Using Subsidies, Fines, and Restitution with Budget Balance to Combat Digital Piracy

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Technical E-companion

EC.1. Firm's Profit Concavity

We begin with providing the following functions which help with proving the concavity of the firm's profit function. Let

$$\tilde{\gamma}(\tilde{v},e)=\frac{\partial u^S(\tilde{v},e)}{\partial \tilde{v}}-\frac{\partial u^C(\tilde{v},e)}{\partial \tilde{v}}>0 \quad \text{and} \quad \underline{\gamma}(\underline{v},e)=\frac{\partial u^C(\underline{v},e)}{\partial \underline{v}}>0,$$

which are positive from Assumptions 3(b) and 5. Thus, the impact of the firm's decisions on the proportion of subscribers can be written as

$$\frac{\partial SU(\cdot)}{\partial p}=\frac{-1}{\tilde{\gamma}(\tilde{v},e)}<0 \quad \text{and} \quad \frac{\partial SU(\cdot)}{\partial e}=\frac{\dfrac{\partial u^S(\tilde{v},e)}{\partial e}-\dfrac{\partial u^C(\tilde{v},e)}{\partial e}}{\tilde{\gamma}(\tilde{v},e)}>0,$$

in which the sign of the last equation is determined using the first part of Assumptions 3(a) and 4. Moreover, the direct effect of an increase in subsidies and in fines can be written as

$$\frac{\partial SU(\cdot)}{\partial s}=\frac{1}{\tilde{\gamma}(\tilde{v},e)}>0 \quad \text{and} \quad \frac{\partial SU(\cdot)}{\partial f}=\frac{\mu}{\tilde{\gamma}(\tilde{v},e)}>0.$$

Next, the impact of the firm's decisions on the proportion of pirates can be written as

$$\frac{\partial CU(\cdot)}{\partial p}=\frac{1}{\tilde{\gamma}(\tilde{v},e)}>0 \quad \text{and} \quad \frac{\partial CU(\cdot)}{\partial e}=-\frac{\dfrac{\partial u^S(\tilde{v},e)}{\partial e}-\dfrac{\partial u^C(\tilde{v},e)}{\partial e}}{\tilde{\gamma}(\tilde{v},e)}+\frac{\dfrac{\partial u^C(\underline{v},e)}{\partial e}}{\underline{\gamma}(\underline{v},e))}<0,$$

which the sign of the last partial derivative is determined using Assumptions 4 and 5. Assumption 5 ensures a sufficiently elastic response to changes in subsidies, fines, subscription fees, and investment in quality.

Subscription fee. Using the first order condition (FOC) of (4), we have

$$\frac{\partial \phi_1(\cdot)}{\partial p}=\frac{\partial^2 \pi(\cdot)}{\partial p^2}=2\frac{\partial SU(\cdot)}{\partial p}<0,$$

which indicates that the firm's profit is concave in its subscription fee.

Investment in quality. By Assumption 3(c), as the firm's investment increases the marginal utility of subscribers increases faster than the marginal utility of pirates, or

$$\frac{\partial \tilde{\gamma}(\tilde{v},e)}{\partial e}=\frac{\partial^2 u^S(\tilde{v},e)}{\partial \tilde{v}\partial e}-\frac{\partial^2 u^C(\tilde{v},e)}{\partial \tilde{v}\partial e}>0.$$

Therefore, we have

$$\frac{\partial \phi_2(\cdot)}{\partial e} = \frac{\partial^2 \pi(\cdot)}{\partial e^2} = p\frac{\partial^2 SU(\cdot)}{\partial e^2} + \alpha\mu f\frac{\partial^2 CU(\cdot)}{\partial e^2} - 1,$$

where $\partial^2 SU(\cdot)/\partial e^2$ is

$$\frac{\frac{\partial^2 u^S(\tilde{v},e)}{\partial e^2} - \frac{\partial^2 u^C(\tilde{v},e)}{\partial e^2}}{\tilde{\gamma}(\tilde{v},e)} + \frac{\frac{\partial \tilde{v}(\cdot)}{\partial e}\left[\frac{\partial^2 u^S(\tilde{v},e)}{\partial \tilde{v}\partial e} - \frac{\partial^2 u^C(\tilde{v},e)}{\partial \tilde{v}\partial e}\right]}{\tilde{\gamma}(\tilde{v},e)} - \frac{\frac{\partial \tilde{\gamma}(\tilde{v},e)}{\partial e}\left[\frac{\partial u^S(\tilde{v},e)}{\partial e} - \frac{\partial u^C(\tilde{v},e)}{\partial e}\right]}{\tilde{\gamma}(\tilde{v},e)^2} < 0$$

which is always negative. Moreover, $\partial^2 CU(\cdot)/\partial e^2$ is

$$-\frac{\partial^2 SU(\cdot)}{\partial e^2} + \frac{\frac{\partial^2 u^C(\underline{v},e)}{\partial e^2} + \frac{\partial \underline{v}(\cdot)}{\partial e}\frac{\partial^2 u^C(\underline{v},e)}{\partial \underline{v}\partial e}}{\underline{\gamma}(\underline{v},e)} - \frac{\frac{\partial^2 u^C(\underline{v},e)}{\partial \underline{v}\partial e}\frac{\partial u^C(\underline{v},e)}{\partial \underline{v}}}{\underline{\gamma}(\underline{v},e)^2}.$$

Substituting for p, using the FOC of (4) with respect to the subscription fee, $\partial^2\pi(\cdot)/\partial e^2$ can be written as

$$-\frac{SU(\cdot)}{\frac{\partial SU(\cdot)}{\partial p}}\frac{\partial^2 SU(\cdot)}{\partial e^2} + \alpha\mu f\left[\frac{\frac{\partial^2 u^C(\underline{v},e)}{\partial e^2} + \frac{\partial \underline{v}(\cdot)}{\partial e}\frac{\partial^2 u^C(\underline{v},e)}{\partial \underline{v}\partial e}}{\underline{\gamma}(\underline{v},e)} - \frac{\frac{\partial^2 u^C(\underline{v},e)}{\partial \underline{v}\partial e}\frac{\partial u^C(\underline{v},e)}{\partial \underline{v}}}{\underline{\gamma}(\underline{v},e)^2}\right] - 1 < 0.$$

Thus, $\partial\phi_2(\cdot)/\partial e < 0$, and the firm's profit is concave in its investment in quality.

EC.2. Proofs of Lemmas, Theorems, and Propositions

Proof of Lemma 1

LEMMA 1. *In the piracy region, an increase in each policy instrument (a) increases the subscription fee; and (b) decreases the firm's investment in quality if the negative direct effect dominates the positive indirect effect through the subscription fee.*

Proof. Using the envelope theorem and by differentiating the FOCs of (4) with respect to subsidies, we get

$$\frac{\partial \phi_1(\cdot)}{\partial s} = \frac{\partial SU(\cdot)}{\partial s} > 0 \quad \text{and} \quad \frac{\partial \phi_2(\cdot)}{\partial s} = [p - \alpha\mu f]\frac{\frac{\partial \tilde{v}(\cdot)}{\partial s}\left[\frac{\partial^2 u^S(\tilde{v},e)}{\partial \tilde{v}\partial e} - \frac{\partial^2 u^C(\tilde{v},e)}{\partial \tilde{v}\partial e}\right]}{\tilde{\gamma}(\tilde{v},e)} \le 0.$$

Using the implicit function rule

$$\frac{\partial p(\cdot)}{\partial s} = -\frac{\partial \phi_1(\cdot)}{\partial s}\Big/\frac{\partial \phi_1(\cdot)}{\partial p} > 0 \quad \text{and} \quad \frac{\partial e(\cdot)}{\partial s} = -\frac{\partial \phi_2(\cdot)}{\partial s}\Big/\frac{\partial \phi_2(\cdot)}{\partial e} \le 0,$$

indicating that the direct effect of an increase in subsidies on the subscription fee and on the firm's investment in quality is positive and (weakly) negative, respectively.

Using the envelope theorem and by differentiating the FOCs of (4) with respect to fines, we get

$$\frac{\partial \phi_1(\cdot)}{\partial f} = \frac{\partial SU(\cdot)}{\partial f} + \alpha\mu\frac{\partial CU(\cdot)}{\partial p} = \mu\frac{\partial SU(\cdot)}{\partial s}[1+\alpha] > 0 \quad \text{and}$$

$$\frac{\partial \phi_2(\cdot)}{\partial f} = \alpha\mu\frac{\partial CU(\cdot)}{\partial e} + [p-\alpha\mu f]\frac{\frac{\partial \tilde{v}(\cdot)}{\partial f}\left[\frac{\partial^2 u^S(\tilde{v},e)}{\partial \tilde{v}\partial e} - \frac{\partial^2 u^C(\tilde{v},e)}{\partial \tilde{v}\partial e}\right]}{\tilde{\gamma}(\tilde{v},e)} + \alpha\mu f\frac{\frac{\partial \underline{v}(\cdot)}{\partial f}\frac{\partial^2 u^C(\underline{v},e)}{\partial \underline{v}\partial e}}{\frac{\partial u^C(\underline{v},e)}{\underline{v}}} < 0.$$

Following the implicit function rule we get

$$\frac{\partial p(\cdot)}{\partial f} = -\frac{\partial \phi_1(\cdot)}{\partial f}\Big/\frac{\partial \phi_1(\cdot)}{\partial p} = \frac{\mu[1+\alpha]}{2} > 0 \quad \text{and} \quad \frac{\partial e(\cdot)}{\partial f} = -\frac{\partial \phi_2(\cdot)}{\partial f}\Big/\frac{\partial \phi_2(\cdot)}{\partial p} < 0,$$

indicating that the direct effect of an increase in fines on the subscription fee and on the firm's investment in quality is positive and negative, respectively.

Using the envelope theorem and by differentiating the FOCs of (4) with respect to restitution, we get

$$\frac{\partial \phi_1(\cdot)}{\partial \alpha} = \mu f\frac{\partial CU(\cdot)}{\partial p} > 0 \quad \text{and} \quad \frac{\partial \phi_2(\cdot)}{\partial \alpha} = \mu f\frac{\partial CU(\cdot)}{\partial e} < 0.$$

Following the implicit function rule we get

$$\frac{\partial p(\cdot)}{\partial \alpha} = -\frac{\partial \phi_1(\cdot)}{\partial \alpha}\Big/\frac{\partial \phi_1(\cdot)}{\partial p} = \frac{\mu(\cdot)f}{2} > 0 \quad \text{and} \quad \frac{\partial e(\cdot)}{\partial \alpha} = -\frac{\partial \phi_2(\cdot)}{\partial \alpha}\Big/\frac{\partial \phi_2(\cdot)}{\partial e} < 0,$$

indicating that the direct effect of an increase in restitution on the subscription fee and on the firm's investment in quality is positive and negative, respectively.

To find the total effect of our policy instruments on the firm's decisions, we examine how the subscription fee and the investment in quality affect each other. By differentiating $\phi_1(\cdot) = 0$ and $\phi_2(\cdot) = 0$ with respect to e and p, respectively, we get

$$\frac{\partial^2 \pi(\cdot)}{\partial e \partial p} = \frac{\partial \phi_1(\cdot)}{\partial e} = \frac{\partial SU(\cdot)}{\partial e} + [p-\alpha\mu f]\frac{\frac{1}{\tilde{\gamma}(\tilde{v},e)}\left[\frac{\partial^2 u^S(\tilde{v},e)}{\partial \tilde{v}\partial e} - \frac{\partial^2 u^C(\tilde{v},e)}{\partial \tilde{v}\partial e}\right]}{\tilde{\gamma}(\tilde{v},e)} > 0,$$

$$\frac{\partial^2 \pi(\cdot)}{\partial p \partial e} = \frac{\partial \phi_2(\cdot)}{\partial p} = \frac{\partial SU(\cdot)}{\partial e} + [p-\alpha\mu f]\frac{\frac{1}{\tilde{\gamma}(\tilde{v},e)}\left[\frac{\partial^2 u^S(\tilde{v},e)}{\partial \tilde{v}\partial e} - \frac{\partial^2 u^C(\tilde{v},e)}{\partial \tilde{v}\partial e}\right]}{\tilde{\gamma}(\tilde{v},e)} > 0.$$

Using the above equations, we can obtain the determinant of the Jacobian matrix,

$$\det(J) = \frac{\partial^2 \pi(\cdot)}{\partial p^2}\frac{\partial^2 \pi(\cdot)}{\partial e^2} - \frac{\partial^2 \pi(\cdot)}{\partial p \partial e}\frac{\partial^2 \pi(\cdot)}{\partial p \partial e} = 2\frac{\partial SU(\cdot)}{\partial p}\frac{\partial^2 \pi(\cdot)}{\partial e^2} - \frac{\partial^2 \pi(\cdot)}{\partial p \partial e}\frac{\partial^2 \pi(\cdot)}{\partial p \partial e},$$

which we take as positive and is true with a sufficiently large scaled first-order term from Assumption 5.

We first find the total impact of an increase in subsidies on the subscription fee,

$$\det(J)\frac{dp(\cdot)}{ds} = -\frac{\partial^2\pi(\cdot)}{\partial s\partial p}\frac{\partial^2\pi(\cdot)}{\partial e^2} + \frac{\partial^2\pi(\cdot)}{\partial e\partial p}\frac{\partial^2\pi(\cdot)}{\partial s\partial e} > 0, \tag{EC.1}$$

which is always positive because a sufficient condition for $dp(\cdot)/ds > 0$ is

$$\frac{\partial u^S(\tilde{v},e)}{\partial e} - \frac{\partial u^C(\tilde{v},e)}{\partial e} \leq 1,$$

which is always true from Assumption 5.

We next find the total impact of an increase in subsidies on the firm's investment in quality,

$$\det(J)\frac{de(\cdot)}{ds} = -\frac{\partial^2\pi(\cdot)}{\partial p^2}\frac{\partial^2\pi(\cdot)}{\partial s\partial e} + \frac{\partial^2\pi(\cdot)}{\partial s\partial p}\frac{\partial^2\pi(\cdot)}{\partial p\partial e}, \tag{EC.2}$$

which is positive if

$$\frac{\partial u^S(\tilde{v},e)}{\partial e} - \frac{\partial u^C(\tilde{v},e)}{\partial e} \geq SU(\cdot)\left[\frac{\partial^2 u^S(\tilde{v},e)}{\partial\tilde{v}\partial e} - \frac{\partial^2 u^C(\tilde{v},e)}{\partial\tilde{v}\partial e}\right].$$

We next examine the total impact of an increase in restitution on the firm's subscription fee,

$$\det(J)\frac{dp(\cdot)}{d\alpha} = -\frac{\partial^2\pi(\cdot)}{\partial\alpha\partial p}\frac{\partial^2\pi(\cdot)}{\partial e^2} + \frac{\partial^2\pi(\cdot)}{\partial\alpha\partial e}\frac{\partial^2\pi(\cdot)}{\partial e\partial p} > 0, \tag{EC.3}$$

which is always positive following the same logic we used to show the positive effect of subsidies on the subscription fee. The total impact of an increase in restitution on the firm's investment in quality is

$$\det(J)\frac{de(\cdot)}{d\alpha} = -\frac{\partial^2\pi(\cdot)}{\partial p^2}\frac{\partial^2\pi(\cdot)}{\partial\alpha\partial e} + \frac{\partial^2\pi(\cdot)}{\partial\alpha\partial p}\frac{\partial^2\pi(\cdot)}{\partial e\partial p}, \tag{EC.4}$$

which indicates that an increase in restitution increases the firm's investment in quality if

$$\frac{\partial u^S(\tilde{v},e)}{\partial e} - \frac{\partial u^C(\tilde{v},e)}{\partial e} \leq SU(\cdot)\left[\frac{\partial^2 u^S(\tilde{v},e)}{\partial e\partial\tilde{v}} - \frac{\partial^2 u^C(\tilde{v},e)}{\partial e\partial\tilde{v}}\right] + 2\frac{\dfrac{\partial u^C(\underline{v},e)}{\partial e}}{\dfrac{\partial u^C(\underline{v},e)}{\partial\underline{v}}}\left[\frac{\partial u^S(\tilde{v},e)}{\partial\tilde{v}} - \frac{\partial u^C(\tilde{v},e)}{\partial\tilde{v}}\right].$$

We examine the total impact of an increase in fines on the firm's subscription fee,

$$\det(J)\frac{dp(\cdot)}{df} = -\frac{\partial^2\pi(\cdot)}{\partial f\partial p}\frac{\partial^2\pi(\cdot)}{\partial e^2} + \frac{\partial^2\pi(\cdot)}{\partial e\partial p}\frac{\partial^2\pi(\cdot)}{\partial f\partial e} > 0, \tag{EC.5}$$

which is always positive following the same logic we used to show the positive effect of subsidies on the subscription fee. We finally explore the total impact of an increase in fines on the firm's investment in quality,

$$\det(J)\frac{de(\cdot)}{df} = -\frac{\partial^2\pi(\cdot)}{\partial p^2}\frac{\partial^2\pi(\cdot)}{\partial f\partial e} + \frac{\partial^2\pi(\cdot)}{\partial f\partial p}\frac{\partial^2\pi(\cdot)}{\partial p\partial e}, \tag{EC.6}$$

which is non-monotonic. For $de(\cdot)/df$ to be positive the impact of investment in quality on the proportion of subscribers must be relatively greater than the magnitude of its impact on the proportion of pirates.

We next prove that (7) is a sufficient condition for digital piracy to reduce the firm's investment in quality. The firm's investment in quality increases with subsidies if

$$\frac{\partial SU(\cdot)}{\partial e} > \left[\frac{\partial SU(\cdot)}{\partial e}\right]^2 [p(\cdot) - \alpha\mu f]\frac{\partial\tilde{\gamma}(\cdot)}{\partial e},$$

or equally,

$$\frac{\partial u^S(\tilde{v},e)}{\partial e} - \frac{\partial u^C(\tilde{v},e)}{\partial e} > SU(\cdot)\left[\frac{\partial^2 u^S(\tilde{v},e)}{\partial\tilde{v}\partial e} - \frac{\partial^2 u^C(\tilde{v},e)}{\partial\tilde{v}\partial e}\right]$$

Simplifying (EC.6), it is easy to show that if the above equation holds, then $de(\cdot)/df > 0$. □

Proof of Theorem 1

THEOREM 1. *In the piracy region, (a) if the firm's investment in quality increases with subsidies and restitution, then the aggregate proportion of subscribers and pirates increases with subsidies and restitution, respectively, and (b) if the firm's investment in quality decreases with fines, then the aggregate proportion of subscribers and pirates decreases with fines.*

Proof of Part (a). Using Lemma 1, the total impact of subsidies on the proportion of subscribers and pirates can be expressed as

$$\frac{\mathrm{d}SU(\cdot)}{\mathrm{d}s} = \frac{\partial SU(\cdot)}{\partial s} + \frac{\mathrm{d}p(\cdot)}{\mathrm{d}s}\frac{\partial SU(\cdot)}{\partial p} + \frac{\mathrm{d}e(\cdot)}{\mathrm{d}s}\frac{\partial SU(\cdot)}{\partial e} \quad \text{and} \tag{EC.7}$$

$$\frac{\mathrm{d}CU(\cdot)}{\mathrm{d}s} = \frac{\partial CU(\cdot)}{\partial s} + \frac{\mathrm{d}p(\cdot)}{\mathrm{d}s}\frac{\partial CU(\cdot)}{\partial p} + \frac{\mathrm{d}e(\cdot)}{\mathrm{d}s}\frac{\partial CU(\cdot)}{\partial e}. \tag{EC.8}$$

By adding (EC.7) to (EC.8) we get

$$\frac{\mathrm{d}SU(\cdot)}{\mathrm{d}s} + \frac{\mathrm{d}CU(\cdot)}{\mathrm{d}s} = \frac{\mathrm{d}e(\cdot)}{\mathrm{d}s}\left[\frac{\partial SU(\cdot)}{\partial e} + \frac{\partial CU(\cdot)}{\partial e}\right],$$

which is negative if $de(\cdot)/ds < 0$. In other words, if the direct effect of subsidies on the firm's investment outweighs the indirect effect through the subscription fee, then an increase in subsidies always decreases the aggregate proportion of subscribers and pirates.

The total impact of restitution on the proportion of subscribers and pirates can be defined as

$$\frac{\mathrm{d}SU(\cdot)}{\mathrm{d}\alpha} = \frac{\mathrm{d}p(\cdot)}{\mathrm{d}\alpha}\frac{\partial SU(\cdot)}{\partial p} + \frac{\mathrm{d}e(\cdot)}{\mathrm{d}\alpha}\frac{\partial SU(\cdot)}{\partial e} \quad \text{and} \tag{EC.9}$$

$$\frac{\mathrm{d}CU(\cdot)}{\mathrm{d}\alpha} = \frac{\mathrm{d}p(\cdot)}{\mathrm{d}\alpha}\frac{\partial CU(\cdot)}{\partial p} + \frac{\mathrm{d}e(\cdot)}{\mathrm{d}\alpha}\frac{\partial CU(\cdot)}{\partial e}. \tag{EC.10}$$

Similarly, by adding (EC.9) to (EC.10) we have

$$\frac{\mathrm{d}SU(\cdot)}{\mathrm{d}\alpha}+\frac{\mathrm{d}CU(\cdot)}{\mathrm{d}\alpha}=\frac{\mathrm{d}e(\cdot)}{\mathrm{d}\alpha}\left[\frac{\partial SU(\cdot)}{\partial e}+\frac{\partial CU(\cdot)}{\partial e}\right],$$

which is negative if $de(\cdot)/d\alpha<0$. In other words, if the direct effect of subsidies on the firm's investment outweighs the indirect effect through the subscription fee, then an increase in restitution always decreases the aggregate proportion of subscribers and pirates.

Proof of Part (b). The total impact of fines on the proportion of subscribers and pirates can be defined as

$$\frac{\mathrm{d}SU(\cdot)}{\mathrm{d}f}=\frac{\partial SU(\cdot)}{\partial f}+\frac{\mathrm{d}p(\cdot)}{\mathrm{d}f}\frac{\partial SU(\cdot)}{\partial p}+\frac{\mathrm{d}e(\cdot)}{\mathrm{d}f}\frac{\partial SU(\cdot)}{\partial e}\quad\text{and}\tag{EC.11}$$

$$\frac{\mathrm{d}CU(\cdot)}{\mathrm{d}f}=\frac{\partial CU(\cdot)}{\partial f}+\frac{\mathrm{d}p(\cdot)}{\mathrm{d}f}\frac{\partial CU(\cdot)}{\partial p}+\frac{\mathrm{d}e(\cdot)}{\mathrm{d}f}\frac{\partial CU(\cdot)}{\partial e}.\tag{EC.12}$$

By adding (EC.11) to (EC.12) we get

$$\frac{\mathrm{d}SU(\cdot)}{\mathrm{d}f}+\frac{\mathrm{d}CU(\cdot)}{\mathrm{d}f}=\left[\frac{\partial SU(\cdot)}{\partial f}+\frac{\partial CU(\cdot)}{\partial f}\right]+\frac{\mathrm{d}e(\cdot)}{\mathrm{d}f}\left[\frac{\partial SU(\cdot)}{\partial e}+\frac{\partial CU(\cdot)}{\partial e}\right],$$

which is negative if an increase in fines reduces the firm's investment in quality. □

Proof of Corollary 1

COROLLARY 1. *In the piracy region, subsidies, fines, and restitution are not necessarily effective in fighting digital piracy.*

Proof. The total impact of higher subsidies on the proportion of pirates is shown in (EC.8). In this equation, the first term is negative, the second term is positive because $dp(\cdot)/ds>0$, and the last term is non-monotonic. Moreover, the total impact of imposing higher fines on pirates is shown in (EC.12). Although, the first two terms are negative and positive, respectively, the last term is non-monotonic. □

Proof of Theorem 2

THEOREM 2. *In the no piracy region with threat of copying, if the impact of fines on the firm's investment in quality is positive and sufficiently large, then the proportion of subscribers increases in fines.*

Proof. We begin with defining the KKT conditions. The first three KKT conditions are FOCs of (9) with respect to the subscription fee, quality, and the Lagrangian multiplier,

$$\frac{\partial L(e,p,f,\gamma)}{\partial p}=p\frac{\partial SU_{np}(e,p)}{\partial p}+SU_{np}(e,p)-\gamma\frac{\partial\overline{v}(e,p)}{\partial p}=0,\tag{EC.13}$$

$$\frac{\partial L(e,p,f,\gamma)}{\partial e}=p\frac{\partial SU_{np}(e,p)}{\partial e}-e-\gamma\left[\frac{\partial\overline{v}(e,p)}{\partial e}-\frac{\partial\underline{v}(e,p)}{\partial e}\right]=0,\tag{EC.14}$$

$$\frac{\partial L(e,p,f,\gamma)}{\partial\gamma}=\overline{v}(e,p)-\underline{v}(e,f)=0,\tag{EC.15}$$

where (EC.13), (EC.14), and (EC.15) are the FOCs of the Lagrangian function with respect to the subscription fee, quality, and the Lagrangian multiplier. The next conditions are feasibility ((*i*) and (*ii*)) and complementary slackness (*iii*),

$$(i)\ \overline{v}(e,p)-\underline{v}(e,f)\leq 0, \qquad (ii)\ \gamma\geq 0, \qquad (iii)\ \gamma\left[\overline{v}(e,p)-\underline{v}(e,f)\right]=0.$$

Because $\overline{v}(e,p)-\underline{v}(e,f)=\phi_4(e,p,g)$ implicitly defines the firm's subscription fee,

$$\frac{\partial p(\cdot)}{\partial f}=\frac{\partial\phi_4(e,p,f)/\partial f}{\partial\phi_4(e,p,f)/\partial p}=-\frac{\partial\underline{v}(e,f)/\partial f}{\partial\overline{v}(e,p)/\partial p}>0 \quad \text{and}$$
$$\frac{\partial p(\cdot)}{\partial e}=\frac{\partial\phi_4(e,p,f)/\partial e}{\partial\phi_4(e,p,f)/\partial p}=-\frac{\partial\overline{v}(e,p)/\partial e-\partial\underline{v}(e,f)/\partial e}{\partial\overline{v}(e,p)/\partial p}>0.$$

Therefore, (EC.14) can be written as

$$\frac{\partial L(e,p,f,\gamma)}{\partial e}=\frac{\partial p(\cdot)}{\partial e}SU_{np}(e,p)+p\frac{\partial SU_{np}(e,p)}{\partial e}+p\frac{\partial p(\cdot)}{\partial e}\frac{\partial SU_{np}(e,p)}{\partial p}-e=0. \tag{EC.16}$$

By totally differentiating (EC.16) with respect to f, we get

$$\begin{aligned}\frac{d^2L(e,p,f,\gamma)}{df\,de}&=-\frac{\dfrac{\partial^2u^C(\cdot)}{\partial\underline{v}\partial e}\dfrac{\partial\underline{v}(\cdot)}{\partial f}}{\dfrac{\partial u^C(\cdot)}{\partial\underline{v}}}\left[SU_{np}(e,p)+p\frac{\partial SU_{np}(e,p)}{\partial p}\right]+2\frac{\partial p(\cdot)}{\partial e}\frac{\partial SU_{np}(e,p)}{\partial p}\frac{\partial p(\cdot)}{\partial f}\\&\quad+\frac{\partial p(\cdot)}{\partial f}\frac{\partial SU_{np}(e,p)}{\partial e}-\frac{\partial p(\cdot)}{\partial f}\frac{\partial^2u^S(\cdot)}{\partial\overline{v}\partial e}\frac{\partial SU_{np}(e,p)}{\partial p}\\&=\frac{\partial p(\cdot)}{\partial f}\left[2\frac{\partial p(\cdot)}{\partial e}\frac{\partial SU_{np}(e,p)}{\partial p}+\frac{\partial SU_{np}(e,p)}{\partial e}-\frac{\partial^2u^S(\cdot)}{\partial\overline{v}\partial e}\frac{\partial SU_{np}(e,p)}{\partial p}\right],\end{aligned}$$

where in the square bracket of the last line, the first term is negative, the second term is positive, and the third term is weakly positive. Therefore, increasing fines in this region leads the firm to increase its investment in quality if

$$2\frac{\partial u^C(\cdot)/\partial e}{\partial u^C(\cdot)/\partial\underline{v}}+\frac{\partial^2u^S(\cdot)/\partial\overline{v}\partial e}{\partial u^S(\cdot)/\partial\overline{v}}>\frac{\partial u^S(\cdot)/\partial e}{\partial u^S(\cdot)/\partial\overline{v}}.$$

By totally differentiating $TU_{np}(e,p)=SU_{np}(e,p)$ with respect to f, we get

$$\begin{aligned}\frac{\mathrm{d}TU_{np}(e,p)}{\mathrm{d}f}&=\frac{\partial TU_{np}(e,p)}{\partial e}\frac{\mathrm{d}e(\cdot)}{\mathrm{d}f}+\frac{\partial TU_{np}(e,p)}{\partial p}\frac{\partial p(\cdot)}{\partial f}+\frac{\partial TU_{np}(e,p)}{\partial p}\frac{\partial p(\cdot)}{\partial e}\frac{\mathrm{d}e(\cdot)}{\mathrm{d}f}\\&=\left[\frac{\partial TU_{np}(e,p)}{\partial e}+\frac{\partial TU_{np}(e,p)}{\partial p}\frac{\partial p(\cdot)}{\partial e}\right]\frac{\mathrm{d}e(\cdot)}{\mathrm{d}f}+\frac{\partial TU_{np}(e,p)}{\partial p}\frac{\partial p(\cdot)}{\partial f}.\end{aligned}$$

From (EC.13), (EC.14), and (EC.15), the above equation can be written as,

$$\frac{\mathrm{d}TU_{np}(e,p)}{\mathrm{d}f}=\frac{\partial u^C(\underline{v},e)/\partial e}{\partial u^C(\underline{v},e)/\partial\underline{v}}\frac{\mathrm{d}e(\cdot)}{\mathrm{d}f}-\frac{\mu}{\partial u^C(\underline{v},e)/\partial\underline{v}},$$

which is positive if

$$\frac{\mathrm{d}e(\cdot)}{\mathrm{d}f}>\frac{\mu}{\dfrac{\partial u^C(\underline{v},e)}{\partial e}},$$

indicating that if the impact of fines on the firm's investment in quality is positive and sufficiently large then the proportion of subscribers increases with fines. □

Proof of Theorem 3

Theorem 3. *In the piracy region, imposing higher fines on detected pirates can improve social welfare.*

Proof. To prove the theorem, we first identify conditions under which each regime arises (**Step 1**). We then discuss the policy-maker's optimal intervention in each regime (**Step 2**).

Step 1.

We begin the proof by determining conditions under which increasing subsidies decreases social welfare ($dSW(s,f,\alpha)/ds < 0$) and the policy-maker sets a zero subsidy (conditions we refer to as the *no subsidy regime*). We then determine conditions under which increasing restitution decreases social welfare ($dSW(s,f,\alpha)/d\alpha < 0$) and the policy-maker sets a zero restitution (conditions we refer to as the *no restitution regime*). We next identify in what space the policy-maker may set positive subsidies and restitution at the same time (conditions we refer to as the *restitution-subsidies regime*).

Suppose we define the change in social welfare from an increase in quality as

$$\Phi(\cdot) = \int_{\tilde{v}(\cdot)}^{1} \frac{\partial u^S(v,e(\cdot))}{\partial e} dv + \int_{\underline{v}(\cdot)}^{\tilde{v}(\cdot)} \frac{\partial u^C(v,e(\cdot))}{\partial e} dv - e(\cdot),$$

which we take to be positive from appropriate scaling of $e(\cdot)$.

Following (13), an increase in subsidies leads to an increase in social welfare, $dSW(s,f,\alpha)/ds > 0$, if

$$\frac{de(\cdot)}{ds} > \frac{\frac{\partial SU(\cdot)}{\partial p}\left[1 - \frac{dp(\cdot)}{ds}\right][p(\cdot) - s - \mu f]}{\frac{\partial SU(\cdot)}{\partial e}[p(\cdot) - s - \mu f] + \frac{\partial TU(\cdot)}{\partial e}\mu f + \Phi(\cdot)}, \quad \text{(EC.17)}$$

where the denominator of the right-hand side is positive. However, the numerator is non-monotonic and depends on the magnitude of $dp(\cdot)/ds$. From the proof of Lemma 1, the firm's investment in quality decreases with subsidies if

$$\frac{\frac{\partial u^S(\tilde{v},e)}{\partial e} - \frac{\partial u^C(\tilde{v},e)}{\partial e}}{\frac{\partial u^S(\tilde{v},e)}{\partial \tilde{v}} - \frac{\partial u^C(\tilde{v},e)}{\partial \tilde{v}}} < SU(\cdot) \frac{\frac{\partial^2 u^S(\tilde{v},e)}{\partial e \partial \tilde{v}} - \frac{\partial^2 u^C(\tilde{v},e)}{\partial e \partial \tilde{v}}}{\frac{\partial u^S(\tilde{v},e)}{\partial \tilde{v}} - \frac{\partial u^C(\tilde{v},e)}{\partial \tilde{v}}}. \quad \text{(EC.18)}$$

Moreover, $dp(\cdot)/ds < 1$ when (EC.18) holds. Under such conditions, (EC.17) is violated and the policy-maker sets a zero subsidy and the no subsidy regime arises. Cancelling the denominators on each side of (EC.18), the right hand side defines the threshold T_1 in the text.

From (14) an increase in restitution leads to an increase in social welfare, or equally $dSW(s,f,\alpha)/d\alpha > 0$, if

$$\frac{de(\cdot)}{d\alpha} > \frac{-\dfrac{\partial SU(\cdot)}{\partial p}\dfrac{dp(\cdot)}{d\alpha}[p(\cdot)-s-\mu f]}{\dfrac{\partial SU(\cdot)}{\partial e}[p(\cdot)-s-\mu f]+\dfrac{\partial TU(\cdot)}{\partial e}\mu f+\Phi(\cdot)}, \tag{EC.19}$$

where the right-hand side is positive because the numerator and denominator are positive. This implies that the above inequality is violated if the firm's investment in quality decreases in restitution ($de(\cdot)/d\alpha < 0$). From the proof of Lemma 1, the firm's investment in quality decreases with restitution if

$$\frac{\dfrac{\partial u^S(\tilde{v},e)}{\partial e}-\dfrac{\partial u^C(\tilde{v},e)}{\partial e}}{\dfrac{\partial u^S(\tilde{v},e)}{\partial \tilde{v}}-\dfrac{\partial u^C(\tilde{v},e)}{\partial \tilde{v}}} > SU(\cdot)\frac{\dfrac{\partial^2 u^S(\tilde{v},e)}{\partial e\partial \tilde{v}}-\dfrac{\partial^2 u^C(\tilde{v},e)}{\partial e\partial \tilde{v}}}{\dfrac{\partial u^S(\tilde{v},e)}{\partial \tilde{v}}-\dfrac{\partial u^C(\tilde{v},e)}{\partial \tilde{v}}}+2\underbrace{\frac{\dfrac{\partial u^C(\underline{v},e)}{\partial e}}{\dfrac{\partial u^C(\underline{v},e)}{\partial \underline{v}}}}_{\text{positive}}. \tag{EC.20}$$

Under such conditions, (EC.19) is violated indicating that the policy-maker cannot improve social welfare by setting a positive restitution. Thus, the policy-maker sets a zero restitution and the no restitution regime arises. With suitable adjustment and cancelling the denominators on each side of (EC.20), the right hand side defines the threshold T_2 in the text.

Finally, an increase in fines increases social welfare, $dSW(s,f,\alpha)/df > 0$, if

$$\frac{de(\cdot)}{df} > \frac{\dfrac{\partial SU(\cdot)}{\partial p}\left[\mu-\dfrac{dp(\cdot)}{df}\right][p(\cdot)-s-\mu f]-\mu f\dfrac{\partial TU(\cdot)}{\partial f}}{\dfrac{\partial SU(\cdot)}{\partial e}[p(\cdot)-s-\mu f]+\dfrac{\partial TU(\cdot)}{\partial e}\mu f+\Phi(\cdot)}. \tag{EC.21}$$

The *no restitution regime* emerges when (EC.20) holds. In this regime, we have

$$\frac{de(s,f,\alpha)}{ds} > 0 \qquad \text{and} \qquad \frac{de(s,f,\alpha)}{d\alpha} < 0.$$

The *subsidy-restitution regime* appears when (EC.18) and (EC.20) are violated simultaneously. In this regime, we have

$$\frac{de(s,f,\alpha)}{ds} > 0 \qquad \text{and} \qquad \frac{de(s,f,\alpha)}{d\alpha} > 0.$$

Finally, the *no subsidy regime* arises when (EC.18) holds. In this regime, we have

$$\frac{de(s,f,\alpha)}{ds} < 0 \qquad \text{and} \qquad \frac{de(s,f,\alpha)}{d\alpha} > 0.$$

Step 2.

We now explore the policy-maker's optimal intervention in each regime.

Step 2.1. No Restitution Regime

We first examine the optimal intervention in the no restitution regime.

Approach I. In this regime, an increase in restitution decreases the firm's investment in quality. Thus, the left-hand side of (EC.19) is negative while its right-hand side is positive indicating that social welfare decreases with restitution and $\alpha = 0$.

We can also show that in this regime $dp(s,f,\alpha)/ds < 1$ and $de(s,f,\alpha)/ds > 0$ which indicate that the right-hand side of (EC.17) is negative while its left-hand side is positive indicating that social welfare increases with subsidies. Thus, if the policy-maker imposes fines on detected pirates ($f > 0$), then the revenues from fines are used to subsidize legal purchases ($s > 0$). Due to the budget constraint, if the policy-maker does not impose fines on detected pirates ($f = 0$), then it is not possible to subsidize legal purchases ($s = 0$). In other words, in the no restitution regime there are two possible outcomes: 1) the policy-maker does not intervene, $f = s = 0$, or 2) the policy-maker intervenes and sets positive fines and subsidies, $f > 0$ and $s > 0$.

In what follows, we derive a sufficient condition for each possible outcome. In this regime, because $\alpha = 0$, the total impact of an increase in subsidies on the firm's investment in quality can be written as

$$\frac{de(\cdot)}{ds} = \frac{\partial SU(\cdot)}{\partial s}\left[\frac{\partial SU(\cdot)}{\partial e} - SU(\cdot)\frac{\frac{\partial \tilde{\gamma}(\cdot)}{\partial e}}{\tilde{\gamma}(\cdot)}\right] > 0.$$

Furthermore, when $\alpha = 0$, the total impact of an increase in fines on the firm's investment in quality is

$$\frac{de(s,f)}{df} = \mu \frac{de(s,f)}{ds} > 0. \qquad \text{(EC.22)}$$

Similarly, we can show that in this regime

$$\frac{dp(s,f)}{df} = \mu \frac{dp(s,f)}{ds} > 0. \qquad \text{(EC.23)}$$

Therefore, the total impact of an increase in fines on social welfare can be written as

$$\begin{aligned}\frac{dSW(s,f)}{df} &= \mu \frac{de(\cdot)}{ds}\left[\Phi(\cdot) + \frac{\partial SU(\cdot)}{\partial e}[p(\cdot) - \mu f - s] + \frac{\partial TU(\cdot)}{\partial e}\mu f\right] + \mu f \frac{\partial TU(\cdot)}{\partial f} \\ &\quad + \mu\left[1 - \frac{dp(\cdot)}{ds}\right]\frac{\partial SU(\cdot)}{\partial s}[p(\cdot) - \mu f - s] = \mu \underbrace{\frac{dSW(s,f)}{ds}}_{\text{positive}} + \underbrace{\mu f \frac{\partial TU(\cdot)}{\partial f}}_{\text{negative}},\end{aligned}$$

which indicates that in the no restitution regime

$$\underbrace{\frac{dSW(s,f)}{ds}}_{\textit{positive}} = \frac{1}{\mu}\frac{dSW(s,f)}{df} - f\frac{\partial TU(\cdot)}{\partial f}.$$

We next differentiate the budget constraint when it binds, $sSU(s,f) - \mu f CU(s,f) = 0 = \psi(s,f,\alpha=0)$, with respect to subsidies and fines. Substituting $\psi(\cdot)$ for $\psi(s,f,\alpha=0)$, we have

$$\frac{d\psi(\cdot)}{ds} = SU(\cdot) + \left[1 - \frac{dp(\cdot)}{ds}\right]\frac{\partial SU(\cdot)}{\partial s}[s+\mu f] + \frac{de(\cdot)}{ds}\left[\frac{\partial SU(\cdot)}{\partial e}[s+\mu f] - \frac{\partial TU(\cdot)}{\partial e}\mu f\right] > 0 \quad \text{and}$$

$$\frac{d\psi(\cdot)}{df} = \left[\mu - \frac{dp(\cdot)}{df}\right]\frac{\partial SU(\cdot)}{\partial s}[s+\mu f] + \frac{de(\cdot)}{df}\left[\frac{\partial SU(\cdot)}{\partial e}[s+\mu f] - \frac{\partial TU(\cdot)}{\partial e}\mu f\right] - \mu f\frac{\partial TU(\cdot)}{\partial f} - \mu CU.$$

Using (EC.22) and (EC.23), we can rewrite $d\psi(\cdot)/df$ as a function of $d\psi(\cdot)/ds$,

$$\begin{aligned}\frac{d\psi(\cdot)}{df} &= \mu\left[\left[1 - \frac{dp(\cdot)}{ds}\right]\frac{\partial SU(\cdot)}{\partial s}[s+\mu f] + \frac{de(\cdot)}{ds}\left[\frac{\partial SU(\cdot)}{\partial e}[s+\mu f] - \frac{\partial TU(\cdot)}{\partial e}\mu f\right]\right] - \mu f\frac{\partial TU(\cdot)}{\partial f} - \mu CU \\ &= \mu\left[\frac{\partial\psi(\cdot)}{\partial s} - SU(\cdot)\right] - \mu f\frac{\partial TU(\cdot)}{\partial f} - \mu CU = \underbrace{\mu\frac{\partial\psi(\cdot)}{\partial s}}_{\text{positive}} - \mu\left[\underbrace{TU(\cdot)}_{\text{positive}} + \underbrace{f\frac{\partial TU(\cdot)}{\partial f}}_{\text{negative}}\right].\end{aligned}$$

Rearranging the last equation, in the no restitution regime,

$$\underbrace{\frac{d\psi(\cdot)}{ds}}_{positive} = \frac{1}{\mu}\frac{d\psi(\cdot)}{df} + TU(\cdot) + f\frac{\partial TU(\cdot)}{\partial f}.$$

To explore the conditions under which the policy-maker's intervention improves social welfare, in addition to the direct effect of fines on social welfare, we must consider their indirect effect through subsidies. Thus, the total impact of an increase in fines on social welfare is

$$\frac{dSW(s,f)}{df} - \frac{d\psi(s,f,\alpha=0)/df}{d\psi(s,f,\alpha=0)/ds}\frac{dSW(s,f)}{ds},$$

which is non-negative if

$$\begin{aligned}&\frac{dSW(s,f)}{df}\left[\frac{1}{\mu}\frac{d\psi(\cdot)}{df} + TU(\cdot) + f\frac{\partial TU(\cdot)}{\partial f}\right] - \frac{d\psi(\cdot)}{df}\left[\frac{1}{\mu}\frac{dSW(s,f)}{df} - f\frac{\partial TU(\cdot)}{\partial f}\right] \\ &= \frac{dSW(s,f)}{df}\left[TU(\cdot) + f\frac{\partial TU(\cdot)}{\partial f}\right] - f\frac{\partial TU(\cdot)}{\partial f}\frac{d\psi(\cdot)}{df} \geq 0.\end{aligned}$$

A sufficient condition for the above equation to hold is

$$|\zeta_f| = \left|\frac{f}{TU}\frac{\partial TU(\cdot)}{\partial f}\right| < 1.$$

In general, the policy-maker sets $f, s > 0$ if

$$\frac{dSW(s,f)/df}{d\psi(s,f,\alpha=0)/df} \geq \frac{\zeta_f}{1+\zeta_f} = \frac{|\zeta_f|}{|\zeta_f|-1},$$

or equally

$$\frac{dSW(s,f)}{df} \geq \underbrace{\frac{|\zeta_f|}{|\zeta_f|-1}\frac{d\psi(s,f,\alpha=0)}{df}}_{Negative}, \tag{EC.24}$$

which indicates that even if an increase in fines directly decreases social welfare, it might be socially optimal to impose fines on detected pirates (as it indirectly increases social welfare through subsidies).

Approach II. An alternative way to understand *Approach I* and see whether the policy-maker's intervention improves social welfare is to check if the following sufficient condition holds: the policy-maker's intervention is socially optimal if that the first dollar of fines improves social welfare. Under such conditions, revenue from one dollar of fines on pirates can be redistributed to subsidize legal purchases which is also social welfare maximizing.

Suppose at $f=0$, social welfare is

$$SW_0 = \int_{\tilde{v}_0}^{1} u^S(v,e_0)dv + \int_{\tilde{v}_0}^{\underline{v}_0} u^C(v,e_0)dv - e_0,$$

and at $f=\$1$ social welfare is

$$SW_1 = \int_{\tilde{v}_1}^{1} u^S(v,e_1)dv + \int_{\tilde{v}_1}^{\underline{v}_1} u^C(v,e_1)dv - e_1.$$

Increasing the level of fines from zero to one implies that $e_1 \geq e_0$, $\tilde{v}_0 \geq \tilde{v}_1$, and $\underline{v}_1 \geq \underline{v}_0$. By subtracting SW_0 from SW_1, we get

$$\delta SW = \int_{\tilde{v}_0}^{1} \left[u^S(v,e_1) - u^S(v,e_0)\right] dv + \int_{\tilde{v}_1}^{\tilde{v}_0} \left[u^S(v,e_1) - u^C(v,e_0)\right] dv + \int_{\underline{v}_1}^{\tilde{v}_1} \left[u^C(v,e_1) - u^C(v,e_0)\right] dv$$
$$+ \int_{\underline{v}_0}^{\underline{v}_1} -u^C(v,e_0)dv - [e_1 - e_0].$$

By treating $e_1 - e_0$ as a small increase in investment, we have

$$\delta SW = \int_{\tilde{v}_0}^{1} \frac{\partial u^S(v,e)}{\partial e} dv + \int_{\tilde{v}_1}^{\tilde{v}_0} \left[u^S(v,e_1) - u^C(v,e_0)\right] dv + \int_{\underline{v}_1}^{\tilde{v}_1} \frac{\partial u^C(v,e)}{\partial e} dv - \int_{\underline{v}_0}^{\underline{v}_1} u^C(v,e_0)dv,$$

which is positive if

$$\int_{\tilde{v}_1}^{1} \frac{\partial u^S(v,e)}{\partial e} dv + \int_{\underline{v}_1}^{\tilde{v}_1} \frac{\partial u^C(v,e)}{\partial e} dv > \int_{\underline{v}_0}^{\underline{v}_1} u^C(v,e_0)dv.$$

Step 2.2. No Subsidy Regime

The total impact of an increase in subsidies on social welfare is positive, $dSW(s,f,\alpha)/ds > 0$, if (EC.17) holds. In this regime, an increase in subsidies decreases the firm's investment in quality indicating that the left-hand side of (EC.17) is negative. Moreover, we can show that in this regime $dp(s,f,\alpha)/ds > 1$ indicating that the right-hand side of (EC.17) is positive. In other words, in the no subsidy regime, the inequality in (EC.17) is violated implying that subsidizing legal purchases reduces social welfare and $s=0$.

In this region an increase in restitution leads the firm to raise its investment in quality and the subscription fee. Therefore, from (EC.19), an increase in restitution can improve social welfare.

Given that the policy-maker never uses subsidies in this regime, we show that depending on the impact of restitution on social welfare, there are two possible outcomes: 1) the policy-maker does not intervene and $f = \alpha = 0$, or 2) the policy-maker intervenes and uses fines and restitution simultaneously, $f > 0$ and $\alpha > 0$.

The total impact of an increase in fines on social welfare is positive, $dSW(s,f,\alpha)/df > 0$, if (EC.21) holds. We investigate the conditions under which (EC.21) inequality holds. We begin our analysis by focusing on conditions under which an increase in restitution decreases social welfare, so that $\alpha = 0$. Substituting $(\cdot)$ for $(s=0,f,\alpha=0)$, we determine the impact of an increase in fines on the firm's investment in quality in the restitution regime,

$$\det(J)\frac{de(\cdot)}{df} = -\frac{\partial^2\pi(\cdot)}{\partial f\partial p}\frac{\partial^2\pi(\cdot)}{\partial e^2} + \frac{\partial^2\pi(\cdot)}{\partial e\partial p}\frac{\partial^2\pi(\cdot)}{\partial f\partial e} = \mu\frac{\partial SU(\cdot)}{\partial p}\left[SU\frac{\frac{\partial\tilde{\gamma}(\cdot)}{\partial e}}{\tilde{\gamma}(\cdot)} - \frac{\partial SU(\cdot)}{\partial e}\right] < 0,$$

which is negative indicating that the left-hand side of (EC.21) is negative. We can also show that under such conditions $dp(\cdot)/df > \mu$ indicating that the right-hands side of (EC.21) is positive. Therefore, (EC.21) is violated which implies that in the no subsidy regime, if $\alpha = 0$, then the policy-maker cannot improve social welfare by imposing fines on detected pirates. In other words, the policy-maker does not intervene.

Next, suppose social welfare increases with restitution. In other words, if $f > 0$, then revenue from fines are redistributed to the firm as restitution, $\alpha = 1$. Substituting $(\cdot)$ for $(s=0,f,\alpha=1)$, we determine the impact of an increase in fines on the firm's investment in quality using the following equation

$$\det(J)\frac{de(\cdot)}{df} = -\frac{\partial^2\pi(\cdot)}{\partial f\partial p}\frac{\partial^2\pi(\cdot)}{\partial e^2} + \frac{\partial^2\pi(\cdot)}{\partial e\partial p}\frac{\partial^2\pi(\cdot)}{\partial f\partial e} = \underbrace{-2\mu^2 f\frac{\partial SU(\cdot)}{\partial p}\frac{\frac{\partial^2 u^C(\underline{v},e)}{\partial\underline{v}\partial e}}{\frac{\partial u^C(\underline{v},e)}{\partial\underline{v}}}}_{\text{positive}}$$

$$\underbrace{-2\mu\frac{\partial SU(\cdot)}{\partial p}\left[\frac{\partial SU(\cdot)}{\partial e} + \frac{\partial CU(\cdot)}{\partial e}\right]}_{\text{positive}} + 2\mu SU(\cdot)\frac{\partial SU(\cdot)}{\partial p}SU\frac{\frac{\partial\tilde{\gamma}(\cdot)}{\partial e}}{\tilde{\gamma}(\cdot)} - 2\mu SU(\cdot)\frac{\partial SU(\cdot)}{\partial p}SU\frac{\frac{\partial\tilde{\gamma}(\cdot)}{\partial e}}{\tilde{\gamma}(\cdot)} > 0,$$

which is positive because the first two terms are positive and the last two terms are netted out. Thus, when $\alpha = 1$, an increase in fines leads the firm to raise its investment in quality. We next show that when $\alpha = 1$ then $dp(\cdot)/df > \mu$. To do so, we need to show the following equation is negative,

$$\det(J)\mu - \det(J)\frac{dp(\cdot)}{df} = \mu\frac{\partial^2\pi(\cdot)}{\partial p^2}\frac{\partial^2\pi(\cdot)}{\partial e^2} - \mu\frac{\partial^2\pi(\cdot)}{\partial p\partial e}\frac{\partial^2\pi(\cdot)}{\partial p\partial e} + \frac{\partial^2\pi(\cdot)}{\partial s\partial p}\frac{\partial^2\pi(\cdot)}{\partial e^2} - \frac{\partial^2\pi(\cdot)}{\partial p\partial e}\frac{\partial^2\pi(\cdot)}{\partial f\partial e}$$

$$= \underbrace{-\frac{\partial^2 \pi(\cdot)}{\partial p \partial e}}_{\text{negative}} \left[\underbrace{\mu \frac{\partial TU(\cdot)}{\partial e} + \mu SU(\cdot) \frac{\frac{\partial \tilde{\gamma}(\cdot)}{\partial e}}{\tilde{\gamma}(\cdot)} - 2\mu^2 f \frac{\partial TU(\cdot)}{\partial f} \frac{\frac{\partial^2 u^C(\underline{v}, e)}{\partial \underline{v} \partial e}}{\frac{\partial u^C(\underline{v}, e)}{\partial \underline{v}}}}_{\text{positive}} \right],$$

which is negative because the terms inside the bracket are positive. We can argue that in the restitution regime when $\alpha = 1$, (EC.21) holds. Therefore, when social welfare increases with restitution the policy-maker improves social welfare by setting $f > 0$ and $\alpha > 0$.

Step 2.3. Restitution-Subsidies Regime

Like the no restitution regime, in this regime we can show $dp(s, f, \alpha)/ds < 1$ indicating that the right-hand side of (EC.17) is negative and its left-hand side is positive. Thus, if the policy-maker imposes fines on detected pirates, then it can always improve social welfare by redistributing revenue from fines to subsidize legal purchases. Therefore, if

$$\frac{dSW(s, f, \alpha)/ds}{d\psi(s, f, \alpha)/ds} \geq \frac{dSW(s, f, \alpha)/d\alpha}{d\psi(s, f, \alpha)/d\alpha}, \tag{EC.25}$$

then revenue from fines should be redistributed to subsidize legal purchases. Otherwise, revenue from fines should be redistributed as restitution. When (EC.25) is true, the same analysis discussed in the no restitution regime also holds in the restitution-subsidies regime. Therefore, if (EC.25) holds, then there are two possible outcomes: (*i*) the policy-maker intervenes and sets positive fines and subsidies, $f > 0$, $s > 0$, and $\alpha = 0$; or (*ii*) the policy-maker does not intervene, $f = s = \alpha = 0$. □

Proof of Proposition 1

PROPOSITION 1. *With additive utility, imposing fines increases the firm's investment in quality, regardless of how the policy-maker redistributes revenue from fines.*

Proof. By differentiating the optimal subscription fee in the piracy region with respect to fines, we get $de^*/df = \eta\mu[1+\alpha]/\left[2 - \eta^2[1-\beta]\right]$ which is always positive regardless of whether the policy-maker redistributes revenue from fines as subsidies or restitution. □

Proof of Proposition 2

PROPOSITION 2. *With additive utility if the quality degradation between legitimate and pirated versions is*

(a) large, $0 < \beta \leq T_s$, *then the optimal instruments are fines and subsidies,*

(b) moderate, $T_s < \beta \leq \min\{T_\alpha, T_{lp}\}$, *then the optimal instruments are fines and restitution,*

(c) small, $\min\{T_\alpha, T_{lp}\} < \beta \leq 1$, *then the optimal instruments are large fines that deter digital piracy.*

Proof. To characterize the overall optimum, we first need to determine the relationship between T_a defined in (28), T_s defined in (31), and T_{lp} in (34). We also define SW_α^*, SW_s^*, and SW_{lp}^* as the optimal social welfare in (26), (29), and (32), respectively.

Focusing on the piracy region, we showed that if $T_s < \beta < 1$, then employing fines and redistributing fine revenue as subsidies is not feasible. We also showed that if $T_\alpha < \beta < 1$, then employing fines and redistributing fine revenue as restitution is not feasible.

Focusing on the no piracy region with threat of copying, we showed that if $0 < \beta < T_{lp}$, then the optimal level of fines, f_{lp}, is smaller than the minimum level of fines that deter piracy. Thus, if $0 < \beta < T_{lp}$, then the no piracy region with threat of copying does not emerge.

Comparing T_s with T_α and T_{lp} indicates that $T_s < T_\alpha$ and $T_s < T_{lp}$. The latter inequality indicates that the no piracy region with threat of copying does not emerge if $0 < \beta < T_s$ and the policy-maker maximizes social welfare by imposing fines on detected pirates and redistributing revenue from such fines as either subsidies or restitution. By comparing SW_α^* and SW_s^*, we find that $SW_\alpha^* < SW_s^*$ indicating that if $0 < \beta < T_s$ then fines and subsidies are the optimal policy instruments.

We next determine the relationship between T_α with T_{lp}. Comparing these thresholds indicates $T_\alpha < T_{lp}$ if $\sqrt{0.802} < \eta < 1$ (case 1) and $T_\alpha > T_{lp}$ if $0 < \eta < \sqrt{0.802}$ (case 2).

In case 1, if $T_s < \beta < T_\alpha$, then using subsidies is infeasible and the no piracy region with threat of copying does not emerge. Thus, the policy-maker maximizes social welfare using fines and redistributing revenue from fines as restitution. If $T_\alpha < \beta < T_{lp}$, then redistributing fine revenue as subsidies or restitution are infeasible. Moreover, under such conditions the policy-maker sets the lowest level of fines that ensure $CU = 0$. Finally, if $T_{lp} < \beta < 1$, then in the no piracy region with threat of copying the policy-maker can improve social welfare by increasing fines up to f_{lp}.

In case 2, if $T_s < \beta < T_{lp}$, then using subsidies is infeasible and the no piracy region with threat of copying does not emerge. Thus, the policy-maker maximizes social welfare using fines and redistributing revenue from fines as restitution. If $T_{lp} < \beta < T_\alpha$ then the policy-maker either uses fines and redistributing revenue from fines as restitution while tolerating some level of piracy or sets large fines so that the no piracy region with threat of copying emerges. Comparing the optimal social welfare under the two options shows that $SW_\alpha < SW_{lp}$ indicating that if $T_{lp} < \beta < T_\alpha$ the policy-maker maximizes social welfare by removing piracy. Finally, if $T_\alpha < \beta < 1$ then the piracy region never emerges and the policy-maker maximizes social welfare by setting large fines, f_{lp}. □

Proof of Proposition 3

PROPOSITION 3. *With additive utility the policy-maker's intervention decreases consumer surplus and increases the firm's profit.*

Proof. By differentiating (35) with respect to fines and after some manipulation, we find that in the piracy region a sufficient condition for $dCS_s(f)/df < 0$ is $0 < \beta < 1-\eta^2$ which always holds when fines and subsidies are optimal instruments. Given the firm's optimal decisions when $\alpha = 0$, by substituting $s(f)$ in the firm's profit and differentiating the outcome with respect to fines, we find that in the piracy region $d\pi_s(f)/df > 0$.

We next differentiate (36) with respect to fines,

$$\frac{dCS_\alpha(f)}{df} = \frac{\mu^2 f[\eta^4[1+3\beta]-4\eta^2[1+\beta]+4]}{\beta\big[2-\eta^2[1-\beta]\big]^2} - \frac{\mu[4-\eta^2[3+\beta]]}{\beta\big[2-\eta^2[1-\beta]\big]^2} < 0,$$

which is negative in the piracy region. We next differentiate the firm's profit in (18) with respect to fines by setting $s = 0$ and $\alpha = 1$,

$$\frac{d\pi_\alpha(f)}{df} = \frac{2\mu\big[\beta-\mu f[2-\eta^2]\big]}{\beta\big[2-\eta^2[1-\beta]\big]} > 0,$$

which is positive in the piracy region.

Finally, by differentiating (37) and (20) with respect to fines, we get

$$\frac{dCS_{lp}(f)}{df} = -\frac{\big[1-\eta^2\big]\mu\big[\beta-\mu f+\eta^2\mu f\big]}{\beta^2} < 0 \quad \text{and} \quad \frac{d\pi_{lp}(f)}{df} = \frac{\mu\big[\beta-2\mu f+\eta^2\mu f\big]}{\beta^2} > 0,$$

which are negative and positive, respectively. Thus, when the policy-maker's intervention decreases consumer surplus and increases the firm's profit. □

Proof of Proposition 4

PROPOSITION 4. *With multiplicative utility, the impact of fines on the firm's investment in quality depends on how the policy-maker redistributes revenue from fines, if doing so at all.*

Proof. The total effect of increased fines on the firm's investment in quality is

$$\frac{d\tilde{e}(\cdot)}{\partial f} = -\left[\frac{f\mu^2\big[4\alpha-\beta\mu f[1+\alpha]^2\big]-\beta\mu s[1-\alpha]}{2\beta[1-\beta]e^2}\right]\left[\frac{\partial^2\pi(\cdot)}{\partial e^2}\right]^{-1}.$$

When the policy-maker does not redistribute revenue from fines as subsidies or restitution, $s = \alpha = 0$, the above equation can be simplified as follows

$$\frac{d\tilde{e}(\cdot)}{\partial f} = \left[\frac{f^2\mu^3\beta\mu f}{2\beta[1-\beta]e^2}\right]\left[\frac{\partial^2\pi(\cdot)}{\partial e^2}\right]^{-1} < 0.$$

When the policy-maker redistributes revenue from fines as subsidies, $s > 0$ and $\alpha = 0$, the above equation can be simplified as follows

$$\frac{d\tilde{e}(\cdot)}{\partial f} = \left[\frac{\beta f\mu^3\big[s+f\big]}{2\beta[1-\beta]e^2}\right]\left[\frac{\partial^2\pi(\cdot)}{\partial e^2}\right]^{-1} < 0.$$

When the policy-maker redistributes revenue from fines as restitution, $s = 0$ and $\alpha = 1$, the above equation can be simplified as follows

$$\frac{d\tilde{e}(\cdot)}{\partial f} = -\left[\frac{4f\mu^2\big[1-\beta\mu f\big]}{2\beta[1-\beta]e^2}\right]\left[\frac{\partial^2\pi(\cdot)}{\partial e^2}\right]^{-1} > 0.$$

Therefore, depending on how the policy-maker redistributes revenue from fines (through restitution or subsidies), if doing so at all, the impact of fines on the firm's investment in quality can be positive or negative. $\square$