial e = 0$ . We can also show that when (40) holds, the firm's profit function is concave,  $\partial^2\pi(e; s, f, \alpha)/\partial e^2 < 0$ . Thus, the firm maximizes its profit by choosing the unique real solution for  $\partial\pi(e; s, f, \alpha)/\partial e = 0$ , denoted by  $\tilde{e}(f, s, \alpha)$ .

Accounting for the direct effect and indirect effect through the subscription fee, we next find the total impact of each policy instrument on the firm's investment in quality. Using the implicit function theorem, the total effects of subsidies and restitution on the firm's investment in quality,  $\tilde{e}(f, s, \alpha) = \tilde{e}(\cdot)$ , are

$$\frac{d\tilde{e}(\cdot)}{ds} = \left[ \frac{s + [1 - \alpha]\mu f}{2[1 - \beta]e^2} \right] \left[ \frac{\partial^2\pi(\cdot)}{\partial e^2} \right]^{-1} \quad \text{and} \quad \frac{d\tilde{e}(\cdot)}{d\alpha} = \left[ \frac{\mu f s}{2[1 - \beta]e^2} + \frac{\mu^2 f^2 [2 - \beta - \alpha\beta]}{2\beta[1 - \beta]e^2} \right] \left[ \frac{\partial^2\pi(\cdot)}{\partial e^2} \right]^{-1},$$

which are both negative indicating that with multiplicative utility, the indirect effects of subsidies and restitution on the firm's investment in quality through the subscription fee dominate their respective direct effects.

**PROPOSITION 4.** *With multiplicative utility, the impact of fines on the firm's investment in quality depends on how the policy-maker redistributes revenue from fines, if doing so at all.*

The total effects of increased fines on the firm's investment in quality,

$$\frac{d\tilde{e}(\cdot)}{df} = - \left[ \frac{f\mu^2 [4\alpha - \beta\mu f[1 + \alpha]^2] - \beta\mu s[1 - \alpha]}{2\beta[1 - \beta]e^2} \right] \left[ \frac{\partial^2\pi(\cdot)}{\partial e^2} \right]^{-1},$$

is ambiguous. Depending on the policy-maker's optimal instruments, the firm's investment in quality can increase or decrease with fines.

The effect of fines on quality, under the budget balance constraint, hinges on how the revenue from fines is redistributed to society, if it is redistributed at all. When fines on detected pirates are

the only instrument available, the total effect of fines on the firm's investment in quality is given by,

$$\frac{d\tilde{e}(f|\alpha=0, s=0)}{df} = \left[ \frac{\beta\mu^3 f^2}{2\beta[1-\beta]e^2} \right] \left[ \frac{\partial^2 \pi(\cdot)}{\partial e^2} \right]^{-1} < 0, \quad (41)$$

which is negative indicating that in without budget balance imposing fines on detected pirates reduces the firm's investment in quality. This finding aligns with [Lahiri and Dey \(2013\)](#).

When our novel features are considered, fines on detected pirates do not necessarily reduce the firm's investment in quality. Specifically, when the policy-maker uses the revenue from fines as restitution ( $s=0$  and  $\alpha=1$ ), the effect becomes,

$$\frac{d\tilde{e}(f|\alpha=1, s=0)}{df} = - \left[ \frac{4f\mu^2[1-\beta\mu f]}{2\beta[1-\beta]e^2} \right] \left[ \frac{\partial^2 \pi(\cdot)}{\partial e^2} \right]^{-1} > 0, \quad (42)$$

which is always positive. Comparing equations (41) and (42) highlights the importance of budget balance. Without budget balance, fines directly lower the firm's investment in quality. With budget balance, however, (42) demonstrates that fines, when used as restitution, indirectly increase the firm's investment in quality, eventually outweighing the direct negative effect.

Comparing our findings in Propositions 1 and 4 reveals that the choice of functional form can change intermediate results.

*No Piracy Region With Threat of Copying.* With the budget balance constraint, because in this region piracy does not occur, we have  $s = \alpha = 0$ . Therefore, the condition characterizing this region,  $\underline{v} = \bar{v}$ , can be written as  $p = \mu f / \beta$ . Thus, the firm's profit in this region is

$$\max_e \left\{ \pi_{np}(e; f) = \frac{\mu f}{\beta} \left[ 1 - \frac{\mu f}{\beta} \right] - \frac{e^2}{2} \right\}. \quad (43)$$

From the FOC of (43), the profit maximizing investment in quality is  $e(f) = f^{2/3} \mu^{2/3} / \beta^{2/3}$ .

*No Piracy Region Without Threat of Copying.* This region is characterized by  $\underline{v} > \bar{v}$ , or  $\mu f > \beta p$ , and the firm maximizes

$$\max_{p,e} \left\{ \pi_{np}(p, e; f) = p \left[ 1 - \frac{p}{e} \right] - \frac{e^2}{2} \right\}. \quad (44)$$

From the FOC of (44) with respect to the subscription fee, for a given level of investment in quality, the optimal subscription fee is  $p = e/2$ . Thus, the condition that defines the no piracy region without threat of copying,  $\mu f > \beta p$  can be written as  $2\mu f / \beta > e$ . By substituting the optimal subscription fee into (44), it is straightforward that in this region the firm maximizes its profit by choosing  $e = 1/4$ .

We now summarize the results of our analysis in stage 2 with multiplicative utility. Given the firm's optimal decisions, we rewrite conditions that define each region. We begin with the updating

the condition of piracy region where the level of fines is relatively small. We let  $f_t(\alpha, s)$  be the solution for

$$\frac{f^{2/3}\mu^{2/3}}{\beta^{2/3}} = \frac{2\mu f - \beta\mu f[1 + \alpha] + \beta s}{\beta[1 - \beta]},$$

where the left hand side represents the firm's optimal investment in quality in the no piracy region with threat of copying and right hand side shows the threshold defined in (40). The condition defining the piracy region can be written as  $f < \rho(s, \alpha)$ . In the no piracy region without threat of copying, by substituting optimal investment in quality, that is  $e = 1/4$ , the condition that defines this region can be written as  $\mu f > \beta/8$ .

Thus, the firm's investment in quality in each region can be written as follows

$$e(f, s, \alpha) = \begin{cases} \tilde{e}(f, s, \alpha) & \text{if } f < \rho(s, \alpha) \\ \frac{f^{2/3}\mu^{2/3}}{\beta^{2/3}} & \text{if } \rho(s, \alpha) \leq f \leq \frac{\beta}{8\mu} \\ \frac{1}{4} & \text{if } f > \frac{\beta}{8\mu}. \end{cases}$$

### 5.2.3. Stage 1: The Policy-Maker's Response

We now show that the result of Theorem 3 holds when utility functions are multiplicative.

*No Piracy Region With Threat of Copying.* We begin our analysis in the no piracy region with threat of copying,  $\bar{v}(\cdot) = \underline{v}(\cdot)$ . Substituting for the firm's optimal decisions, the valuation of the user indifferent between subscribing and not using the good is  $\bar{v}_{lp} = [\mu f / \beta]^{1/3}$  and social welfare can be written as

$$\max_f \left\{ SW_{lp}(f) = \int_{\bar{v}_{lp}(f)}^1 v \left[ \frac{\mu f}{\beta} \right]^{\frac{2}{3}} dv - \frac{1}{2} \left[ \frac{\mu f}{\beta} \right]^{\frac{4}{3}} \right\}. \quad (45)$$

From the FOC of (45), in the no piracy region with threat of copying social welfare increases with fines. Thus, the policy-maker's choice of fine is the upper bound that defines the no piracy region with threat of copying, that is  $f_{lp} = \beta/[8\mu]$ . By substituting for the optimal level of fines,  $f_{lp} = \beta/[8\mu]$ , in (45), the maximum social welfare is  $SW_{lp}^* = 0.0625$ .

*Piracy Region.* We turn our attention to the piracy region,  $\bar{v}(\cdot) > \underline{v}(\cdot)$ . In this region when the policy-maker imposes fines on detected pirates, it redistributes revenue from fines either as restitution to support the firm ( $\alpha = 1$  and  $s = 0$ ) or subsidies to support subscribers ( $\alpha = 0$  and  $s > 0$ ). Lahiri and Dey (2013) show that with multiplicative utility, in the piracy region imposing fines reduces social welfare. Therefore, we drop the case where the policy-maker does not redistribute fines from our analysis, that is when  $s = \alpha = 0$  and  $f > 0$ .

When fines are redistributed as restitution, the firm's investment in quality is

$$e_\alpha(f) = \frac{\beta[1-\beta] + \sqrt[3]{\xi}}{12\beta} + \frac{\beta[1-\beta]^2}{12\sqrt[3]{\xi}},$$

where  $\xi = 24\sqrt{3}\beta^2\mu f\sqrt{432\mu^2 f^2 + \beta[1-\beta]^3} + 864\beta^2\mu^2 f^2 + \beta^3[1-\beta]^3$ . Moreover, the valuation of the user indifferent between subscribing and copying the good,  $\tilde{v}(\cdot)$ , and the valuation of the user indifferent between copying and not using the good,  $\underline{v}(\cdot)$ , can be written as

$$\tilde{v}_\alpha(f) = \frac{1}{2} \quad \text{and} \quad \underline{v}_\alpha(f) = \frac{12\mu f}{\beta[1-\beta] + \beta^2[1-\beta]^2 - \sqrt[3]{\xi} + \sqrt[3]{\xi}},$$

respectively.

Using the above equations, we now rewrite the policy-maker's problem described in (11) and (12)

$$\max_f \left\{ SW_\alpha(f) = \int_{\tilde{v}_\alpha(f)}^1 v e_\alpha(f) dv + \int_{\underline{v}_\alpha(f)}^{\tilde{v}_\alpha(f)} \beta v e_\alpha(f) dv - \frac{e_\alpha(f)^2}{2} \right\}, \quad (46)$$

where  $SW_\alpha(f)$  represents social welfare when the policy-maker redistributes revenue from fines as restitution. By differentiating (46) with respect to fines, we find that social welfare is strictly increasing in fines. As a result, the policy-maker increases the level of fines up to a point where piracy is deterred indicating that the policy-maker never has revenue from fines to redistribute as restitution. Therefore, following the discussion after Proposition 4, in the the piracy region the firm's investment in quality decreases with fines because  $\alpha = 0$ .

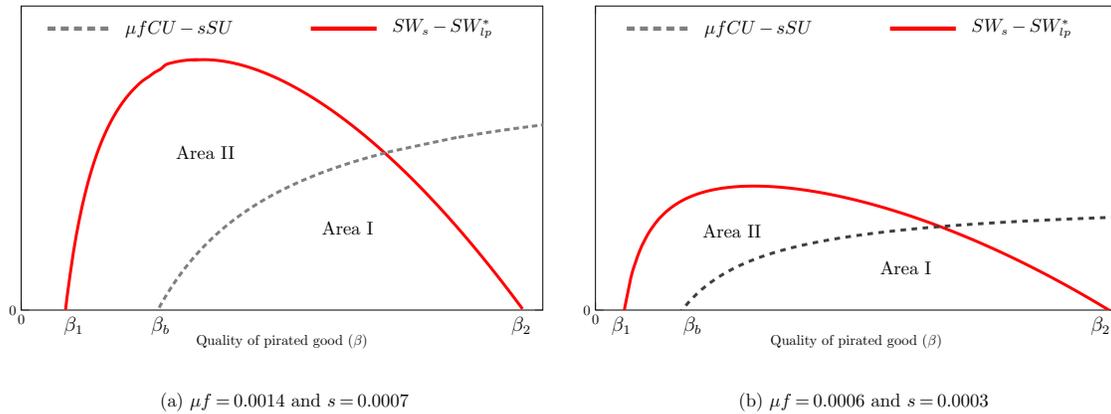
When the policy-maker redistributes revenue from fines as subsidies, social welfare problem can be written as

$$\max_f \left\{ SW(f, s) = \int_{\tilde{v}(f, s)}^1 v e(f, s) dv + \int_{\underline{v}(f, s)}^{\tilde{v}(f, s)} \beta v e(f, s) dv - \frac{e(f, s)^2}{2} \right\},$$

$$\mu f CU(s, f) - s SU(s, f) \geq 0. \quad (47)$$

By differentiating the social welfare function with respect to  $s$ , it turns out that with multiplicative utility the policy-maker enhances social welfare by supporting legal purchases through subsidies. Furthermore, an increase in subsidies reduces the left-hand side of the budget balance constraint. Consequently, if the policy-maker chooses to impose fines on detected pirates, all revenue from these fines is allocated to subsidies, ensuring that the budget constraint is binding. Although setting the budget balance constraint to equality allows for determining the subsidy level as a function of the fines, the complexity of the social welfare problem complicates the interpretation of the optimal fine level.

Viewing the optimal social welfare in the no piracy region with threat of copying,  $SW_{lp}^* = 0.0625$ , we use numerical examples to show that the policy-maker can raise social welfare beyond 0.0625



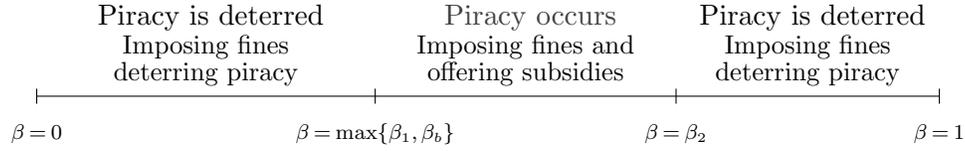
**Figure 8** Numerical examples showing that using subsidies and fines can be welfare-maximizing

by imposing fines and redistributing the revenue as subsidies and by tolerating some level of digital piracy. Figure 8(a) and (b) are associated with our numerical examples  $\mu f = 0.0014$  and  $s = 0.0007$ , and  $\mu f = 0.0006$  and  $s = 0.0003$ , respectively. Substituting for each numerical example, we redraw the budget balance constraint (dotted curves in Figure 8) and show their interaction with the x-axis with  $\beta_b$ . The budget balance constraint is satisfied for  $\beta_b < \beta < 1$ . We next find the difference between social welfare in (47), that is  $SW_s$ , and  $SW_{lp}^* = 0.0625$  for each numerical example. The gap between  $SW_s$  and  $SW_{lp}^*$  (displayed via a solid curve) intersects the x-axis at  $\beta_1$  and  $\beta_2$ .

As shown in Figure 8(a) and (b),  $\forall \beta \in [\beta_1, \beta_2]$  (which are denoted by Area I and Area II) we have  $SW_s > SW_{lp}^*$ . It may appear that in Area I and Area II social welfare is higher when imposing fines and redistributing the revenue as subsidies. However, in our first example, as shown in Figure 8(a), Area II is not feasible because the budget balance constraint is violated if  $\beta_1 < \beta < \beta_b$ . Therefore, accounting for the budget balance constraint, Area I which is associated with  $\beta_b < \beta < \beta_2$  represents the conditions under which  $SW_s > SW_{lp}^*$ , also indicating that  $SW_s^* > SW_{lp}^*$ .

The policy-maker's choice of fines and subsidies affects the relative positions of  $\beta_b$  and  $\beta_1$ . In contrast to the numerical examples shown in Figure 8(a) and (b), there are extreme cases where  $\beta_b$  can be less than  $\beta_1$ . When this occurs, the budget balance constraint is satisfied for all  $\beta \in [\beta_b, \beta_1]$ , but the use of fines and subsidies does not maximize welfare, leading to  $SW_s < SW_{lp}^*$ . Therefore, when  $\beta_b < \beta_1$ , fines and subsidies maximize welfare for all  $\beta \in [\beta_1, \beta_2]$ . Figure 9 shows the policy-maker's optimal intervention accounting for both cases. This figure demonstrates that the optimal strategy involves either eliminating piracy by imposing large fines or allowing some level of piracy through smaller fines, using the revenues from fines as subsidies. Figure 9 outlines a more general condition, that is  $\beta \in [\max \beta_1, \beta_b, \beta_2]$ , under which the combination of fines and subsidies maximizes welfare.

Lahiri and Dey (2013) show that under multiplicative utility, imposing fines on detected pirates decreases social welfare. In contrast, as illustrated in Figures 8 and 9, our welfare analysis with



**Figure 9** Optimal instruments with multiplicative utility functions

multiplicative utility suggests that the policy-maker can maximize social welfare by using the revenue from fines as subsidies. Additionally, we find that although using fines for restitution may not maximize social welfare, it can still lead to improvements. This divergence partially stems from the way fines affect the firm's investment in quality. Redistributing fines as restitution encourages the firm to invest more in quality, which mitigates the negative impact of fines on consumer surplus, ultimately leading to an improvement in social welfare.

### 5.3. Summary of Specific Functional Forms Results

We now summarize the effects of optimal interventions on user segmentation, consumer surplus, firm profit, and social welfare using our specific functional forms. Tables 1 and 2 present the results for additive and multiplicative utility, respectively.

Rather than examining the impact of individual policy instruments in isolation, we evaluate the overall effect of intervention, which may involve a combination of two policy instruments, compared to the case with no intervention. For example, we compare consumer surplus in the absence of any intervention ( $f = 0$ ,  $\alpha = 0$ , and  $s = 0$ ) with consumer surplus under the case where fines and restitution are used ( $f > 0$ ,  $\alpha = 1$ , and  $s = 0$ ).

**Table 1** The impact of optimal intervention when utility functions are additive

	Optimal intervention		
	Piracy region		No piracy region with threat of copying
	Fines & restitution	Fines & subsidies	Fines
<b>User segmentation</b>			
Proportion of subscribers, $SU$	Increased	Increased	Increased/Decreased
Proportion of pirates, $CU$	Decreased	Decreased	Deterred
Proportion of all users, $TU$	Decreased	Decreased	Decreased
<b>Welfare measures</b>			
Consumer surplus, $CS$	Decreased	Decreased	Decreased
Firm's profit, $\pi$	Increased	Increased	Increased
Social welfare, $SW$	Increased	Increased	Increased

With additive utility, Table 1 shows that both optimal interventions in the piracy region have similar effects on user segmentation, consumer surplus, firm profit, and social welfare. This suggests that the method of redistributing fine revenue may not significantly alter outcomes. In the no

piracy region with threat of copying, the optimal intervention has an ambiguous impact on the proportion of subscribers, reflecting the delicate trade-off between the impact of imposing large fines on (i) increasing the cost of piracy and (ii) boosting the firm’s market power by removing its “shadow competitor”. Table 1 also indicates that with additive utility, consumer surplus decreases across all interventions (in both piracy and no-piracy regions), while the firm’s profit and social welfare increase.

Table 2 summarizes the effects of optimal interventions on user segmentation, consumer surplus, the firm’s profit, and social welfare under multiplicative utility. In this case, although both interventions lead to increases in the firm’s profit and social welfare, the impact on consumer surplus is ambiguous. Furthermore, in the no piracy region without threat of copying, imposing large fines to raise piracy costs has a similar effect to extracting subscriber surplus by increasing the firm’s market power.

**Table 2    The impact of optimal intervention when utility functions are multiplicative**

	Optimal intervention	
	Piracy region	No piracy region with threat of copying
	Fines & subsidies	Fines
<b>User segmentation</b>		
Proportion of subscribers, $SU$	Increased	Unaffected
Proportion of pirates, $CU$	Decreased	Deterred
Proportion of all users, $TU$	Decreased	Decreased
<b>Welfare measures</b>		
Consumer surplus, $CS$	Decreased/Increased	Decreased/Increased
Firm’s profit, $\pi$	Increased	Increased
Social welfare, $SW$	Increased	Increased

## 6. Conclusion

Our goal is to investigate the role of a policy-maker in the enforcement of digital piracy prevention through maximizing social welfare by subsidizing legal purchases, imposing fines on detected pirates, and supporting the firm by sharing a fraction of revenues from fines as restitution. We develop a new general formulation in which the policy-maker maximizes social welfare with a constraint that the budget must be balanced. The novel elements we bring to the analysis are restitution as a policy instrument and budget balance.

We partition our analysis into three regions: (i) the piracy region where some users copy the good; (ii) the no piracy region with threat of copying where the firm deters pirates; and (i) the no piracy region without threat of copying where the policy-maker deters pirates. Our findings

indicate that by raising fines the policy-maker can lead the firm to deter piracy. In such a case, although piracy is removed, its threat still exists, limiting the firm's market power.

In the piracy region, our analysis yields a series of results. First, our findings explain contradictory results in the literature concerning the impact of digital piracy on the firm's investment in quality. Prior studies show that if users have the same (different) preference for quality, digital piracy decreases (increases) the firm's investment in quality. However, the results of our general framework show no monotonic relationship between digital piracy and quality. To determine the relationship between digital piracy and quality, in addition users' preferences for quality, we need to understand how digital piracy affects the firm's market power. We find that although fines and subsidies reduce digital piracy, they may have opposing effects on the firm's investment in quality. More importantly, we show that, independent of how a given policy instrument affects the firm's investment in quality, its impact on social welfare remains fairly constant. In contrast to prior studies (e.g., [Chen and Png 2003](#), [Lahiri and Dey 2013](#), [Belleflamme and Peitz 2010](#)), we find that higher fines and redistributing fine revenue with greater subsidies does not necessarily discourage users from copying. In the no piracy region with threat of copying, we find that the firm's investment in quality decreases in fines if subscribers' valuation for quality is sufficiently higher than that of pirates.

Although prior studies show that social welfare decreases with fines, we show this intuition does not always hold. Our welfare analyses imply that the optimality of social welfare using fines depends on whether the policy-maker can redistribute revenues from fines to subsidize legal purchases or to support the firm through restitution. In other words, the policy-maker can maximize social welfare when it views fines as transfers between pirates, subscribers, and the firm, and also balances its budget between fines, subsidies, and restitution. In contrast to prior research, our welfare analysis indicates that subsidizing legal purchases is not always welfare maximizing. By employing specific functional forms, we demonstrate that this outcome can arise when utility functions are additive. Our results indicate that under certain conditions, a policy-maker may prefer redistributing revenue from fines as restitution instead of providing subsidies.

To provide more detailed insights, we also develop a functional form with additive utility to determine algebraic conditions under which each region described above is realized and show that imposing fines on detected pirates enhances social welfare. Our analysis shows that although the social welfare maximizing fines comes at the expense of consumer surplus, it may sometimes deter digital piracy. This effect occurs because the policy-maker intervention directly reduces the surplus of pirates and indirectly decreases the surplus of subscribers by increasing the subscription fee. We also introduce a different model employing multiplicative utility to explore the effects of choosing specific functional forms on intermediate outcomes. For example, with additive utility we show that the firm's investment in quality increases with fines, irrespective of whether the policy-maker

redistributes revenue from fines as subsidies or restitution. In contrast, with multiplicative utility the impact of fines on the firm's investment in quality depends on how the policy-maker redistributes revenue from fines – that is, whether using restitution or subsidies, if choosing to do so at all.

Even though using a general framework provides more general results and captures a wide range of settings to which these results apply, our work includes some limitations. In our setting, the probability of detecting a pirate is treated as exogenous. According to the literature, this probability depends on ISPs' investment in monitoring users of digital goods. Thus, the first extension is to endogenize detection probability by adding a new stage into our model where an ISP maximizes profits by setting the level of detection probability. Second, our focus is on demand-side enforcement that is penalizing pirates through detection and fines. However, the policy-maker can design incentive mechanisms so that ISPs are motivated to reduce the accessibility of illegal versions of digital goods. Another area to study is how the policy-maker can fight digital piracy by using supply-side enforcement in the context of a balanced budget and restitution. Comparing the effectiveness of demand-side and supply-side enforcement can be another extension of our study. Finally, we may examine the impact of the policy-maker's instruments when firms offer a free version of the digital good in addition to the priced one. In combating digital piracy, employing versioning is considered a potential strategy. However, the provision of a lower-quality version may impact the sales of higher-quality versions. To understand the influence of versioning on users' inclination towards piracy, it is crucial to explore how users value the low-quality version compared to pirated content. If users value the low-quality version more than pirated content, then offering such a version can decrease piracy, but it may lead to a reduction in subscription fees revenue. Conversely, if the valuation for the low-quality version is lower than that of pirated content, then providing a low-quality version can reduce piracy without adversely affecting subscription revenue.

## Acknowledgments

We thank Duy Dao, Raymond Patterson, and Business Technology Management colloquium participants at the Haskayne School of Business for helpful comments. We also thank the Social Sciences and Humanities Research Council of Canada (SSHRC) for financial support. The authors are grateful for excellent editing help from Jeanette Burman.

## References

- Adermon A, Liang CY (2014) Piracy and music sales: The effects of an anti-piracy law. *Journal of Economic Behavior & Organization* 105:90–106.
- August T, Tunca TI (2008) Let the pirates patch? an economic analysis of software security patch restrictions. *Information Systems Research* 19(1):48–70.

- Bae SH, Choi JP (2006) A model of piracy. *Information Economics and Policy* 18(3):303–320.
- Baird A, Miller CJ, Raghu T, Sinha RK (2016) Product line extension in consumer software markets in the presence of free alternatives. *Information Systems Research* 27(2):282–301.
- Belleflamme P, Peitz M (2010) Digital piracy: theory. *Available at SSRN* .
- Bhattacharjee S, Gopal R, Marsden JR, Sankaranarayanan R, Telang R (2009) To theme or not to theme: Can theme strength be the music industry’s “killer app”? *Decision Support Systems* 48(1):141–149.
- Bhattacharjee S, Gopal RD, Lertwachara K, Marsden JR (2006) Consumer search and retailer strategies in the presence of online music sharing. *Journal of Management Information Systems* 23(1):129–159.
- Business Software Alliance (2018) Software management: Security imperative, business opportunity. *Business Software Alliance, Washington, DC* .
- Chellappa RK, Shivendu S (2005) Managing piracy: Pricing and sampling strategies for digital experience goods in vertically segmented markets. *Information Systems Research* 16(4):400–417.
- Chen Yn, Png I (2003) Information goods pricing and copyright enforcement: Welfare analysis. *Information Systems Research* 14(1):107–123.
- Cho WY, Ahn BH (2010) Versioning of information goods under the threat of piracy. *Information Economics and Policy* 22(4):332–340.
- Conner KR (1995) Obtaining strategic advantage from being imitated: When can encouraging “clones” pay? *Management Science* 41(2):209–225.
- Copyright Law of the US (2010) Intellectual property: Observations on efforts to quantify the economic effects of counterfeit and pirated goods. *Report GAO-10-423* .
- Danaher B, Smith MD, Telang R (2014a) Piracy and copyright enforcement mechanisms. *Innovation Policy and the Economy* 14(1):25–61.
- Danaher B, Smith MD, Telang R, Chen S (2014b) The effect of graduated response anti-piracy laws on music sales: evidence from an event study in France. *The Journal of Industrial Economics* 62(3):541–553.
- Dey D, Kim A, Lahiri A (2019) Online piracy and the “longer arm” of enforcement. *Management Science* 65(3):1173–1190.
- Eisend M (2019) Explaining digital piracy: a meta-analysis. *Information Systems Research* 30(2):636–664.
- Gopal RD, Gupta A (2010) Trading higher software piracy for higher profits: The case of phantom piracy. *Management Science* 56(11):1946–1962.
- Guo H, Zhao X, Hao L, Liu D (2019) Economic analysis of reward advertising. *Production and Operations Management* 28(10):2413–2430.
- Guo L, Meng X (2015) Digital content provision and optimal copyright protection. *Management Science* 61(5):1183–1196.

- Herings PJJ, Peeters R, Yang MS (2018) Piracy on the internet: Accommodate it or fight it? a dynamic approach. *European Journal of Operational Research* 266(1):328–339.
- International Federation of the Phonographic Industry (2018) In-depth study on global music listening habits. *International Federation of the Phonographic Industry, London* .
- Jain S (2008) Digital piracy: A competitive analysis. *Marketing science* 27(4):610–626.
- Johar M, Kumar N, Mookerjee V (2012) Content provision strategies in the presence of content piracy. *Information Systems Research* 23(3-part-2):960–975.
- Lahiri A, Dey D (2013) Effects of piracy on quality of information goods. *Management Science* 59(1):245–264.
- Lu S, Wang X, Bendle N (2020) Does piracy create online word of mouth? an empirical analysis in the movie industry. *Management Science* 66(5):2140–2162.
- Martin KD, Sykes BL, Shannon S, Edwards F, Harris A (2018) Monetary sanctions: Legal financial obligations in us systems of justice. *Annual Review of Criminology* 1:471–495.
- Mo J, Park J, Im N, Park J, Kim H (2017) Why internet service provider and content provider do not collaborate via monitoring of digital piracy. *Socio-Economic Planning Sciences* 60:49–61.
- Novos IE, Waldman M (1984) The effects of increased copyright protection: An analytic approach. *Journal of political economy* 92(2):236–246.
- Oberholzer-Gee F, Strumpf K (2007) The effect of file sharing on record sales: An empirical analysis. *Journal of political economy* 115(1):1–42.
- Polinsky AM, Shavell S (1979) The optimal tradeoff between the probability and magnitude of fines. *The American Economic Review* 69(5):880–891.
- Polinsky AM, Shavell S (2000) The economic theory of public enforcement of law. *Journal of economic literature* 38(1):45–76.
- Shavell S (1987) The optimal use of nonmonetary sanctions as a deterrent. *The American Economic Review* 584–592.
- Smith MD, Telang R (2009) Competing with free: The impact of movie broadcasts on DVD sales and internet piracy. *Mis Quarterly* 321–338.
- Smith MD, Telang R (2016) Windows of opportunity: the impact of piracy and delayed international availability on DVD sales. *Available at SSRN 2784759* .
- Sundararajan A (2004) Managing digital piracy: Pricing and protection. *Information Systems Research* 15(3):287–308.
- Wu Sy, Chen Py (2008) Versioning and piracy control for digital information goods. *Operations Research* 56(1):157–172.