Equivalence of Taxes and Subsidies in the Control of Production Externalities

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We are always better off having many policies that can achieve a given objective because it extends the criteria that can be included in policy selection. This paper studies the equivalence between taxes and subsidies in the control of negative production externalities. In our models, under the tax regime, firms that take no treatment action to mitigate the damage caused by their negative externalities are punished, whereas under the subsidy regime, firms are rewarded for externality treatment activities. We employ a formulation where firms differ in the vintage of their production technology and as a result differ in profitability, negative externality generation, and the cost of treatment. We consider three measures as policy objectives: total output, total damage from negative externalities, and social welfare. We find reasonable conditions where, with an appropriate setting of uniform lump-sum and unit subsidies, the policy maker can achieve a pair of policy objectives equivalent to those obtained using unit taxes. Thus, either tax or subsidy regimes can be used to achieve desired levels of one or two policy objectives, allowing other factors such as fairness, equity, or international trade issues to be considered in policy selection.

(*Public Policy; Externalities; Taxes; Subsidies; Environmental Policy; Social Welfare; Production Technology*)

1. Overview

The choice between reward and punishment has always been difficult. Because our understanding of symmetry among these incentives depends on the definition of symmetry, this choice has been further clouded. The important question for policy selection is whether reward and punishment can achieve tantamount objectives. If equivalent economic objectives can be realized through rewards or punishments then additional criteria can be incorporated in the selection of policy, thereby increasing the range of factors considered. These additional criteria include fairness, equity, and international trade issues. The purpose of this paper is to examine the equivalence of taxes and subsidies in the control of negative production externalities. Specifically, we compare unit taxes with subsidies made up of unit and uniform lump-sum amounts. We determine whether measures of one or more of three key policy objectives (total output, total damage from negative externalities, and social welfare) that are obtained from a given tax regime, can also be achieved by the subsidy regime.

We employ an approach that is characterized by firms that differ in the vintage of their production technologies. A given production technology generates negative externalities that depend on the level of output and the vintage of the technology. We allow individual firms to choose whether to take actions to reduce their negative externalities where their incentives to do so come in the form of either a tax regime or a subsidy regime. Actions take the form of treatment of the negative externalities generated. Thus, firms differ in profitability, negative externalities produced per unit of output, and the cost of negative externality treatment. The tax regime is a unit tax, that is, a tax per unit of negative externality produced. The subsidy regime is a uniform fixed transfer plus a subsidy per unit of negative externality treated. Under the tax regime, firms that do not treat their externalities are taxed; under a subsidy regime, firms that engage in externality treatment are

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subsidized. Therefore, in our model, taxes and subsidies are not symmetric by design, for taxes are used as punishments for not taking action, and subsidies are rewards for taking action. While the structure of the tax regime is similar to previous models (e.g., Brock and Evans 1985), our formulation differs from earlier models that consider subsidies as mirror images of taxes.

The novel aspects of our approach are that firms differ in the vintage of their production technologies, that firms choose whether to participate in a given regime, and that mitigation of the negative externalities includes treatment as well as reduced levels of output. In addition, the problem of unlimited market entry to collect the uniform lump-sum subsidy, a key issue in previous analyses, is less of an issue in our model because firms must treat their negative externalities in order to receive the subsidy, thus, they must take actions; for example, installation of emissions control equipment.

Historically, policy determination has been a two-step process. In the first step standards or objectives are set. In the second step a regulatory system, made up of economic instruments or direct controls, is designed to achieve the requirements set in the first step (Cropper and Oates 1992). Policy objectives are usually stated in terms of total output or total damage from negative externalities. In the United States, for example, the Amendments to the Clean Air Act in 1970 and to the Clean Water Act of 1972 set target levels of aggregate negative externalities. International agreements are often framed in terms of these measures. When levels of total damage from negative externalities are mandated by government, industry's counter arguments focus on the effects on total output. There is an additional reason why total damage from negative externalities, in particular, is a critical objective: measurements of social costs and benefits may not have readily available market measures, and policy makers have been reluctant to employ monetary measures of value and quality of life (Cropper and Oates 1992). As a result, total output and total damage from negative externalities appear as objectives in economic policy models. For example, Milliman and Prince (1989) view total industry emissions as the regulatory control variable in their model.

Our analysis considers total output and total damage from negative externalities as policy objectives. We also consider the traditional policy objective, social welfare, recognizing that policy makers often trade-off output and externalities in an attempt to maximize some measure of social welfare.

Economic instruments (subsidies, taxes, and tradable permits), along with direct controls, are the options for designing a regulatory system. The choice between these options is governed by their ability to meet regulatory objectives, and if more than one option can achieve the objectives, then other criteria can be included in policy selection. Indeed, this is why understanding equivalence between options to achieve different objectives is important. The additional criteria that can be included are varied. Legal and political concerns may be critical. Taxes, as formulated in this paper and as modelled in Brock and Evans (1985), as well as tradable permits increase firm operating costs and extract transfer payments from affected firms (Milliman and Prince 1989). These payments are often blocked by firms in the legal and political arenas, arguing that they are losing a historical property right—for example, the right to pollute (Polinsky 1979). While achieving equivalent objectives, our model yields differences in the proportion of firms that engage in externality treatment and differences in the distribution of firm output, depending on whether a subsidy or tax regime is applied, further extending the criteria that can be included in policy selection.

There are other reasons why subsidies, in particular, are an attractive policy instrument. There is a recognition that producers of negative externalities differ. For polluters, more stringent standards have been legislated for new sources than for older ones, and small firms and industries have been exempted altogether (Crandall 1983). While a tax regime could accommodate varying tax rates to address these differences, unequal application of a tax regime may not be politically acceptable, whereas a subsidy regime may be. Inter-firm differences are also problematic for tradable permits because it is unclear how to define a marketable right. Moreover, tradable permits become less efficient when there is strategic behavior by traders, when there is extensive market reporting requirements and when there is thin trading. The history of emissions trading shows that tradable permit markets have yielded few trades and in one noticeable exception, the trading of lead rights in the mid-1980s, there were various irregularities and illegal procedures (Cropper and Oates 1992).

In the context of perfectly competitive industries, subsidies have been formulated as uniform lump-sum payments to firms, less a unit tax multiplied by output (for example, Baumol and Oates 1988). Thus, tax and subsidy policy differed only by a fixed transfer, the uniform lump-sum payment. The lack of, or presence of, an impact of the fixed transfer on individual firm output was the criterion by which tax and subsidy symmetry versus nonsymmetry was determined (Polinsky 1979). The dominance of taxes versus subsidies that resulted from this work centered on the entry of firms into the market simply to collect the lump-sum payment. An infinite number of identical firms, a feature assumed in models of perfectly competitive markets, compounded the issue of entry to collect the fixed subsidy. This latter problem has been mitigated by requiring that fixed subsidies be paid to existing firms only.

We argue that the question of symmetry is less important than the issue of whether the two alternative regimes can produce equivalent policy objectives. Our discussion is developed from the standpoint that policy objectives attained by a given tax regime can be achieved by a subsidy regime. We show that there are straightforward necessary conditions under which an individual objective obtained under a given tax regime can be achieved by a subsidy regime. Moreover, these conditions are consistent across objectives—equal total output requires three conditions, equal total damage from negative externalities requires two of these conditions, and equal social welfare does not require any conditions. In addition, we provide relatively unrestrictive necessary and sufficient conditions for any pair of policy objectives obtained under a tax regime to be achieved under a subsidy regime.

Our tax regime subsumes the standard of optimality against which control instruments are often measured, that is, the Pigovian tax per unit of negative externality equal to the marginal damage at the Pareto Optimum (Burrows 1979). We show that equivalent social welfare can be obtained either through taxes or through subsidies with an appropriate choice of subsidy components. Prior results suggest that under a tax total output, total damage from negative externalities and firm profits fall, in contrast to a subsidy regime where these quantities rise (Baumol and Oates 1888, Cropper and Oates 1992, Polinsky 1979). Our models confirm the former, that is, that under a tax, total output and total damage from negative externalities fall. Under a subsidy, however, we find that setting the unit subsidy lower than the marginal cost of externality treatment also reduces total output, reduces total damage from negative externalities, and may reduce firm profits.

The remainder of the paper is organized as follows. We first outline our assumptions and notation used in the models. We then construct and solve models of the industry under alternative tax and subsidy regimes. Subsequently, we examine the conditions under which equality of individual policy objectives can be obtained under the two regimes. We then develop the conditions where the alternative regimes can be used to achieve equal measures of two policy objectives simultaneously—our main result. A summary of the findings and their implications for policy concludes the paper.

2. Assumptions and Notation

In this section we enumerate our assumptions and introduce the notation used in the remainder of the paper.

ASSUMPTION 1. *Firms differ in their production technologies.*

Consider an industry where firms are characterized by their production technology. Let $\theta \in R$ represent a firm with a given production technology, where θ follows the density $f(\theta)$ that is positive over the support $[\underline{\theta}, \overline{\theta}]$ and is zero elsewhere. Thus, $F(\underline{\theta}) = 0$ and $F(\overline{\theta}) = 1$, where $\underline{\theta}$ and $\overline{\theta}$ are the best and worst technology firms respectively. We interpret θ as representing a firm's technology *vintage* where a larger θ indicates an older, less efficient, technology.

ASSUMPTION 2. *The negative externalities produced by an individual firm are increasing at an increasing rate in output and are increasing in the vintage of their production technology.*

The production of output, x, and the vintage of the firm's production technology combine to produce negative externalities, $q(x, \theta)$, where Assumption 2 is represented by the partial derivatives[1]

$$\frac{\partial q(x,\theta)}{\partial x} > 0, \quad \frac{\partial^2 q(x,\theta)}{\partial x^2} > 0 \quad \text{and} \quad \frac{\partial q(x,\theta)}{\partial \theta} > 0.$$

ASSUMPTION 3. *Firms have the choice of whether to mitigate the damage from their negative externalities through treatment.*

We assume the treatment technology allows firms to treat all their negative externalities if they deal with any. We do not require, however, that all the harmful effects are eliminated as we allow for damage done from the remaining negative externalities. This requirement is reasonable, for example, if receipt of the subsidy or exemption from the tax is conditional on installing and running all emissions through emissions-treatment equipment.

ASSUMPTION 4. *Firms' pretax regime or presubsidy regime profits are concave in output and are decreasing in the vintage of their production technology.*

The reduced-form profit function for firm θ, not considering its externalities, is $\mathrm{PR}(x, \theta)$. Assumption 4 is exemplified by the following partial derivatives:

$$\frac{\partial^2 \mathrm{PR}(x,\theta)}{\partial x^2} < 0 \quad \text{and} \quad \frac{\partial \mathrm{PR}(x,\theta)}{\partial \theta} < 0.$$

ASSUMPTION 5. *The cost of treating negative externalities is increasing at an increasing rate in the amount of negative externality and is increasing in the vintage of the production technology.*

Writing the cost of treating $q(x, \theta)$ units of negative externalities as $C(q(x, \theta), \theta)$, Assumption 5 is captured by the partial derivatives

$$\frac{\partial C(q(x,\theta),\theta)}{\partial q} > 0, \quad \frac{\partial^2 C(q(x,\theta),\theta)}{\partial q^2} > 0 \quad \text{and}$$
$$\frac{\partial C(q(x,\theta),\theta)}{\partial \theta} > 0.$$

ASSUMPTION 6. *Firms with older vintages of production technology (a) produce greater negative externalities from marginal increases in output; (b) are less profitable, pretax regime or presubsidy regime, at the margin; and (c) have higher marginal costs of treating their negative externalities.*

These conditions can be embodied by the cross partial derivatives

$$\frac{\partial^2 q(x,\theta)}{\partial x \partial \theta} > 0, \quad \frac{\partial^2 \mathrm{PR}(x,\theta)}{\partial x \partial \theta} < 0 \quad \text{and} \quad \frac{\partial^2 C(q(x,\theta),\theta)}{\partial q \partial \theta} > 0,$$

respectively. Assumption 6 is the key to obtaining many of the results because it links the vintage of production technology to the marginal effects of output.

ASSUMPTION 7. *(a) The total damage from negative externalities is increasing in the volume of the negative externality, and (b) treatment reduces the marginal value of damage.*

Let $\omega(Q_u(\cdot), Q(\cdot))$ be the total damage from negative externalities, where $Q_u(\cdot)$ and $Q(\cdot)$ represent aggregate untreated and treated externalities, respectively, and arguments are either t or S, s indicating whether a tax or subsidy regime is in effect. Defining $\partial\omega(Q_u(\cdot), Q(\cdot))/\partial Q_u$ as the marginal damage from untreated externalities and $\partial\omega(Q_u(\cdot), Q(\cdot))/\partial Q$ as the marginal damage from treated externalities, conditions (a) and (b) can be stated as[2]

$$\frac{\partial \omega(Q_u(\cdot), Q(\cdot))}{\partial Q_u} > \frac{\partial \omega(Q_u(\cdot), Q(\cdot))}{\partial Q} > 0.$$

While we can specify conditions of the damage function $\omega(Q_u(\cdot), Q(\cdot))$, the difficulty associated with determining and evaluating actual measurements of this function is why policy makers specify their objectives in terms of output and externalities rather than social welfare.

3. The Tax Alternative

3.1. Firms That Treat Their Negative Externalities

Under the tax regime those firms which choose to deal with their negative externalities incur costs of externality treatment but are not taxed. Their profit function is

[1] We assume all functions are continuously differentiable where required.

[2] Assumptions 1 through 7, in addition to the structure of the subsidy alternative in §4, are similar to those used in a companion paper examining the trade-offs between uniform lump-sum and unit subsidies as incentives (Levi and Nault 1995).

$$\Pi(x, \theta) = \mathrm{PR}(x, \theta) - C(q(x, \theta), \theta).$$

Optimum choice of output gives the necessary first-order condition

$$\frac{\partial\, \mathrm{PR}(x, \theta)}{\partial x} - \frac{\partial C(q(x, \theta), \theta)}{\partial q} \frac{\partial q(x, \theta)}{\partial x} = \Psi_t(x, \theta) = 0. \tag{1}$$

Using Assumptions 2, 4, and 5, the second derivative condition,

$$\frac{\partial^2\, \mathrm{PR}(x, \theta)}{\partial x^2} - \frac{\partial^2 C(q(x, \theta), \theta)}{\partial q^2} \left[\frac{\partial q(x, \theta)}{\partial x}\right]^2 - \frac{\partial C(q(x, \theta), \theta)}{\partial q} \frac{\partial^2 q(x, \theta)}{\partial x^2} = \frac{\partial \Psi_t(x, \theta)}{\partial x} < 0,$$

is sufficient to ensure a maximum. Equation (1) implicitly defines optimum outputs for firms that treat their negative externalities under the tax regime as a function of the vintage of their production technology, $x_t(\theta)$. Differentiating (1) with respect to the vintage of the production technology results in

$$\frac{\partial \Psi_t(x, \theta)}{\partial \theta} = \frac{\partial^2\, \mathrm{PR}(x, \theta)}{\partial x \partial \theta} - \frac{\partial^2 C(q(x, \theta), \theta)}{\partial q \partial \theta} \frac{\partial q(x, \theta)}{\partial x} - \frac{\partial^2 C(q(x, \theta), \theta)}{\partial q^2} \frac{\partial q(x, \theta)}{\partial x} \frac{\partial q(x, \theta)}{\partial \theta} - \frac{\partial C(q(x, \theta), \theta)}{\partial q} \frac{\partial^2 q(x, \theta)}{\partial x \partial \theta},$$

which from Assumptions 2, 4, 5, and 6 is negative. Using the implicit function rule we can sign the derivative of optimum firm output for firms that treat their externalities,

$$x_t'(\theta) = -\frac{\partial \Psi_t(x, \theta)/\partial \theta}{\partial \Psi_t(x, \theta)/\partial x} < 0.$$

Thus, less efficient firms, those with older production technology, produce less output.

3.2. Firms That Do Not Treat Their Negative Externalities

Firms that do not treat their negative externalities are faced with a unit tax t on the amount of negative externality they produce, $q(x, \theta)$. Their profit function is

$$\Pi(x, \theta) = \mathrm{PR}(x, \theta) - t\, q(x, \theta).$$

Optimum choice of output yields the necessary first-order condition

$$\frac{\partial\, \mathrm{PR}(x, \theta)}{\partial x} - t \frac{\partial q(x, \theta)}{\partial x} = \Psi(x, t, \theta) = 0, \tag{2}$$

where, from Assumptions 2 and 4, the second-order condition,

$$\frac{\partial^2\, \mathrm{PR}(x, \theta)}{\partial x^2} - t \frac{\partial^2 q(x, \theta)}{\partial x^2} = \frac{\partial \Psi(x, t, \theta)}{\partial x} < 0,$$

is sufficient for a maximum. For those firms that do not treat their negative externalities under the tax regime, (2) implicitly defines optimum outputs as a function of the unit tax and the vintage of their production technologies, $x(t, \theta)$. Employing (2),

$$\frac{\partial \Psi(x, t, \theta)}{\partial t} = -\frac{\partial q(x, \theta)}{\partial x} < 0,$$

from Assumption 2 and

$$\frac{\partial \Psi(x, t, \theta)}{\partial \theta} = \frac{\partial^2\, \mathrm{PR}(x, \theta)}{\partial x \partial \theta} - t \frac{\partial^2 q(x, \theta)}{\partial x \partial \theta} < 0,$$

from Assumption 6(b) and (c). Using the implicit function rule we can determine the effects of changes in the unit tax and the vintage of production technology on optimum output for firms that do not treat their negative externalities,

$$\frac{\partial x(t, \theta)}{\partial t} = -\frac{\partial \Psi(x, t, \theta)/\partial t}{\partial \Psi(x, t, \theta)/\partial x} < 0, \quad \text{and}$$

$$\frac{\partial x(t, \theta)}{\partial \theta} = -\frac{\partial \Psi(x, t, \theta)/\partial \theta}{\partial \Psi(x, t, \theta)/\partial x} < 0.$$

Therefore, output falls as the unit tax is increased and less efficient firms produce less output.

3.3. Industry Structure Under the Tax Regime

Taking these optimum outputs into account, individual firms choose whether to be taxed or to avoid taxes by treating their negative externalities. The firm that is indifferent between treatment and being taxed, $\tilde{\theta}$, is implicitly defined by

$$\mathrm{PR}(x_t(\tilde{\theta}), \tilde{\theta}) - C(q(x_t(\tilde{\theta}), \tilde{\theta}), \tilde{\theta}) - \mathrm{PR}(x(t, \tilde{\theta}), \tilde{\theta}) + t\, q(x(t, \tilde{\theta}), \tilde{\theta}) = \Phi(t, \tilde{\theta}) = 0.$$

We assume $\Pi(x(t, \bar{\theta}), \bar{\theta}) > 0$ so that the most inefficient firm can make profit producing positive output while absorbing the unit tax. Employing the implicit function rule and the first-order conditions (1) and (2) to cancel terms, the following condition is necessary for firms with vintages of production technology $\theta < \tilde{\theta}$ to treat their externalities, and firms with vintages of production technology $\theta > \tilde{\theta}$ to be taxed:

$$\frac{\partial \Phi(t, \tilde{\theta})}{\partial \tilde{\theta}} = \frac{\partial \, \mathrm{PR}(x_t(\tilde{\theta}), \tilde{\theta})}{\partial \tilde{\theta}} - \frac{\partial \, \mathrm{PR}(x(t, \tilde{\theta}), \tilde{\theta})}{\partial \tilde{\theta}} - \frac{\partial C(q(x_t(\tilde{\theta}), \tilde{\theta}), \tilde{\theta})}{\partial \tilde{\theta}} + t \frac{\partial q(x(t, \tilde{\theta}), \tilde{\theta})}{\partial \tilde{\theta}} - \frac{\partial C(q(x_t(\tilde{\theta}), \tilde{\theta}), \tilde{\theta})}{\partial q} \frac{\partial q(x_t(\tilde{\theta}), \tilde{\theta})}{\partial \tilde{\theta}} < 0. \quad (3)$$

Condition (3) is likely to be satisfied in general, because from $x_t(\theta)$ and $x(t, \theta)$, and from Assumptions 2, 4, and 6 (b), the first pair and last pair of terms are offsetting in sign over different values of t, leaving the third term as dominant. From Assumption 5 this term is negative. From further analysis of (3) we derive our first result.

LEMMA 1. *Sufficient conditions for firms with newer vintages of production technology to prefer treating their externalities are (a) the unit tax is less than or equal to the marginal cost of treatment for the indifferent firm, and (b) the marginal pre-tax regime profits are constant in the vintage of production technology for the indifferent firm.*

PROOF. From condition (b) the first two terms in (3) cancel. From Assumption 5,

$$\frac{\partial C(q(x_t(\tilde{\theta}), \tilde{\theta}), \tilde{\theta})}{\partial \tilde{\theta}} > 0.$$

Thus, the first line of (3) is negative. From Assumption 6(c),

$$\frac{\partial q(x(t, \tilde{\theta}), \tilde{\theta})}{\partial \tilde{\theta}} < \frac{\partial q(x_t(\tilde{\theta}), \tilde{\theta})}{\partial \tilde{\theta}}. \qquad \square$$

Condition (b) in Lemma 1 is satisfied by a reduced-form profit function that is additively separable in vintage of production technology and output. The model is least sensitive to condition (b) of Lemma 1 because our results continue to be valid as long as $\partial \, \mathrm{PR}(x, \theta)/\partial\theta\partial x$ is not large relative to the magnitude of the derivatives in our other assumptions.

Firms with newer vintage production technologies have lower marginal costs of externality treatment. Thus, when the unit tax is less than or equal to the marginal cost of treatment for the indifferent firm, all firms with newer vintage technologies prefer to treat their negative externalities rather than be taxed. For firms with older vintage production technologies the argument is reversed.

Normalizing the number of firms to unity, total output under the tax regime is

$$X(t) = \int_{\underline{\theta}}^{\tilde{\theta}(t)} x_t(\theta) f(\theta) d\theta + \int_{\tilde{\theta}(t)}^{\bar{\theta}} x(t, \theta) f(\theta) d\theta.$$

Aggregate treated and untreated externalities are

$$Q(t) = \int_{\underline{\theta}}^{\tilde{\theta}(t)} q(x_t(\theta), \theta) f(\theta) d\theta, \quad \text{and}$$

$$Q_u(t) = \int_{\tilde{\theta}(t)}^{\bar{\theta}} q(x(t, \theta), \theta) f(\theta) d\theta,$$

respectively.

4. The Subsidy Alternative

4.1. Firms That Treat Their Negative Externalities

The profit of a firm that chooses to treat its negative externalities is

$$\Pi(x, \theta) = \mathrm{PR}(x, \theta) + S + s\, q(x, \theta) - C(q(x, \theta), \theta),$$

where S is a uniform lump-sum subsidy and s is the unit subsidy applied to the amount of treated negative externality, $q(x, \theta)$. The first-order condition for profit maximization by choice of level of output is

$$\frac{\partial \, \mathrm{PR}(x, \theta)}{\partial x} + s \frac{\partial q(x, \theta)}{\partial x} - \frac{\partial C(q(x, \theta), \theta)}{\partial q} \frac{\partial q(x, \theta)}{\partial x} = \Psi(x, s, \theta) = 0. \quad (4)$$

The second-order condition for profits to be maximized is

$$\frac{\partial^2 \Pi(x, \theta)}{\partial x^2} = \frac{\partial \Psi(x, s, \theta)}{\partial x} = \frac{\partial^2 \, \mathrm{PR}(x, \theta)}{\partial x^2} - \frac{\partial^2 C(q(x, \theta), \theta)}{\partial q^2} \left[\frac{\partial q(x, \theta)}{\partial x}\right]^2 + \left[s - \frac{\partial C(q(x, \theta), \theta)}{\partial q}\right] \frac{\partial^2 q(x, \theta)}{\partial x^2} < 0.$$

The first two terms are negative from Assumptions 2, 4, and 5. In order for this condition to be satisfied the unit subsidy, s, cannot exceed the marginal cost of externality treatment by enough to offset these first two terms. We assume this restriction is satisfied so that the second-order condition holds. We state the sufficient condition as Lemma 2 because, similar to Lemma 1, the condition involves the marginal cost of externality treatment. The proof follows directly from our assumptions and is not stated.

LEMMA 2. *A sufficient condition for concavity of profits for firms that treat their externalities under the subsidy regime is that the unit subsidy is less than or equal to the marginal cost of externality treatment.*

The intuition is obvious. Should the unit subsidy exceed the marginal cost of treatment by a sufficient amount as to offset production inefficiencies, firms would produce output simply to treat the negative externalities and receive a positive margin for the treatment activities.

The optimum output function for firms that treat their negative externalities under the subsidy regime is a function of the unit subsidy and the vintage of their production technology, $x(s, \theta)$. Using (4)

$$\frac{\partial \Psi(x, s, \theta)}{\partial s} = \frac{\partial q(x, \theta)}{\partial x} > 0,$$

from Assumption 2 and

$$\frac{\partial \Psi(x, s, \theta)}{\partial \theta} = \frac{\partial^2 \mathrm{PR}(x, \theta)}{\partial x \partial \theta} - \left[\frac{\partial^2 C(q(x, \theta), \theta)}{\partial q^2}\frac{\partial q(x, \theta)}{\partial \theta} + \frac{\partial^2 C(q(x, \theta), \theta)}{\partial q \partial \theta}\right] \frac{\partial q(x, \theta)}{\partial x} + \left[s - \frac{\partial C(q(x, \theta), \theta)}{\partial q}\right]\frac{\partial^2 q(x, \theta)}{\partial x \partial \theta}. \tag{5}$$

Working forward, from Assumption 6(b) the first term in (5) is negative. Directly from Assumptions 2, 5, and 6(c) the second term in (5) is also negative. Using Assumption 6(a), and assuming the unit subsidy does not exceed the marginal cost of dealing with the externalities by enough to offset the negative value of the first two terms, we can take $\partial \Psi(x, s, \theta)/\partial \theta$ as negative. Lemma 2 again provides a sufficient condition for $\partial \Psi(x, s, \theta)/\partial \theta < 0$. Following the implicit function rule we can determine the effects of changes in the unit subsidy and in the vintage of production technology on optimum firm outputs for firms that treat their externalities under the subsidy regime:

$$\frac{\partial x(s, \theta)}{\partial s} = -\frac{\partial \Psi(x, s, \theta)/\partial s}{\partial \Psi(x, s, \theta)/\partial x} > 0, \quad \text{and}$$

$$\frac{\partial x(s, \theta)}{\partial \theta} = -\frac{\partial \Psi(x, s, \theta)/\partial \theta}{\partial \Psi(x, s, \theta)/\partial x} < 0.$$

Thus, the unit subsidy increases output, and firms with less efficient production technology produce less output.

4.2. Firms That Do Not Treat Their Negative Externalities

For firms that choose not to treat their negative externalities under the subsidy regime the profit function is simply $\Pi(x, \theta) = \mathrm{PR}(x, \theta)$. Output is set such that

$$\frac{\partial\, \mathrm{PR}(x, \theta)}{\partial x} = 0 = \Psi_s(x, \theta), \tag{6}$$

provided that

$$\frac{\partial \Psi_s(x, \theta)}{\partial x} = \frac{\partial^2 \mathrm{PR}(x, \theta)}{\partial x^2} < 0,$$

which holds from Assumption 4. (6) implicitly defines optimum output for firms that do not treat their negative externalities under the subsidy regime, $x_s(\theta)$. From Assumption 6(b), for these firms we know

$$\frac{\partial \Psi_s(x, \theta)}{\partial \theta} = \frac{\partial^2 \mathrm{PR}(x, \theta)}{\partial x \partial \theta} < 0.$$

Therefore, using the implicit function rule, $x_s'(\theta) < 0$, that is, firms with older vintage production technology produce less output.

4.3. Industry Structure Under the Subsidy Regime

As under the tax regime, each firm maximizes profit by choice of whether to treat its negative externalities:

$$\max\{\mathrm{PR}(x(s, \theta), \theta) + S + s\, q(x(s, \theta), \theta) - C(q(x(s, \theta), \theta), \theta), \mathrm{PR}(x_s(\theta), \theta)\}.$$

Similar to the prior section, we assume that the firm with the oldest vintage production technology can make positive profits without treating its negative externalities, $\mathrm{PR}(x_s(\bar{\theta}), \bar{\theta}) > 0$.

The firm which is indifferent between treating its negative externalities and not taking action is implicitly defined by

$$\mathrm{PR}(x(s,\tilde{\theta}),\tilde{\theta}) + S + s\,q(x(s,\tilde{\theta}),\tilde{\theta}) - C(q(x(s,\tilde{\theta}),\tilde{\theta}),\tilde{\theta}) - \mathrm{PR}(x_s(\tilde{\theta}),\tilde{\theta}) = 0 = \Phi(S,s,\tilde{\theta}).$$

Differentiating with respect to $\tilde{\theta}$, cancelling terms using the necessary optimality conditions (4) and (6), and rearranging terms

$$\frac{\partial\Phi(S,s,\tilde{\theta})}{\partial\tilde{\theta}} = \frac{\partial\,\mathrm{PR}(x(s,\tilde{\theta}),\tilde{\theta})}{\partial\tilde{\theta}} - \frac{\partial\,\mathrm{PR}(x_s(\tilde{\theta}),\tilde{\theta})}{\partial\tilde{\theta}} + \left[s - \frac{\partial C(q(x(s,\tilde{\theta}),\tilde{\theta}),\tilde{\theta})}{\partial q}\right]\frac{\partial q(x(s,\tilde{\theta}),\tilde{\theta})}{\partial\tilde{\theta}} - \frac{\partial C(q(x(s,\tilde{\theta}),\tilde{\theta}),\tilde{\theta})}{\partial\tilde{\theta}}. \tag{7}$$

Separation of $[\underline{\theta},\bar{\theta}]$ into two continuous segments where firms with newer vintage production technologies treat their negative externalities requires that (7) be negative. From Assumption 5, the last term is negative because less efficient firms have higher externality treatment costs. In general, the difference between the first and second terms and the third term of (7) have opposite signs. This is because if the subsidy is less than the marginal cost of treatment for the indifferent firm, then $x(s,\theta) < x_s(\theta)$ and using Assumption 2 the third term is negative, while the difference between the first and second terms is positive from Assumption 6(b). We assume (7) is negative so that firms with production technology vintages $\theta < \tilde{\theta}$ treat their externalities and firms with technology vintages $\theta > \tilde{\theta}$ do not. Lemma 3 provides sufficient conditions for $\partial\Phi(S,s,\tilde{\theta})/\partial\tilde{\theta} < 0$.

LEMMA 3. *Sufficient conditions for firms with newer vintages of production technology to prefer treating their externalities are (a) the unit subsidy is less than or equal to the marginal cost of treatment for the indifferent firm, and (b) the marginal pre-subsidy regime profits are constant in the vintage of production technology for the indifferent firm.*

PROOF. From condition (a) the third term of (7) is negative. From condition (b) the first two terms cancel. The last term in (7) is negative from Assumption 5. □

Condition (a) in Lemma 3 is weaker than Lemma 2 because it only applies to the indifferent firm. Condition (b) in Lemma 3 is identical to condition (b) in Lemma 1.

Separation of firms into one group that treats their negative externalities and another that does not is intuitive from condition (a) of Lemma 3. Because firms with older vintage production technologies have greater marginal costs of treatment, they prefer to give up the subsidy while those with newer vintage production technologies do not. If the condition in Lemma 2 holds, then treatment of externalities is costly for all firms at the margin. This marginal cost, however, is mitigated by the fixed transfer—the uniform lump-sum subsidy. The size of the fixed subsidy determines the amount of participation in treatment of negative externalities.

Normalizing the number of firms to unity, total output under the subsidy regime is

$$X(S,s) = \int_{\underline{\theta}}^{\tilde{\theta}(S,s)} x(s,\theta)f(\theta)d\theta + \int_{\tilde{\theta}(S,s)}^{\bar{\theta}} x_s(\theta)f(\theta)d\theta.$$

Aggregate amounts of treated and untreated externality are

$$Q(S,s) = \int_{\underline{\theta}}^{\tilde{\theta}(S,s)} q(x(s,\theta),\theta)f(\theta)d\theta, \quad \text{and}$$

$$Q_u(S,s) = \int_{\tilde{\theta}(S,s)}^{\bar{\theta}} q(x_s(\theta),\theta)f(\theta)d\theta,$$

respectively.

5. Equality of Each of the Policy Objectives Under the Two Regimes

5.1. Total Output

Consider first the effects of the two alternative regimes on output. Individual firm outputs are determined by the first-order conditions (1), (2), (4), and (6). These first-order conditions yield the optimum value functions $x_t(\theta)$, $x(t,\theta)$, $x(s,\theta)$ and $x_s(\theta)$, respectively. The range of the unit amounts, t and s, are restricted to being positive and by the conditions in Lemmas 1 and 2, that is

$$0 < t < \frac{\partial C(q(x_t(\tilde{\theta}),\tilde{\theta}),\tilde{\theta})}{\partial q}, \quad \text{and}$$

$$0 < s < \frac{\partial C(q(x,\theta),\theta)}{\partial q}.$$

Equations (2) and (6) imply that for firms with older vintage production technologies

$$x_s(\theta) > x(t, \theta), \tag{8}$$

and Equations (1) and (4) imply that for firms with newer vintage production technologies

$$x(s, \theta) > x_t(\theta). \tag{9}$$

Recalling the equations for total output under the tax and subsidy regimes, Lemma 4 provides necessary conditions for the equality of $X(t)$ and $X(S, s)$.

LEMMA 4. *The necessary conditions for equality of total output under given tax and subsidy regimes are (a) the unit tax is less than the difference between the marginal cost of treatment and the unit subsidy, (b) a greater proportion of firms treat their negative externalities under the subsidy regime than under the tax regime, and (c) the firm with the oldest vintage technology weakly prefers not to treat its negative externalities.*

PROOF. Condition (a) is equivalent to

$$t < \frac{\partial C(q(x, \theta), \theta)}{\partial q} - s.$$

This implies $x(t, \theta) > x(s, \theta)$. Condition (b) is equivalent to $\tilde{\theta}(t) < \tilde{\theta}(S, s)$. Together these conditions imply that over the interval $[\tilde{\theta}(t), \tilde{\theta}(S, s)]$ firms produce greater output under the tax regime. Firms outside this interval produce greater output under the subsidy regime. Condition (c) is equivalent to $\tilde{\theta}(S, s) \le \overline{\theta}$. This latter condition is necessary to ensure that the excess production in $[\underline{\theta}, \tilde{\theta}(t)]$ under the subsidy versus tax regime can be offset by excess production under the tax versus subsidy regime in the remaining support of θ. □

As well as the levels of t and s, the choice of S under the subsidy regime directly determines $\tilde{\theta}(S, s)$. Thus, through the uniform lump-sum subsidy, it is possible to control the proportion of firms that participate in the subsidy regime, and achieve total output equal to a given tax regime.

5.2. Total Damage from Negative Externalities

Turning to externality generation, for a given firm the amount of negative externality generated is a function of both output and production technology vintage, $q(x, \theta)$. Directly from the relative output levels (8) and (9), for firms with older vintage production technologies the relationship between the tax and subsidy regimes for untreated externalities is

$$q(x_s(\theta), \theta) > q(x(t, \theta), \theta),$$

and for firms with newer vintage production technologies the relationship between the tax and subsidy regimes for treated externalities is

$$q(x(s, \theta), \theta) > q(x_t(\theta), \theta).$$

Lemma 5 states necessary conditions for the equality of total damage from negative externalities under a tax regime and under a subsidy regime.

LEMMA 5. *The necessary conditions for equality of total damage from negative externalities under given tax and subsidy regimes are (a) a greater proportion of firms treat their negative externalities under the subsidy regime than under the tax regime, and (b) the firm with the oldest vintage technology weakly prefers not to treat its negative externalities.*

PROOF. Condition (a) is equivalent to $\tilde{\theta}(t) < \tilde{\theta}(S, s)$. This condition implies that under the subsidy regime, firms in the interval $[\tilde{\theta}(t), \tilde{\theta}(S, s)]$ treat their negative externalities whereas under the tax regime, firms in this interval do not. Condition (b) is equivalent to $\tilde{\theta}(S, s) \le \overline{\theta}$. □

Conditions (a) and (b) in Lemma 5 are the same as conditions (b) and (c) in Lemma 4, respectively. Condition (a) from Lemma 4 is not necessary here because the additional damage that results from not treating negative externalities under the tax scheme may outweigh damage from the higher output of firms that treat their negative externalities under the subsidy scheme in the interval $[\tilde{\theta}(t), \tilde{\theta}(S, s)]$.

5.3. Social Welfare

Social welfare is based on the benefit function

$$B(\cdot) = \mathrm{CS}(X(\cdot)) + \mathrm{PS}(\cdot) - \omega(Q_u(\cdot), Q(\cdot)),$$

where the possible arguments, t or S, s, indicate which regime is in effect. That is, social welfare is the sum of consumer and producer surplus less the total damage from negative externalities. The direct effect of either the tax or subsidy regime is a net transfer, and therefore does not affect the benefit function. The consumer surplus is a function of total output and its derivative is

$$CS'(X(\cdot)) > 0.$$

The producer surplus excluding the payment of taxes or receipt of subsidies under the two regimes is

$$\mathrm{PS}(t) = \int_{\underline{\theta}}^{\tilde{\theta}(t)} \Pi(x_t(\theta), \theta) f(\theta) d\theta + \int_{\tilde{\theta}(t)}^{\bar{\theta}} \mathrm{PR}(x(t, \theta), \theta) f(\theta) d\theta;$$

$$\mathrm{PS}(S, s) = \int_{\underline{\theta}}^{\tilde{\theta}(S,s)} [\mathrm{PR}(x(s, \theta), \theta) - C(q(x(s, \theta), \theta), \theta)] f(\theta) d\theta + \int_{\tilde{\theta}(S,s)}^{\bar{\theta}} \Pi(x_s(\theta), \theta) f(\theta) d\theta.$$

Compare the factors that determine components of social welfare across production technology vintages for the two regimes. For firms with older vintage production technologies

$$x_s(\theta) > x(t, \theta), \quad q(x_s(\theta), \theta) > q(x(t, \theta), \theta) \quad \text{and}$$
$$\Pi(x_s(\theta), \theta) > \mathrm{PR}(x(t, \theta), \theta),$$

where the former two inequalities are from the previous sections and the last inequality is because taxed firms underproduce. For firms with newer vintage production technologies

$$x(s, \theta) > x_t(\theta), \quad q(x(s, \theta), \theta) > q(x_t(\theta), \theta), \quad \text{and}$$
$$\mathrm{PR}(x(s, \theta), \theta) - C(q(x(s, \theta), \theta), \theta) < \Pi(x_t(\theta), \theta).$$

The first two inequalities are from the previous sections, and the latter is because under the subsidy regime the marginal cost of treatment is partially subsidized and firms underproduce. Thus, with no further conditions the subsidy regime favors increased consumer surplus and increased damage from negative externalities, which affect social welfare in opposite directions. Producer surplus favors neither regime. As a result, there are no conditions of the type required in Lemmas 4 and 5 to obtain equality of social welfare under a tax regime and under a subsidy regime. Specifically, there need not be a larger proportion of firms treating their externalities under the subsidy regime than under the tax regime.

6. Equality of Two Policy Objectives Under the Two Regimes

Lemmas 4 and 5 provide necessary conditions under which, for a given tax regime, equal levels of output and damage from negative externalities can be obtained from a subsidy regime. No such conditions are required to obtain equality of social welfare under the two regimes. We now consider whether levels of two, or all three, of total output, total damage from negative externalities, and social welfare, which arise under a given tax regime, can be achieved by a single subsidy regime simultaneously.

For a given tax regime, the subsidy regime that equalizes output or equalizes damage from negative externalities or equalizes social welfare is unlikely to be unique. The function representing the set of subsidies that yield total output equal to that obtained from a given tax regime, $S^o(s; t)$, is implicitly defined by the condition

$$\gamma(S, s; t) = X(S, s) - X(t) = 0.$$

Taking total differentials and setting one component of the subsidy regime to zero gives

$$d\gamma|_{ds=0} = \frac{\partial X(S, s)}{\partial S} dS = \left[x(s, \tilde{\theta}) f(\tilde{\theta}) \frac{\partial \tilde{\theta}}{\partial S} - x_s(\tilde{\theta}) f(\tilde{\theta}) \frac{\partial \tilde{\theta}}{\partial S} \right] dS, \quad \text{and}$$

$$d\gamma|_{dS=0} = \frac{\partial X(S, s)}{\partial s} ds = \left[x(s, \tilde{\theta}) f(\tilde{\theta}) \frac{\partial \tilde{\theta}}{\partial s} + \int_{\underline{\theta}}^{\tilde{\theta}} \frac{\partial x(s, \theta)}{\partial s} f(\theta) d\theta - x_s(\tilde{\theta}) f(\tilde{\theta}) \frac{\partial \tilde{\theta}}{\partial s} \right] ds,$$

where the arguments to $\tilde{\theta}(S, s)$ are dropped to conserve space. It is straightforward to show

$$q(x(s, \tilde{\theta}), \tilde{\theta}) \frac{\partial \tilde{\theta}}{\partial S} = \frac{\partial \tilde{\theta}}{\partial s} > 0,$$

so that

$$d\gamma|_{dS=0} = \left[q(x(s, \tilde{\theta}), \tilde{\theta}) \frac{\partial X(S, s)}{\partial S} + \alpha \right] ds, \quad \text{where}$$
$$\alpha = \int_{\underline{\theta}}^{\tilde{\theta}} \frac{\partial x(s, \theta)}{\partial s} f(\theta) d\theta > 0.$$

The function representing the set of subsidies that yield total damage from negative externalities equal to that obtained from a given tax regime, $S^d(s; t)$, is implicitly defined by

$$\lambda(S, s; t) = \omega(Q_u(S, s), Q(S, s)) - \omega(Q_u(t), Q(t)) = 0.$$

Again, taking total differentials and setting one component of the subsidy regime to zero gives

$$d\lambda|_{ds=0} = \frac{\partial\omega(\cdot)}{\partial Q_u}\frac{\partial Q_u(S, s)}{\partial S}dS + \frac{\partial\omega(\cdot)}{\partial Q}\frac{\partial Q(S, s)}{\partial S}dS, \text{ and}$$

$$d\lambda|_{dS=0} = \frac{\partial\omega(\cdot)}{\partial Q_u}\frac{\partial Q_u(S, s)}{\partial s}ds + \frac{\partial\omega(\cdot)}{\partial Q}\frac{\partial Q(S, s)}{\partial s}ds,$$

where $\omega(Q_u(S, s), Q(S, s))$ is represented by $\omega(\cdot)$. Expanding each differential,

$$d\lambda|_{ds=0} = \frac{\partial\omega(\cdot)}{\partial Q_u}\left[-q(x_s(\tilde{\theta}), \tilde{\theta})f(\tilde{\theta})\frac{\partial\tilde{\theta}}{\partial S}\right]dS + \frac{\partial\omega(\cdot)}{\partial Q}\left[q(x(s, \tilde{\theta}), \tilde{\theta})f(\tilde{\theta})\frac{\partial\tilde{\theta}}{\partial S}\right]dS, \quad \text{and}$$

$$d\lambda|_{dS=0} = \frac{\partial\omega(\cdot)}{\partial Q_u}\left[-q(x_s(\tilde{\theta}), \tilde{\theta})f(\tilde{\theta})\frac{\partial\tilde{\theta}}{\partial s}\right]ds + \frac{\partial\omega(\cdot)}{\partial Q}\left[q(x(s, \tilde{\theta}), \tilde{\theta})f(\tilde{\theta})\frac{\partial\tilde{\theta}}{\partial s} + \int_{\underline{\theta}}^{\tilde{\theta}}\frac{\partial q(x(s, \theta), \theta)}{\partial x}\frac{\partial x(s, \theta)}{\partial s}f(\theta)d\theta\right]ds.$$

This last differential can be restated as

$$d\lambda|_{dS=0} = q(x(s, \tilde{\theta}), \tilde{\theta})\left[\frac{\partial\omega(\cdot)}{\partial Q_u}\frac{\partial Q_u(S, s)}{\partial S} + \frac{\partial\omega(\cdot)}{\partial Q}\frac{\partial Q(S, s)}{\partial S} + \frac{\partial\omega(\cdot)}{\partial Q}\sigma\right]ds, \quad \text{where}$$

$$\sigma = \int_{\underline{\theta}}^{\tilde{\theta}}\frac{\partial q(x(s, \theta), \theta)}{\partial x}\frac{\partial x(s, \theta)}{\partial s}f(\theta)d\theta > 0.$$

Set the magnitude of dS to a multiple of ds, specifically $dS = q(x(s, \tilde{\theta}), \tilde{\theta})\, ds$. Then

$$d\gamma|_{dS=0} = d\gamma|_{ds=0} + \alpha\, ds, \quad \text{and} \tag{10}$$

$$d\lambda|_{dS=0} = d\lambda|_{ds=0} + \frac{\partial\omega(\cdot)}{\partial Q}\sigma\, ds, \tag{11}$$

indicate the trade-offs for total output and total damage from negative externalities, respectively, between changes in S and s, for a given tax regime. If they exist, both $S^o(s; t)$ and $S^d(s; t)$ are downward sloping functions.

Having determined the differentials for the main component of consumer surplus (total output) and for total damage from negative externalities, we define the function representing the set of subsidy regimes that yield equal producer surplus, to that obtained from a given tax regime, $S^{ps}(s; t)$, implicitly by

$$\delta(S, s; t) = \mathrm{PS}(S, s) - \mathrm{PS}(t) = 0.$$

Taking total differentials and setting one component of the subsidy regime to zero gives

$$d\delta|_{ds=0} = \frac{\partial\,\mathrm{PS}(S, s)}{\partial S}dS = \left[[\mathrm{PR}(x(s, \tilde{\theta}), \tilde{\theta}) - C(q(x(s, \tilde{\theta}), \tilde{\theta}), \tilde{\theta})]f(\tilde{\theta})\frac{\partial\tilde{\theta}}{\partial S} - \Pi(x_s(\tilde{\theta}), \tilde{\theta})f(\tilde{\theta})\frac{\partial\tilde{\theta}}{\partial S}\right]dS, \quad \text{and}$$

$$d\delta|_{dS=0} = \frac{\partial\,\mathrm{PS}(S, s)}{\partial s}ds = \left[[\mathrm{PR}(x(s, \tilde{\theta}), \tilde{\theta}) - C(q(x(s, \tilde{\theta}), \tilde{\theta}), \tilde{\theta})]f(\tilde{\theta})\frac{\partial\tilde{\theta}}{\partial s} + \int_{\underline{\theta}}^{\tilde{\theta}}\left[\frac{\partial\,\mathrm{PR}(x(s, \theta), \theta)}{\partial x} - \frac{\partial C(q(x(s, \theta), \theta), \theta)}{\partial q}\times\frac{\partial q(x(s, \theta), \theta)}{\partial x}\right]\frac{\partial x(s, \theta)}{\partial s}f(\theta)d\theta - \Pi(x_s(\tilde{\theta}), \tilde{\theta})f(\tilde{\theta})\frac{\partial\tilde{\theta}}{\partial s}\right]ds.$$

Let

$$\eta = \int_{\underline{\theta}}^{\tilde{\theta}}\left[\frac{\partial\,\mathrm{PR}(x(s, \theta), \theta)}{\partial x} - \frac{\partial C(q(x(s, \theta), \theta), \theta)}{\partial q}\frac{\partial q(x(s, \theta), \theta)}{\partial x}\right]\frac{\partial x(s, \theta)}{\partial s}f(\theta)d\theta.$$

From (4), $\eta < 0$. Therefore,

$$d\delta|_{dS=0} = \left[q(x(s, \tilde{\theta}), \tilde{\theta})\frac{\partial\,\mathrm{PS}(S, s)}{\partial S} + \eta\right]ds = d\delta|_{ds=0} + \eta\, ds. \tag{12}$$

The function representing the set of subsidy regimes that yield social welfare equal to that obtained by a given tax regime, $S^{sw}(s; t)$, is implicitly defined by

$$\beta(S, s; t) = B(S, s) - B(t).$$

Taking total differentials and setting one component of the subsidy regime to zero gives

$$\begin{aligned} d\beta|_{ds=0} &= \frac{\partial B(S,s)}{\partial S} dS \\ &= CS'(X(S,s)) \frac{\partial X(S,s)}{\partial S} dS + \frac{\partial\, PS(S,s)}{\partial S} dS \\ &\quad - \frac{\partial \omega(\cdot)}{\partial Q_u} \frac{\partial Q_u(S,s)}{\partial S} dS - \frac{\partial \omega(\cdot)}{\partial Q} \frac{\partial Q(S,s)}{\partial S} dS, \end{aligned}$$

which, by using (10), (11), and (12) is

$$d\beta|_{ds=0} = CS'(X(S,s))\, d\gamma|_{ds=0} + d\delta|_{ds=0} - d\lambda|_{ds=0}.$$

Similarly,

$$d\beta|_{dS=0} = CS'(X(S,s))\, d\gamma|_{dS=0} + d\delta|_{dS=0} - d\lambda|_{dS=0}.$$

By substituting,

$$\begin{aligned} d\beta|_{dS=0} &= CS'(X(S,s))\, d\gamma|_{ds=0} + CS'(X(S,s))\, \alpha\, ds \\ &\quad + d\delta|_{ds=0} + \eta\, ds - d\lambda|_{ds=0} - \frac{\partial \omega(\cdot)}{\partial Q} \sigma\, ds \\ &= d\beta|_{ds=0} + \left[CS'(X(S,s))\, \alpha + \eta - \frac{\partial \omega(\cdot)}{\partial Q} \sigma \right] ds. \end{aligned}$$

Because

$$CS'(X(S,s))\, \alpha + \eta - \frac{\partial \omega(\cdot)}{\partial Q} \sigma$$

cannot be signed, it is not possible to determine the shape of $S^{sw}(s; t)$.

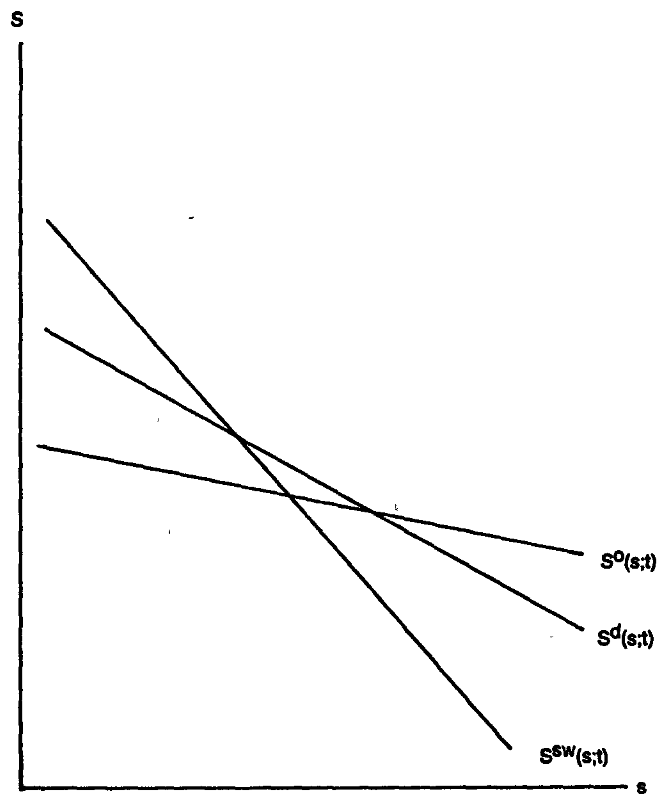
The three functions, $S^o(s; t)$, $S^d(s; t)$, and $S^{sw}(s; t)$ are iso-output, iso-damage, and iso-welfare functions, respectively. Because of the trade-offs between the two components of the subsidy regime and the restrictions on the unit subsidy, these iso-objective functions are each defined only over a limited interval. Define the intervals of these functions as $[s^o_{\max}, s^o_{\min}]$, $[s^d_{\max}, s^d_{\min}]$, and $[s^{sw}_{\max}, s^{sw}_{\min}]$. Let the intersection of any two of the previous intervals be $[s^{i\cap j}_{\max}, s^{i\cap j}_{\min}]$, where $i, j = \{o, d, sw\}$. Theorem 1 provides our main result for any two objectives.

THEOREM 1. *The necessary and sufficient condition for a subsidy regime to produce an equal level of any two objectives (total output, total damage from negative externalities, and social welfare) that are obtained under a given tax regime, is that the two functions describing the subsidy regimes yielding each individual objective, intersect at least once.*

PROOF. If $S^i(s; t) = S^j(s; t)$ in $[s^{i\cap j}_{\max}, s^{i\cap j}_{\min}]$, then at these points, the subsidy regime will produce equal levels of both objectives as the given tax regime (sufficiency). If $S^i(s; t) \neq S^j(s; t)$ in $[s^{i\cap j}_{\max}, s^{i\cap j}_{\min}]$, then there can be no such subsidy regime (necessity). □

An example where each pair of objectives obtained from a given tax regime are produced by a unique subsidy regime is depicted in Figure 1. Summarizing the above insights, for each of two objectives, if the result from a given tax regime can be duplicated by a given subsidy regime, then it is likely to be duplicated by a continuum of subsidy regimes. The continuum of sub-

Figure 1 Any Two Policy Objectives from a Given Tax Regime Can Be Obtained by a Unique Subsidy Regime.



sidy regimes can be characterized as a function in (s, S) space over a closed interval. The functions for each of the two objectives (i) may not cross, (ii) may cross once, or (iii) may cross multiple times, over the intersection of their intervals. If they do not cross, then it is not possible to achieve simultaneously the two objectives from a given tax regime with a subsidy regime. If they cross once, then there is a unique subsidy regime that can achieve the results of a given tax regime, and if they cross multiple times, then there are multiple subsidy regimes that can achieve the results from the given tax regime.

Theorem 1 can be trivially extended to state the condition under which a subsidy regime can produce equal total output, total damage from negative externalities, and social welfare, as is obtained from a given tax regime. This condition requires that the three functions intersect at the same point at least once. However, it is obvious that this is a much stricter requirement than the condition in Theorem 1.

7. Summary

We have shown that there is a basic equivalence between taxes and subsidies to control negative production externalities. That is, through reasonable conditions either punishment or reward can be used to achieve equivalent policy objectives over any one of, or pair of, total output, total damage from negative externalities, and social welfare. Symmetry between taxes and subsidies is not required for equivalence and, in fact, our model takes advantage of the natural asymmetries involving taxation of those not taking action to treat their externalities in the tax regime, and subsidization of those who take externality treatment actions under the subsidy regime.

The mechanics of our model provides policy guidance. From our formulation, under the tax regime all firms are affected—they either treat their externalities or are taxed—unlike the subsidy regime where only treating firms receive the subsidy. This distinction can impact the perceived equity of a given policy. Our analysis indicates that under both regimes newer vintage technology firms treat their externalities (Lemmas 1 and 3). This outcome can be viewed as socially efficient as these firms have lower costs of externality treatment. For those firms that treat their externalities in both regimes and those that do not treat their externalities in both regimes, the subsidy yields greater output and greater negative externalities. Thus, at the level of the firm, subsidies increase production. It follows that to achieve equivalence in total output or in total damage from negative externalities requires that the subsidy regime has more firms treating their negative externalities than a given tax regime. Thus, policy can be chosen based on the proportion of firms that take treatment action. This requirement, however, is not necessary to achieve equivalence of social welfare between the two regimes. Finally, we find that there is a continuum of subsidy regimes that can achieve equivalence with a given tax regime in any one of our objectives. As a result, policy makers have the latitude to calibrate the mix of uniform lump-sum and unit amounts to achieve their targets.

Our results have several important policy implications. The first concerns the main alternative economic instrument to taxes and subsidies: tradable permits. Notwithstanding the poor history of tradable permits in practice, and similar to tradable permits, our tax and subsidy regimes give firms a choice—whether to treat their externalities or not. Favoring subsidies, under a tax regime or under tradable permits, authorities must monitor the entire industry, whereas under a subsidy regime only the proportion of the industry that is receiving the subsidies need be monitored.

The second is that because equivalent policy objectives can be achieved by taxes and subsidies, additional criteria can be included in policy selection. For example, because it is difficult politically and administratively to move from one regime to another, policy continuity may favor taxes. In addition, trading partners may see subsidies for externality treatment as production subsidies, also favoring taxes. Alternatively, domestic industry may view taxes for externality damage as restricting their ability to compete, favoring subsidies. Moreover, because equivalence between taxes and subsidies does not require that individual firm behavior be identical under the two regimes, there is considerable latitude for policy discretion.

The third implication concerns equity and fairness. Accounting for inter-firm differences when employing taxes, the unequal application of taxes across firms may be viewed as inequitable. Moreover, there is the question of whether it is fair, or even legal, to penalize historical production technology decisions by applying externality taxes to established operations, particularly if at the time of acquisition the production technology was given regulatory approval. Therefore, subsidy regimes may be favored because taxes can be viewed as retroactively penalizing firms with old vintage production technology.[3]

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