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Information Technology Substitution Revisited

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Taking advantage of the opportunities created by the price adjusted performance improvement in IT depends in part on the ability of IT capital to substitute for other inputs in production. Studies in the IS literature as well as most economics training that examine substitution of IT capital for other inputs use the Allen elasticity of substitution (AES). We present a less-well-known measure for the elasticity of substitution, the Morishima Elasticity of Substitution (MES). In contrast to the AES which is misleading when there are three or more inputs – such as non-IT capital, labor and IT capital – the MES provides a substitution measure where the scale is meaningful, and the measure differs depending upon which price is changing. This is particularly important for IT capital as prices have been declining and there is evidence that IT capital can substitute for non-IT capital or labor in a qualitatively different way than non-IT capital and labor substitute for each other. Methodologically we also show the impact of imposing local regularity – for example, monotonicity of output from increases in inputs – that we do through Bayesian methods employed to estimate the underlying functions that are used to calculate various measures of substitution. We demonstrate the importance of the MES as an under-recognized measure of substitution and the impact of imposing local regularity using an economy-wide industry-level dataset covering 1998-2009 at the three-digit NAICS level. Our MES results show that reductions in the price of IT capital increase the quantity of IT capital in use but are unlikely to change the input share of IT capital – the value of IT capital as a proportion of the value of all inputs, in contrast to major studies using the AES. In addition, estimates for both elasticities of substitution are more stable after imposing local regularity. Both of these advances – that is, the MES and imposing local regularity – have potential to impact future work on IT productivity, IT pricing, IT cost estimation and any type of analysis that posits the substitution of IT capital for non-IT capital or labor.

Keywords: Information Systems, IT Policy and Management, Economics of IS, IT Substitution, Production Function

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1. Introduction

Critical to our society, economy, and to research in Information Systems (IS) is the impact of information technology (IT) on production, on jobs, and on other capital. One key set of measures to assess this impact is the economic measures that determine tradeoffs between the amounts of labor, non-IT capital and IT capital used in production as the result of wage or price changes – the elasticity of substitution (ES). The objective of this Research Commentary is to present an important and under-recognized ES, the Morishima elasticity of substitution or MES, and describe its benefits – especially for IT as an input – over more frequently used measures. In so doing, we will show why it is critical to isolate which price is changing in any ES measure.

The major studies from the IS literature that examine substitution of IT capital for other inputs concentrate on the Allen elasticity of substitution (AES). When there are three or more inputs – such as non-IT capital, labor and IT capital – the AES has strict limitations: it is not a measure of ease of substitution and can only provide the direction of the change in quantity of one input in response to a price change of the other input (Blackorby and Russell 1981, 1989). Fortunately, Blackorby and Russell (1981) discovered the Japanese article by Morishima (1967), an article that has yet to be translated into English, that introduces the MES. In contrast to the AES, in the context of three or more inputs the scale and sign of the MES is a sufficient statistic for assessing both quantitative and qualitative effects of changes in price on input quantity ratios and on relative input shares. Moreover, the MES allows for different MESs depending on which input price is changing. This matters because the price of IT capital is the only input price that has been consistently decreasing, and because IT capital can have broader effects than typical non-IT capital and labor. Thus, it is possible to determine the effect of a change in the price of an input on the relative share of an input. That is, even though IT capital may be a substitute for other inputs, the increased use of IT in response to a price reduction in IT may result in a reduced relative input share of IT, where relative share is price times quantity of IT divided by price times quantity of another input.

In addition to introducing the MES, we present an under-recognized and potentially important methodological issue: the importance of imposing local regularity such that the estimated function from which the parameters are used to calculate the ES follow its theoretical properties – for example, the monotonicity of output from increases in inputs. Imposing local regularity increases the stability and accuracy of the estimates, reducing the chance of erroneous and misleading results.

Overview of Our Approach We begin by briefly reviewing ESs, and then we explain the two different ES from a production perspective. In this explanation we aggregate much of the relevant economic literature that describes the differences between the AES and MES, the underlying problems with the AES, and how the MES overcomes those problems.

Next we execute an analysis that yields estimates for the AES and MES. In so doing we demonstrate the differences between the two ESs, and perhaps more importantly through our MES estimates we provide evidence for a different level of substitution by IT when the price of IT changes. Using an economy-wide industry-level dataset for 1998-2009 we estimate both the MES and the AES using non-linear regression and Bayesian analyses. We employ one flexible functional form (FFF), the CES-Translog, to estimate a production function with three inputs – non-IT capital, IT capital, and labor – and use the estimated parameters to calculate the AES and MES. We then examine if there are violations of regularity conditions, and when there are we impose regularity conditions using Bayesian methods to re-estimate the FFF and use these newly estimated parameters to recalculate the AES and MES.

2. Background Literature

Prior studies in the IS literature have examined ESs between IT capital and other inputs. In an excellent article in *Management Science* Dewan and Min (1997) used firm-level data from 1988-1992 to estimate Translog and CES-Translog production functions, and then calculated AESs. They found that IT capital is a substitute for non-IT capital and for labor. In another excellent article in this Journal Chwelos, Ramirez, Kraemer and Melville (2010) used firm-level data from 1987-1998 to estimate the Translog and CES-Translog forms, and then calculated AESs. They found that IT capital is a substitute for labor, but in their more recent data found that IT capital and non-IT capital have become complements. They also found that over time more units of IT capital are needed to substitute for a unit of labor. In a working paper Hitt and Snir (1999) matched survey data of organizational practices from 1995-1996 with firm-level data on IT spending and output and inputs from 1987-1994, and estimated a Translog form to calculate AESs. They also found that IT is a substitute for non-IT capital and for labor. Outside of the IS literature Chun and Mun (2006) in the *Southern Economic Journal* used cost function estimates from a selection of US industries for 1984-1999 to calculate ESs. Their AESs indicate that IT capital is a substitute for labor, non-IT equipment, and structures whereas IT is a complement to intermediate inputs. Their MESs show that IT capital and intermediate inputs are substitutes when the price of IT capital changes, but complements when the price of intermediate inputs changes.

Unfortunately, in the published studies in IS detailed above, results from the AES were used in a chain of reasoning to support implications regarding the balance of IT capital and the other inputs, non-IT capital and labor, that in fact the AES cannot be used to show. Indeed, Dewan and Min (1997) state that as a consequence of the AES findings whereby IT capital is a substitute for non-IT capital and labor, the input share of IT capital would increase over time. Chwelos et al. (2010) make a similar prediction given their AES findings that IT capital is a substitute for labor, indicating that the input share of IT capital comes at the expense of labor. As we indicated in the Introduction and will report in detail later, with three or more inputs the AES does not provide any information about input shares, and using the MES we find empirically that an increasing input share for IT capital in response to a decline in the price of IT capital, ceteris paribus, is not correct for the time period covered by our dataset. Moreover, both Dewan and Min, and Chwelos et al., compare the AES between subsets of industries such as manufacturing versus services, and draw conclusions from these comparisons. Again, as we indicated earlier and will see in detail later, with three or more inputs the relative magnitudes of the AES have no meaning, and comparisons between AESs are not useful. Indeed, this over-interpretation of the AES when there are more than two inputs has likely plagued a wide range of research for more than a generation.

Although the AES has been the standard in IS literature and the dominant ES taught and used in economics, research since the early 1980s has shown it is a misleading and relatively uninformative measure when used in analyses with more than two inputs (e.g., Blackorby and Russell 1981, 1989). This research also explains why the MES is a superior measure. One possible reason for why the MES is taking longer to penetrate the literature is that, as mentioned in the Introduction, the original Morishima article from 1967 has yet to be translated into English. In contrast, the AES traces its origins to a classic mathematical economics textbook by Allen in 1938 and was generalized to more than two inputs by Uzawa in 1962 in the well-known *Review of Economic Studies*.

3. Methodology

Elasticities of Substitution Hicks (1932) introduced the Hicks Elasticity of Substitution (HES) to examine the substitutability between inputs. His purpose was to provide a measure of the curvature of the isoquant or ease of substitution, and comparative statics regarding relative input shares. Allen (1938) introduced the AES as a measure of substitution. The information properties of the HES are satisfied by the AES for two inputs. However, for more than two inputs the AES is not a measure of curvature, and provides no information regarding relative input shares (Blackorby and Russell 1981, 1989).

Fortuitously, Blackorby and Russell (1981) discovered the Morishima (1967) article that introduces the MES. For an arbitrary number of inputs, the MES is a measure of the curvature of the isoquant, and provides comparative statics of relative input shares.

3.1. Production and Elasticities of Substitution

We define a production function as $Y = f(x)$ where Y is output in units and x is a vector of inputs containing non-IT capital K, labor L, and IT capital Z. We use capital letters for quantities of Y, K, L, and Z as we use lower case to denote natural logs of these quantities later on. Generically our inputs are over $i \in \{K, L, Z\}$. We take $f(x)$ to have the usual properties: single valued, non-negative and real for all finite x, monotonic, concave, continuous and twice continuously differentiable. Following Chambers (1988) we can define $V(Y) = x : f(x) \geq Y$, which is closed and non-empty, and the lower boundary of $V(Y)$ is defined by the level set $\tilde{V}(Y) = \{x : f(x) = Y\}$. Consequently, through the implicit function rule we can solve for a given x_i in terms of the remaining x and Y.

For constant output we can obtain the *marginal rate of technical substitution* (MRTS), which is the rate at which one input can be substituted for another while holding the level of output constant:

$$\partial x_i/\partial x_j = -\frac{\partial f(x)/\partial x_j}{\partial f(x)/\partial x_i} = -f_j/f_i,$$

where subscripts of f indicate partial derivatives so that f_i is the partial derivative of $f(x)$ with respect to x_i. In a classic work, Hicks (1932) developed an ES between inputs as a percentage change in the input ratio in response to a percentage change in the MRTS:

$$\sigma = \frac{d(x_i/x_j)/[x_i/x_j]}{d(f_j/f_i)/[f_j/f_i]}.$$

In the two-factor case the HES provides two pieces of information. The first is a measure of the curvature of the isoquant, or ease of substitution. The higher the elasticity, the "easier" the substitution of one input for the other - in other words, the lesser the degree of curvature of the isoquant. The second is comparative statics regarding relative input shares (Blackorby and Russell, 1989).

The HES was generalized for the n-factor case by Uzawa (1962) to the AES. In the context of production functions, the AES is given by

$$\sigma_{ij}^A = \frac{\sum_i x_i f_i}{x_i x_j} \frac{\mathbf{H}_{ij}}{\mathbf{H}} \tag{1}$$

where $\mathbf{H}$ is the bordered Hessian determinant of $f(x)$ and $\mathbf{H}_{ji}$ is the cofactor associated with f_{ij}. In the AES factors i and j are substitutes if $\sigma_{ij}^A > 0$, which means that increasing the price of the j^{th} input increases the optimal quantity demanded of input i. If $\sigma_{ij}^A < 0$, then inputs i and j are complements whereby decreasing the price of input j increases the optimal quantity demanded of input i. From the monotonicity of the production function, own price elasticity is negative, $\sigma_{ii}^A < 0$.

The AES embeds some important restrictions. First, the AES is symmetric so that $\sigma_{ij}^A = \sigma_{ji}^A$. Consequently, the AES does not depend on which price is changing. Second, from the properties of $f(x)$, for a given input x_r at least one σ_{rj}^A must be positive so that a given input must be a substitute for at least one other input. In the context of our production function this latter restriction means that σ_{ZK}^A and σ_{ZL}^A cannot both be negative (Chambers, 1988) so that IT capital must be a substitute for either non-IT capital or labor, or both.

Blackorby and Russell (1989) argue that the AES is not informative beyond the cross-price elasticity of demand. The AES in (1) can be expressed as

$$\sigma_{ij}^A = \epsilon_{ij}/s_j,$$

where $\epsilon_{ij} = \partial \ln x_i / \partial \ln p_j$ is the (constant output) cross-price elasticity of demand and s_j is input j's cost share of the producer's total expenditure, $s_j = x_j p_j / \sum_i x_i p_i$. As such, the AES is not a measure of the curvature of the isoquant. They also argue that the AES does not provide information regarding relative input shares, concluding that "*absolute income shares is a property of cross-price elasticities and shares; the AES provides no new information about these shares."*(Blackorby and Russell, 1989: page 884).

In contrast to the AES, the MES can be defined as

$$\sigma_{ij}^M = \frac{f_j}{x_i}\frac{\mathbf{H}_{ij}}{\mathbf{H}} - \frac{f_j}{x_j}\frac{\mathbf{H}_{jj}}{\mathbf{H}}. \tag{2}$$

We can rearrange to express the MES in terms of the AES as follows:

$$\sigma_{ij}^M = \frac{f_j x_j}{\sum_i f_i x_i}[\sigma_{ij}^A - \sigma_{jj}^A].$$

The interpretation of substitutes and complements in the MES relates to the *relative* quantities of inputs, x_i/x_j. In the MES, inputs i and j are substitutes if $\sigma_{ij}^M > 0$, which means that increasing the price of the j^{th} input increases the quantity of input i *relative* to the quantity of input j. That is, for normal goods (or inputs) where an increase in the price of the j^{th} input results in decreased quantity of the j^{th} input, x_j, then $\sigma_{ij}^M > 0$ means the ratio of input quantities, x_i/x_j, increases – recognizing that this can occur with a decreased x_i. Inputs i and j are complements if $\sigma_{ij}^M < 0$: an increase in the price of j decreases not only the quantity of input j but also decreases the quantity of input i such that the ratio x_i/x_j decreases.

The MES is not symmetric, which means that in general $\sigma_{ij}^M \neq \sigma_{ji}^M$. The MES can also classify substitution elasticities differently from the AES. For example, two inputs i and j could be Allen complements, $\sigma_{ij}^A < 0$, while being Morishima substitutes, $\sigma_{ij}^M > 0$, following the description above where x_i may decrease along with x_j. Hence, "$\cdots$ *the Allen measure has a bias toward treating inputs as complements (or, the Morishima measure has a bias toward treating inputs as substitutes).*" (Mundra and Russell, 2004: 35), reflecting the fact that the AES measures changes in absolute quantities and the MES measures changes in relative quantities.

Blackorby and Russell (1989) make the point that the MES is a measure of curvature of the isoquant, or ease of substitution, and that the MES provides comparative static information about relative input shares. Using a cost function formulation (which is dual to our production function constant output formulation) they show that

$$\frac{\partial \ln[x_j p_j / x_i p_i]}{\partial \ln[p_j / p_i]} = 1 - \sigma_{ij}^M .$$

The above equation can be rewritten as

$$\frac{-\partial \ln[x_i p_i / x_j p_j]}{\partial \ln[p_j / p_i]} = 1 - \sigma_{ij}^M . \tag{3}$$

The equation in (3) relates a change in the relative input share, $x_i p_i / x_j p_j$, from an underlying change in the price ratio, p_j / p_i, to the MES. The partial derivative in (3) requires that the percentage change in the price ratio, p_j / p_i, is induced solely by changing the price of input j, because allowing both p_j and p_i to vary would entail variation in all other price ratios, p_k / p_i, $k \neq i$, contrary to the definition of partial differentiation. Thus, consider input j is IT capital where p_j has been falling over time. For a given decrease in the price ratio p_j / p_i induced by a drop in p_j, the share of another input i – say non-IT capital or labor – relative to input j – IT capital – decreases if $\sigma_{ij}^M > 1$ and increases if the $\sigma_{ij}^M < 1$.

According to Chambers (1988), the MES can be written as

$$\sigma_{ij}^M = \frac{\partial \ln[x_i / x_j]}{\partial \ln[p_j]} . \tag{4}$$

Thus, the MES is a *two-factor-one-price* ES because the MES measures relative input adjustment to single-input price changes. On the other hand, the AES measures how a single input adjusts to changes in a single-input price, and thus is a *one-price-one-factor* ES (Chambers, 1988). The MES does not measure relative input adjustment to relative price changes because the effect on the relative input shares of changing p_i instead of p_j is generally different.

3.2. Empirical Methodology

Estimation Form Measuring the AES and the MES requires estimating parameters of a functional form that is sufficiently flexible to provide a suitable fit for most datasets. Following earlier research (Dewan and Min 1997, Chwelos et al. 2010), we employ the CES-Translog production function. The CES-Translog (Pollak, Sickles and Wales 1984) is a FFF, which includes both the CES (Constant Elasticity of Substitution) and the Translog production functions as special cases. Using lower case letters to represent the natural log of our variables, the CES-Translog production function for three inputs is,

$$\begin{aligned} y = \; & \alpha \; - \frac{1}{\rho} ln[\delta_z \, Z^{-\rho} + \delta_k \, K^{-\rho} + (1 - \delta_z - \delta_k) \, L^{-\rho}] + \beta_{zk} \, z \, k + \beta_{zl} \, z \, l + \beta_{kl} \, k \, l + \beta_{zz} \, z^2 \\ & + \beta_{kk} \, k^2 + \beta_{ll} \, l^2 + \epsilon , \end{aligned} \tag{5}$$

where y is value added (as a measure of output), z is IT capital, k is non-IT capital, l is labor, and ϵ is an error term.

Dataset Our dataset consists of the MFP dataset for the three-digit 2002 North American Industry Classification System (NAICS) from 1998 to 2009. We acquired data on capital stock, IT capital stock, and labor input for 59 three-digit NAICS code industries directly from the Bureau of Labor Statistics (BLS) website. There are four categories for IT capital stock: computers, software, communication, and other. The productive capital stock of these four IT categories is aggregated as Z. For the non-IT capital stock (K), we totaled the equipment and structure components of the asset types and subtracted the IT capital stock from it. Both IT capital and non-IT capital are collected in millions of 2005 dollars. The labor input, L is in millions of hours. We also collect current dollar value-added (Y) at the three-digit NAICS code level from the Bureau of Economic Analysis (BEA). Deflating it by the corresponding chain-type quantity indexes for output (2005) yields the real value-added Y in millions of 2005 dollars. In the end we have a balanced panel of 708 observations from 12 years of data on 59 industries. Table 1 contains summary statistics for our dataset. All of our data is publically available.

Industry groups Our dataset covers a broad range of industry groups. Different industry groups may have different fixed effects when estimating the parameters of the CES-Translog production function. Different industry groups may also have very different ESs. Therefore, we classify our dataset into 12 industry groups both for including fixed effects in estimation and for reporting ESs: manufacturing (NAICS code: 31-33), transportation, communications and public utilities (NAICS code: 22, 48-49), wholesale trade (NAICS code: 42), retail trade (NAICS code: 44-45), accommodation, food services and drinking places (NAICS code: 72), finance, insurance, and real estate (NAICS code: 52, 53), entertainment (NAICS code: 51, 71), professional services (NAICS code: 54, 56, 81), educational services, health care and social assistance (NAICS code: 61, 62), forestry (NAICS code: 11), Mining (NAICS: 21), and Construction (NAICS: 23).[1]

3.3. Estimation

We first use non-linear least-squares methods to estimate our CES-Translog form in (5) for our dataset. Then, in order to impose regularity conditions, we use Bayesian-based methods to obtain the parameters.

3.3.1. Non-Linear Least-squares Approach

[1] In addition, we ran our analyses for two subperiods, 1998-2001 and 2002-2009, with the cutoff year 2001/2002 corresponding to the rise and collapse of the IT bubble. There is nothing systematically different in our results between the subperiods, and between the subperiods and the full dataset. The results of our subperiod analyses are available on request.

CES-Translog To obtain converged value for all 10 parameters in the CES-Translog form in (5), we adopt the approach of "Maximization by Parts" (Song, Fan, and Kalbfleisch 2005; Fan, Pastorello, and Renault 2007; Amado and Teravirta 2011) where we divide the 10 parameters into two sets, iteratively fix one set and obtain estimates for the other set until convergence. Specifically, set 1 includes the 3 parameters (ρ, δ_z, and δ_k) from the "CES" part of the CES-Translog form. Set 2 includes the remaining 7 parameters (α, β_{zk}, β_{zl}, β_{kl}, β_{zz}, β_{kk}, and β_{ll}) that are the non-linear parts of the Translog form.

We use 1000 iterations of our process in Figure 1 to generate our final parameter estimates. To start a given iteration, we randomly generate values for set 1 from a uniform distribution on $[-10, 10]$. Keeping these randomly generated values for set 1 as fixed, set 2 is estimated by the non-linear least squares (NL) in Stata SE 10.1 that uses a variation of the Gauss-Newton algorithm for optimization. These newly obtained estimates from set 2 are then fixed and set 1 is estimated by the NL method. The result is a current optima for the current iteration. At the end of each iteration the global optima are replaced by the current optima from the current iteration when they lead to a lower residual sum of squares (RSS). This follows Chwelos et al. (2010) in that we iteratively randomly generate starting values for the parameters to be estimated, execute the non-linear optimization process, and then take the results with the minimum residual sum of squares (RSS). Consequently, among the converged results from all the 1000 iterations, we select the set of results with the minimum RSS as our final estimates for the 10 CES-Translog parameters. We use the 1000 iterations to ensure that our results do not depend on a particular set of starting values and thus are global optima in terms of minimum RSS.[2]

3.3.2. Bayesian Approach

The Impact of Regularity Conditions The CES-Translog form in (5) is a FFF. Many researchers argue that the usefulness of an FFF hinges on whether the estimated parameters satisfy regularity conditions such as non-negativity, monotonicity, and concavity for production functions. Violating any of these conditions causes the second-order conditions for optimizing behavior and duality theory to fail (Barnett, 2002). Inferences obtained from an estimated FFF that violates any regularity condition are unreliable (Barnett and Pasupathy, 2003). Monotonicity violation leads to incorrectly signed elasticities, and curvature violation leads to production possibilities frontiers that are convex to the origin (Caves and Christensen 1980; Diewert and Wales 1987; Morey 1986). We focus on the monotonicity conditions in our study because they directly affect the elasticities, and it is generally agreed that greater amounts of input should not yield less output. To show the

[2] For robustness we executed our NL estimation process multiple times and with different numbers of iterations. The global optima we obtained were highly consistent and the minimum RSS we obtained from these multiple executions varied by less than 6%.

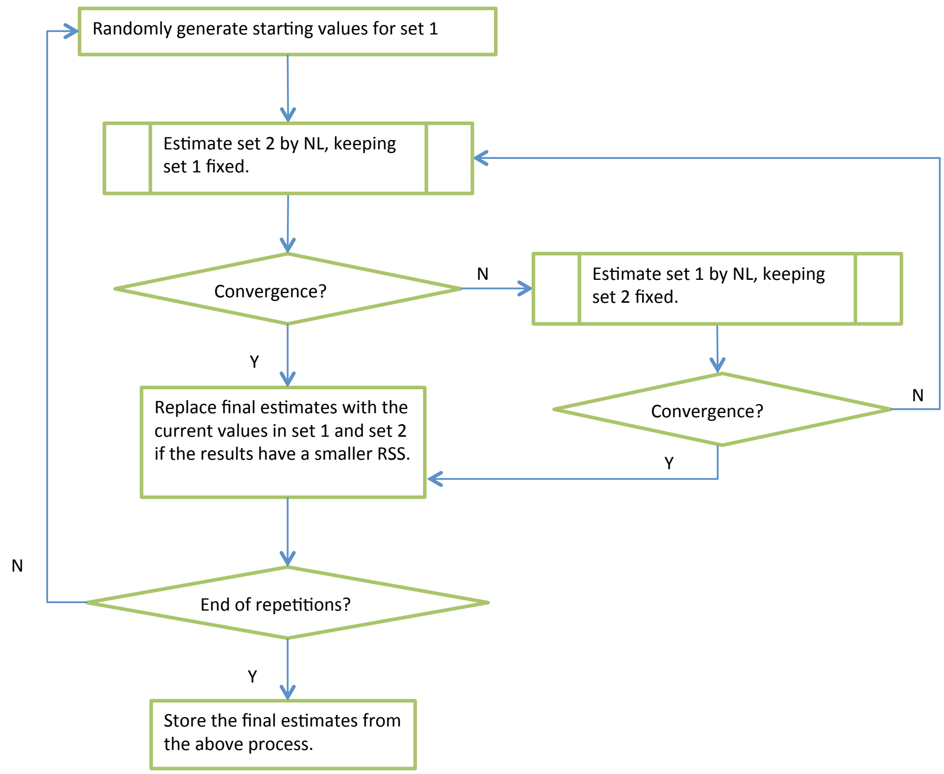

Figure 1: Non-Linear Estimation Flowchart

impact of violating monotonicity on the sign of an ES, consider our form in (5). Monotonicity is satisfied if $f_i \geq 0$, where $i = \{k, l, z\}$. Clearly, both the AES and the MES given by (1) and (2), respectively are a function of $\mathbf{H}$ and f_i. Thus, by definition, the sign of the ES is influenced by the sign of f_i, which in turn determines if the estimated parameters satisfy monotonicity.

We do not impose concavity for two reasons. First, theoretically, concavity is essentially a condition of diminishing returns. However, diminishing returns have been challenged in the case of IT capital both conceptually and by previous research that has documented increasing returns to IT capital (Kudyba and Diwan 2002). Second, implementation imposing both concavity and monotonicity leads to zero acceptance in our Monte Carlo simulation, possibly because of IT having increasing returns and also because imposing a second moment restriction across every observation in a dataset is a severe constraint.

There are two approaches to impose regularity conditions. The first imposes parametric restrictions on the FFF to ensure that conditions hold at all data points. For example, global curvature restrictions can be achieved by using eigenvalue decomposition techniques and Cholesky factorization methods (Diewert and Wales, 1987). Although not the same as imposing regularity, Dewan and

Min (1997) used regularity conditions via parametric restrictions as a selection criterion between Translog and CES-Translog production function. In their dataset, they note the violation rate for Translog is 89% and virtually zero for CES-Translog. Unfortunately, the methods used to impose parametric restrictions undermine the flexibility of most functional forms (Jorgenson and Fraumeni, 1981), and the global imposition of regularity conditions forces many FFFs to exhibit unintended properties. For example, imposing global concavity on the inputs of a Translog cost function may lead to an upward bias in the degree of substitutability, and on a generalised leontief cost function forces all inputs to be substitutes (Diewert and Wales, 1987). Barnett and Pasupathy (2003) argue that imposing regularity conditions globally actually increases the frequency of monotonicity violations.

The second approach imposes regularity conditions locally. Several studies demonstrate that the advantages of local regularity imposition outweigh that of global regularity imposition (Lau, 1978; Diewert and Wales, 1987; Terrell, 1996; O'Donnell and Coelli, 2005). Local imposition of regularity means that constraints are imposed at a single point, at several points, or over a region for which inferences will be drawn (Gallant and Golub, 1984, Salvanes and Tjotta, 1998). Local imposition of regularity conditions can be achieved through sampling theoretic procedures (Jorgenson and Fraumeni, 1981; Ryan and Wales, 1998; Moschini, 1999). More recently, a Bayesian approach using Monte Carlo simulation methods has been proposed to impose regularity locally on the parameters of the estimated FFF.

We adopt the second approach and examine the compliance of the non-linear least squares parameter estimates with monotonicity by examining the marginal products of the three input factors. In particular, we check compliance with the monotonicity conditions for each observation in our dataset and find that about 30% of observations violate at least one of the monotonicity conditions. This violation may undermine the validity of the inferences that are based on the non-linear least squares parameter estimates of the model. Thus, imposing regularity conditions is likely necessary.

CES-Translog Through Bayesian estimation of our CES-Translog model in (5), we have the flexibility and the ability to restrict the sampled parameter estimates (i.e., the generated sample draws, in the context of Monte Carlo simulation) to a sample that satisfies regularity conditions. For our CES-Translog form, we define a vector for the parameters to be estimated, $\beta = [\alpha, \rho, \delta_z, \delta_k, \beta_z, \beta_k, \beta_l, \beta_{zk}, \beta_{zl}, \beta_{kl}, \beta_{zz}, \beta_{kk}, \beta_{ll}]^T$. The objective in the Bayesian estimation method is to properly form the posterior probability distribution function, $g(\beta|D)$, from the observed data D - our measures of value added and inputs.

Bayes Theorem relates the posterior probability to the prior probability,

$$g(\beta|D) \propto L(\beta|D) \times p(\beta),$$

where $L(\beta|D)$ is the likelihood function of the vector of the model parameters β given the observed data D, and $p(\beta)$ is the prior density function of β.

We take the error term in (5), ϵ, to be multivariate normal, with mean 0 and variance-covariance $\sigma^2 I$, where σ is unknown. Had we had no prior knowledge about the parameters, we would have used the noninformative prior. However, in our case, the parameters have to satisfy the regularity conditions, and so we use an informative prior to incorporate these conditions. We follow Coelli, Rao, O'Donnell, and Battese (2005), and propose a joint informative prior for (β, σ):

$$g(\beta,\sigma) \propto \frac{I(\beta)}{\sigma}$$

where $I(\beta)$ is an indicator function that takes the value 1 if β satisfies the conditions and takes the value 0 otherwise.

The CES-Translog form does not have a clear form for the posterior probability density function (pdf) for the parameters. Following Dewan and Min (1997), we assume the posterior is an asymptotic multivariate normal distribution with mean being the estimates from the least-squares approach and with variance-covariance matrix being a diagonal matrix with the variances of the estimators from the least-squares approach on the diagonals. After imposing regularity conditions the posterior is actually a truncated asymptotic multivariate normal distribution because the indicator function forces the posterior pdf to be zero if the parameters fail to satisfy the regularity conditions.

Implementation We use Monte Carlo simulation to implement our Bayesian approach. We randomly draw 10000 times from a multivariate normal distribution for our CES-Translog form. For each draw, we check the regularity conditions for each observation. We accept draws with regularity conditions being satisfied at least 70% of the entire dataset. A threshold of 70% is chosen in order to ensure a reasonable level of acceptance rate. Setting the threshold at 100% results in zero accepted draws. With the threshold set at 70%, the acceptance rate for our dataset is 45.22%. We take the mean and standard error of the accepted draws as the corresponding statistics for the parameter estimates. Figure 2 provides a flowchart showing the process by which we impose regularity conditions in our estimation.

4. Results

In Table 2 we report our parameter estimates for the CES-Translog production functions with both the non-linear least-squares approach and the Bayesian approach. We also provide our estimates for the Cobb-Douglas production function to validate the quality of our dataset. The Cobb-Douglas estimates are consistent with those from the previous literature. For example, our IT capital coefficient is estimated at 0.047. This is comparable with Chwelos et al. (2010), who estimate the

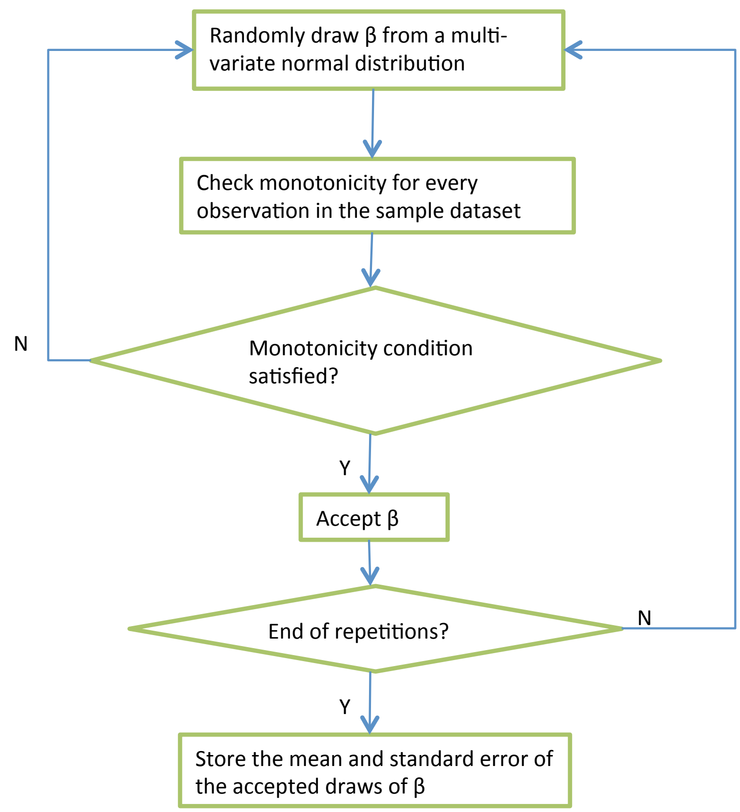

Figure 2: Bayesian Estimation Flowchart

IT coefficient at 0.0636. Our coefficient estimates for non-IT capital and labor are also consistent with the economic conjecture of capital and labor being roughly one third and two thirds of the contribution (and total input share), respectively.

Our results of the AES and the MES are based on their definitions using our CES-Translog estimates as the FFF of the production function (see (1) and (2) for AES and MES definitions, respectively, and the Appendix for the calculation of the elements in the equations). We use the estimated parameters to calculate the ESs in the following steps. First, we calculate the ESs for each observation (industry, year) in the dataset. Second, we calculate the mean ESs for each industry across different years. Third, the industry-group level ESs are aggregated as the weighted average of the industry level ESs measures. The weight is total value-added for each industry.

Then, we develop a bootstrap procedure in STATA to estimate the standard errors of the industry-group level ESs. Specifically, since our goal is to estimate the standard errors at the industry-group level, we bootstrap the observed ESs and re-sample (with replacement) within each industry-group panel. The bootstrap re-sampling is repeated for 200 times. In each repetition, the industry-group ESs are calculated following the above steps. Our final estimates of the standard errors for the industry-group ESs are based on the 200 bootstrapped industry-group ESs. The AES and MES between IT capital and non-IT capital or labor are reported in Table 3. The AES and MES between non-IT capital and labor are reported in Table 4. In both tables, Panel A contains the AES and MES based on the non-linear least-square estimates and Panel B contains the AES and MES based on the Bayesian estimates.

4.1. Comparison of ES estimates based on least-square and Bayesian approaches: The impact of imposing regularity conditions on ES estimates

Compared with the ES estimates based on the least-square approach (Panel A of Tables 3 and 4), the ES estimates based on the Bayesian approach (Panel B of Tables 3 and 4) are remarkably consistent across different industries. Consider the MES between IT capital and non-IT capital (σ_{zk}^{M}) as an example. The MES from the Bayesian approach ranges from 1.12 for the industry group of *Mining* to 1.26 for the industry group of *Accommodation, food services and drinking places*, with the average economy wide estimate being 1.19. This narrow range is in sharp contrast with the same estimate from the non-linear least-square approach ranging from 0.24 for the industry group of *Mining* to 1.57 for the industry group of *Professional services*. The consistency across industries conveniently allows us to draw inferences that apply both economy-wide and across industries.

It is worth noting that imposing monotonicity condition works by removing observations that do not satisfy monotonicity. To the extent that those observations are outliers (or due to data error), there are likely to be dampening effects on the standard error. It is possible that removing observations that do not satisfy monotonicity change the CES-Translog parameters and consequently the ES estimates, but there is no systematic direction of the effect on the ESs. We also examined the standard errors from our bootstrapping process, comparing the standard errors when monotonicity is imposed and when it is not. What we find is that the standard errors are smaller when monotonicity is imposed, but this could be due to the estimation method, Bayesian versus least squares, rather than the observations that are removed.

As mentioned before, it is important to impose regularity conditions, especially monotonicity conditions, when estimating elasticities. Therefore, unless otherwise specified, below we focus on discussing the ES estimates from the Bayesian approach.

As much of the interpretation of ESs involves quantities, we restate that in our study, and most productivity studies, the quantity of IT capital is the stock of IT capital in real dollars. It can be converted into a flow through the rental price methodology but that does not often yield different results because the stock of IT capital is used to derive the rental price. When IT capital is converted to real dollars from nominal dollars the prior years' vintages are deflated by the price deflator. This price is based on market prices that in turn reflects some function of IT performance.

4.2. AES Estimates

Panel B of Tables 3 and 4 shows that the AESs are significant and positive for all industry groups and for the economy wide average. This indicates that each pair of inputs are substitutes in the Allen sense. Thus, if the price of any input increases, there is a corresponding increase in the quantity of each of the other inputs.

In Table 5 we show a comparison of the AES estimates between our findings and those of previous IT substitutability studies, noting that Chun and Mun (2006) are not comparable because they estimate a cost function and use GPO rather than value added and consequently intermediate inputs are part of their analysis. Our AES results are consistent with previous findings of Dewan and Min (1997), and of Hitt and Snir (1999). Our results are also consistent with the findings of Chwelos et al. (2010) except for the AES between IT capital and non-IT capital where they found that IT capital and non-IT capital are complements.

4.3. MES Estimates

In interpreting the results of our MES estimation, it is helpful to review substitutability and complementarity in the Morishima sense. Consider the relationship between IT capital and non-IT capital. The MES in the case of a change in the price of IT capital is σ_{kz}^M. If $\sigma_{kz}^M < 0$, then IT capital and non-IT capital are complements. The result of a decrease in the price of IT capital is to increase not only the quantities of IT and non-IT capital, but also to increase the ratio of the quantity of non-IT capital to IT capital, x_k/x_z (see (4)). Moreover, this also increases the ratio of the input share of non-IT capital to IT capital, $p_k x_k/p_z x_z$.

The more common case, and the case with all of our results, is when the MES is positive and the inputs are substitutes. We show the effects of MES substitutes in Figure 3 where we explain it as a two-step sequence. In the first step, the effect of a decrease in the price of IT capital is to increase the quantity of IT capital due to its own price elasticity and to decrease the quantity ratio of non-IT capital to IT capital, x_k/x_z. Here quantities mean the real dollar value of the IT capital and non-IT capital stock, and in our data in 2005 dollars. Notice this does not necessarily imply that the quantity of non-IT capital falls, but rather that x_z increases by a larger percentage than x_k. In this way, the MES substitutes – here IT capital and non-IT capital – can both have increases in quantity as a consequence of a decrease in price of one of the inputs. This situation with both quantities increasing is essentially what Chwelos et al. (2010) found with IT capital and non-IT capital as AES complements in their post-Internet period.

In addition to the above interpretation of the quantity ratio, in the second step the MES also tells us how the ratio of input shares change in response to a price change by comparing MES with unity (see (3)). If $\sigma_{kz}^M < 1$, then the result of a decrease in the price of IT capital, p_z is to increase the quantity of IT capital, x_z and also to increase the ratio of input share of non-IT capital to IT capital, $p_k x_k / p_z x_z$. Thus, in post-IT capital price change dollar terms the IT capital input share is lower relative to that of non-IT capital – and this occurs because the ratio of quantities, x_k/x_z, falls less than the price of IT capital. However, if $\sigma^M_{kz} > 1$, then a price decrease in IT capital decreases the ratio of input shares of non-IT capital to IT capital because the ratio of quantities falls more than the price of IT capital – a more common interpretation of substitution and typically the case when IT capital has a high own price elasticity.

Interpreting MES Substitution: $\sigma^M_{kz} > 0$

Price of IT capital: p_z (set to 1 initially)
Price of non-IT capital: $p_k = 1$ (numeraire)
Real dollar value of IT capital (quantity): x_z
Real dollar value of non-IT capital (quantity): x_k
Ratio of input share of non-IT capital to IT capital: $\frac{p_k x_k}{p_z x_z}$

Step 1: $p_z \downarrow \Rightarrow x_z \uparrow$ (own price elasticity)

$\Rightarrow \frac{x_k}{x_z} \downarrow \Rightarrow \%\Delta x_k < \%\Delta x_z$

Step 2: If $\sigma^M_{kz} < 1 \Rightarrow \frac{p_k x_k}{p_z x_z} \uparrow \Rightarrow \frac{x_k}{x_z}$ falls less than p_z

If $\sigma^M_{kz} > 1 \Rightarrow \frac{p_k x_k}{p_z x_z} \downarrow \Rightarrow \frac{x_k}{x_z}$ falls more than p_z

Figure 3: Interpreting MES Substitution

From Panel B of Tables 3 and 4, we can see that all MES estimates are significant and positive, meaning that each pair of inputs are substitutes in the Morishima sense. That is to say that, if the price of any particular input increases, then there is a corresponding increase in the quantity ratio of each of the other inputs to that particular input.

Panel B of Tables 3 and 4 also shows the significance from unity for each of the MES estimates, which is marked by "+++", "++", and "+" for 1%, 5%, and 10% significance levels, respectively. Most of the MES estimates are significantly different from unity, with a few exceptions on those of IT capital and labor. However, the economy-wide averages are all significantly different from unity.[3]

[3] It is worth noting that the MES can be interpreted with price going either up or down. After examining the price deflators of IT, non-IT capital and labor obtained from BLS, we observe a general pattern of falling prices for IT and

4.3.1. MES results when the price of IT is changing Begin by examining the MESs in Panel B of Table 3, σ_{kz}^{M} and σ_{lz}^{M}. Over the time period of our dataset the price of IT capital has been decreasing, which for normal inputs implies the quantity of IT capital, x_z, has been increasing. For all 12 industries and economy-wide we find that $0 < \sigma_{kz}^{M} < 1$. Thus, IT capital and non-IT capital are substitutes in the Morishima sense, and the ratio of their quantities, x_k/x_z is decreasing. It is not possible to determine whether the quantity of non-IT capital, x_k is increasing or decreasing. However, even though IT capital and non-IT capital are substitutes in the Morishima sense, we also find that the ratio of the input share of non-IT capital to the input share of IT capital is increasing – meaning that $p_k x_k / p_z x_z$ is increasing. Consequently, the fall in price of IT capital, noting that the price of non-IT capital is constant for the purposes of interpreting σ_{kz}^{M}, outweighs the changes in the relative quantities of IT and non-IT capital. As such, this shows that substitution in the Morishima sense is *less elastic* when the MES is less than unity.

We also find that for 9 of the 12 industries, and economy-wide, $0 < \sigma_{lz}^{M} < 1$. Thus, the relationship between IT capital and labor is similar to that between IT and non-IT capital when the price of IT capital changes (falls) – substitution in the Morishima sense is *less elastic*. We attribute this to the special characteristics of IT as an input to production that we discussed in the introduction: IT enables complementary technological and organization innovations, and enhances the efficiency of labor and non-IT capital through indirect effects and various types of spillovers.

In summary, our MES results show that the real input share of IT capital is stable or slightly shrinking in response to a decline in the price of IT capital at least over the period covered by our dataset. This conclusion regarding input shares of IT capital is consistent with observations from the BLS: "*Computers have seemingly accounted for a steady increasing proportion of investment in equipment. In terms of current (read: real) dollar investment, however, computer and computer peripherals have maintained a constant share of current dollar investment in equipment. ⋯ The increasing prevalence of computers and its constant share of equipment investment may be due to the fact that computers have also embodied rapidly changing technology and have exhibited unprecedented declines in price.*" (BLS, 2007: 7)

4.3.2. MES results when the price of non-IT capital is changing Now consider the MESs between IT capital and non-IT capital in Panel B of Table 3, σ_{zk}^{M}, and between labor and non-IT capital in Table 4, σ_{lk}^{M}. For the most part, over the time period of our dataset the prices of non-IT capital have been increasing, which for normal inputs and absent any effects from other input prices implies that the quantity of non-IT capital, x_k has been decreasing. For all 12 industries

rising prices for non-IT and labor over the period covered by our data. We use this price pattern in our interpretations of MES in the following sections. The price deflator data series for the three inputs are available upon request.

and economy-wide we find that $1 < \sigma^M_{zk}, \sigma^M_{lk}$, so that non-IT capital is a substitute in the Morishima sense for both IT capital and labor. Thus, an increase in the price of non-IT capital increases the quantity of IT capital relative to non-IT capital, x_z/x_k, and the quantity of labor relative to non-IT capital, x_l/x_k. In addition, because these MESs are greater than unity the relative input share of IT capital to non-IT capital, $p_z x_z/p_k x_k$, and the relative input share of labor to non-IT capital, $p_l x_l/p_k x_k$, also increases. These relative input share increases are only possible when the quantity of IT capital and of labor relative to non-IT capital increases more than the price of non-IT capital, showing that substitution in the Morishima sense is *more elastic* when the MES is greater than unity.

4.3.3. MES results when the price of labor is changing Now consider the MESs between IT capital and labor in Panel B of Table 3, σ^M_{zl}, and between non-IT capital and labor in Table 4, σ^M_{kl}. In general over the time period of our dataset the price of labor has been increasing, which as described above for normal inputs and absent any effects from other input prices implies that the quantity of labor, x_l has been decreasing. First, examine the MESs for IT capital and labor. Across industries and economy-wide $0 < \sigma^M_{zl}$, so that IT capital is a substitute for labor when the price of labor changes and the relative quantity of IT capital to labor, x_z/x_l, increases. However, some industries have $0 < \sigma^M_{zl} < 1$, and for some (and economy-wide) $1 < \sigma^M_{zl}$. Thus, for the former set of industries substitution in the Morishima sense is *less elastic* whereby the relative input share of IT capital to labor is lower, and in the latter set substitution in the Morishima sense is *more elastic* whereby the relative input share of IT capital to labor is higher.

Second, examine the MESs for non-IT capital and labor. Here we find across industries and economy-wide that $1 < \sigma^M_{kl}$. Thus, non-IT capital is a substitute for labor when the price of labor changes and the relative quantity of IT capital to labor, x_k/x_l, increases. In addition, substitution in the Morishima sense is *more elastic* whereby the relative input share of non-IT capital to labor is higher.

Finally, consider the response of IT capital to changes in the price of non-IT capital versus labor. σ^M_{zk} is always greater than unity and σ^M_{zl} is around unity and is less than σ^M_{zk} in 10 of 12 industries. What this means is if the prices of non-IT capital and labor change by the same percentage, then the quantity and input share of IT capital relative to non-IT capital changes more than it does relative to labor. In other words, the relative quantity and input share of IT capital is more elastic in response to a change in the price of non-IT capital than it is to the price of labor.

5. Contributions, Limitations and Opportunities for Future Research

We make three main contributions in this Commentary. First, we introduce the MES which does not suffer from the drawbacks of the more-heavily used AES. The key advantages are that the

MES differs depending on which input price is changing, that it represents changes in the relative quantities of inputs, that the scale is meaningful in that a larger MES represents a greater degree of elasticity, and that depending on whether the MES is greater than unity it can determine the changes in relative input shares. These latter two advantages allow for a comparison of price versus quantity effects. These advantages are particularly important for IT research in that, in contrast to other inputs, the price of IT has been falling over a substantial period, and the relationships of IT with other capital and labor is richer than the relationships between other inputs. Consequently, the MES allows us to measure the impact of changes in the price of IT capital on relative quantities and input shares differently than the impact of changes in other input prices on IT capital, while giving meaning to less versus more elastic.

Second, we introduce the idea of imposing regularity conditions locally. In much theory-based empirical research the estimation model has underlying assumptions that are necessary in order to interpret the parameter estimates as elements of the focal theory. These underlying assumptions are what we call regularity conditions, and even imposing the most basic of these regularity conditions can improve the stability and validity of the estimates.

Third, we execute a state-of-the-art analysis to estimate MESs – which is substantial. We also demonstrate how to impose regularity conditions, and show how imposing regularity improves the stability of the estimates of our different elasticities. Finally, our MES results clearly display how its flexibility allows for insights about the relationship between IT and other inputs. For the most part our MES results show that substitution between IT and other inputs is less elastic when the price of IT changes, and more elastic when the prices of other inputs change. These results support the reasoning that as IT becomes less expensive and more IT is used, IT displaces less non-IT capital and labor than is the case when the price of another input increases and more IT is used. This latter point is evidenced from our analysis: a drop in the price of IT capital and an increase in the price of non-IT capital have the same increasing effect on the quantity ratio of IT capital to non-IT capital. However, these two opposite price changes on IT capital and non-IT capital have opposing effects on the relative input share of IT capital to non-IT capital.

The first two contributions are important as they highlight the advantages of an under-recognized measure, issues with the more-heavily used measure in the literature, and a methodology that can improve stability of empirical estimates. They also provide measures and methods along with more refined interpretations that can provide more insight, and can be readily used in future research. Our third contribution serves as part of the Research Commentary as we demonstrate how to implement the measure and methods, and interpret the results. This third contribution also serves as a regular research contribution by adding novel results to the literature on the substitution of

IT for other inputs, and more broadly to the literature on the relationship of IT to other inputs when determining the value of IT.

There are some limitations to what can be learnt from our analyses that are endemic to any ES-type analysis. One is that we do not separate out switching or substitution costs between inputs. When firms make decisions about switching between inputs, for example IT capital for labor, they weigh the switching costs versus the benefit of taking advantage of price changes. These costs are implicitly embedded in the data we use. Similarly, our analyses does not model how quickly firms can substitute one input for another except that they can do so in the time period covered by our observations – for example, with annual data this would be a year.

Implications for Future Research Our analysis has implications for future research in the substitution of IT for other inputs, and will benefit future researchers in a range of other work. This is not only because the MES is a better measure than the AES, and that imposing regularity locally improves estimation, but also because IT is an input with special characteristics. First, MESs can be compared between different subclasses of firms or industries. Consequently, it is possible to see if, for example, the change in the input share balance between two inputs such as IT capital and labor is more elastic in one industry or the other. As we saw above, this is not the case with the AES in spite of work that has made inferences from AESs of different magnitudes. Indeed, based on industry MESs we make such comparisons in our results section. This makes possible some important opportunities for future research.

Second is understanding the resulting changes in input shares that come from the behavior of IT prices. Roughly following Moore's law, the price of IT capital has been consistently falling over time. For example, see Nordhaus (2002) and Berndt and Rappaport (2001). The continuous improvements in the price/performance ratio of IT and the empirical evidence of a significant IT impact on productivity continue to motivate increased investment in IT. What we do not know is whether this increased investment in IT exceeds the decline in value of prior vintages of IT capital such that as an input IT capital commands a growing or shrinking input share over time. In contrast to the work using the AES, our results suggest that in response to decreases in the price of IT capital, the IT capital input share is stable or even slightly shrinking, and this is definitely an area for future work. Indeed, our results about IT capital input shares being stable or shrinking because of the prices declines in IT over time invites a different response to Robert Solow's famous comment "You can see the computer age everywhere but in the productivity statistics" (Solow 1987: 36): computers are everywhere because of price declines, but these same price declines keep the real input share of IT capital roughly the same.

Finally, unlike most other factors of production, IT implementations have multiple effects on the production process. Brynjolfsson and Hitt (2000) argue that IT is a general-purpose technology that facilitates complementary technological or organizational innovations that eventually cause dramatic productivity improvements. Farrell (2003) argues that IT increases output because it enhances the efficiency of labor and asset utilization. Supporting these arguments about IT capital being an input that has broader effects, IT has been shown to contribute to organizational capital (Brynjolfsson and Hitt, 2003), to indirect effects enhancing the efficiency of non-IT capital and labor (Mittal and Nault, 2009), to interorganzational spillovers both up and down the supply chain (Cheng and Nault, 2007; 2012) and to knowledge spillovers (Tambe and Hitt, 2014). To better understand these different effects, MESs can be compared between different inputs. This opens up the potential of comparing different subclasses of IT capital or even IT capital and IT operating expenses, together with non-IT capital and labor. This allows access to a deeper understanding about how IT combines with other inputs to create value.

An important future research opportunity comparing MESs between different inputs is the evolution of IT outsourcing. IT outsourcing was prevalent from 1998-2009. This transformation of IT capital to operating expense reduces the amount of IT capital we measure, but also reduces value added – the dependent variable in our production function estimation. Consequently, there is no observation we can make in our analyses that addresses this type of substitution. To do this within the analyses we propose would require separating the IT operating expense from the rest of the intermediate inputs, and then examine the substitution between IT capital and IT operating expenses. Furthermore, it is possible to separate the different categories of IT capital such as computers, software and telecommunications. An interesting future research opportunity is to examine the substitution between these different categories of IT capital as well as IT operating expenses.

More generally, a wide array of future research in IS has the potential to be informed by our analysis: research in IT productivity, IT substitution, IT cost-benefit analyses, and more broadly in the impact of IT implementations at all levels. In our view, the MES/regularity advances we are reporting are compelling as the currently-favored AES is misleading; are insightful as the alternative MES is more informative; and are actionable in that whenever the AES can be computed the MES can be computed as well based on the same production or cost function estimates.

6. Conclusion

Our analysis and prior research has shown that the MES provides a substitution measure where the scale is meaningful, and the measure differs depending upon which price is changing. This is in contrast to the AES that is misleading when there are three or more inputs. Given that the calculation of the MES follows the same process as the AES, we conclude that the MES should be used exclusively in studies that examine ESs with more than two inputs. Our empirical analyses also show that imposing local regularity – at least first-order features such as non-decreasing output from increases in inputs – increases stability in the estimates. Here we conclude that imposing local regularity conditions using approaches such as the Bayesian methods we used should be employed when possible to estimate the underlying functions that are used to calculate measures of substitution.

Both of these advances, the MES and imposing local regularity, have potential to improve future work on IT productivity, IT pricing, IT cost estimation and any type of analysis that posits the substitution of IT capital for non-IT capital or labor.

7. Appendix: Elements of the MES and AES

In the context of production functions, the AES and the MES are defined as in (1) and (2), respectively. For inputs IT capital (Z), non-IT capital (K), and labor (L), the determinant of the bordered Hessian, $\mathbf{H}$, is defined as

$$\mathbf{H} = \begin{vmatrix} 0 & f_z & f_k & f_l \\ f_z & f_{zz} & f_{zk} & f_{zl} \\ f_k & f_{kz} & f_{kk} & f_{kl} \\ f_l & f_{lz} & f_{lk} & f_{ll} \end{vmatrix}$$

where the elements are the first and second derivatives of the production function. The first and second derivatives of the CES-Translog form in (5) are

$$f_z = \frac{Y}{Z}[\frac{\delta_z Z^{-\rho}}{T} + 2\beta_{zz} z + \beta_{zl} l + \beta_{zk} k],$$

$$f_k = \frac{Y}{K}[\frac{\delta_k K^{-\rho}}{T} + 2\beta_{kk} k + \beta_{kl} l + \beta_{zk} z],$$

$$f_l = \frac{Y}{L}[\frac{\delta_l L^{-\rho}}{T} + 2\beta_{ll} l + \beta_{kl} k + \beta_{zl} z],$$

$$f_{zz} = \frac{f_z^2}{Y} - \frac{f_z}{Z} + \frac{Y}{Z^2}[\frac{\rho\delta_z^2 Z^{-2\rho}}{T^2} - \frac{\rho\delta_z Z^{-\rho}}{T} + 2\beta_{zz}],$$

$$f_{kk} = \frac{f_k^2}{Y} - \frac{f_k}{K} + \frac{Y}{K^2}[\frac{\rho\delta_k^2 K^{-2\rho}}{T^2} - \frac{\rho\delta_k K^{-\rho}}{T} + 2\beta_{kk}],$$

$$f_{ll} = \frac{f_l^2}{Y} - \frac{f_l}{L} + \frac{Y}{L^2}[\frac{\rho\delta_l^2 L^{-2\rho}}{T^2} - \frac{\rho\delta_l L^{-\rho}}{T} + 2\beta_{ll}],$$

$$f_{zk} = \frac{f_z f_k}{Y} + \frac{Y}{ZK}[\frac{\rho\delta_z\delta_k Z^{-\rho} K^{-\rho}}{T^2} + \beta_{zk}],$$

$$f_{zl} = \frac{f_z f_l}{Y} + \frac{Y}{ZL}[\frac{\rho\delta_z\delta_l Z^{-\rho} L^{-\rho}}{T^2} + \beta_{zl}],$$

$$f_{kl} = \frac{f_k f_l}{Y} + \frac{Y}{KL}[\frac{\rho\delta_k\delta_l K^{-\rho} L^{-\rho}}{T^2} + \beta_{kl}],$$

where $T = \delta_z Z^{-\rho} + \delta_k K^{-\rho} + (1 - \delta_z - \delta_k) L^{-\rho}$

By Youngs' theorem, $f_{ij} = f_{ji}$. In the expression of the AES and MES, a cofactor $\mathbf{H}_{ij}$ is defined as $\mathbf{H}_{ij} = (-1)^{i+j}|M_{ij}|$, where $|M_{ij}|$ is the determinant of the minor matrix that is obtained from deleting the i^{th} row and the j^{th} column of the bordered Hessian. The required minor matrices that are needed to form the different cofactors are:

$$\mathbf{M_{kz}} = \begin{vmatrix} 0 & f_k & f_l \\ f_z & f_{zk} & f_{zl} \\ f_l & f_{lk} & f_{ll} \end{vmatrix}, \quad \mathbf{M_{lz}} = \begin{vmatrix} 0 & f_k & f_l \\ f_z & f_{zk} & f_{zl} \\ f_k & f_{kk} & f_{kl} \end{vmatrix}, \quad \mathbf{M_{zk}} = \begin{vmatrix} 0 & f_z & f_l \\ f_k & f_{kz} & f_{kl} \\ f_l & f_{lz} & f_{ll} \end{vmatrix}$$

$$\mathbf{M_{zl}} = \begin{vmatrix} 0 & f_z & f_k \\ f_k & f_{kz} & f_{kk} \\ f_l & f_{lz} & f_{lk} \end{vmatrix}, \quad \mathbf{M_{kl}} = \begin{vmatrix} 0 & f_z & f_k \\ f_z & f_{zz} & f_{zk} \\ f_l & f_{lz} & f_{lk} \end{vmatrix}, \quad \mathbf{M_{lk}} = \begin{vmatrix} 0 & f_z & f_l \\ f_z & f_{zz} & f_{zl} \\ f_k & f_{kz} & f_{kl} \end{vmatrix}$$

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References

Allen, R.G.D.. 1938. Why Information Should Influence Productivity. *Mathematical Analysis for Economists* New York: St. Martin's.

Amado, C., T. Teravirta. 2011. Modelling Volatility by Variance Decomposition. *CREATES Research Paper.* Aarhus University, Denmark. bf 2011:1.

Barnett, W.. 2002. Tastes and Technology: Curvature is not Sufficient for Regularity. *Journal of Econometrics* **108** 199-202.

Barnett, W. and M. Pasupathy. 2003. Regularity of the Generalized Quadratic Production Model: A Counterexample. *Econometric Reviews* **22:2** 135-154.

Bauwens, L., Lubrano, M., and Richard J. 1999. *Bayesian Inference in Dynamic Econometric Models, Advanced Texts in Econometrics* Oxford University Press Inc., New York.

Berndt, R. and Rappaport, N. 2002. Price and Quality of Desktop and Mobile Personal Computers: A Quarter Century Historical Overview. *American Economic Review* **91**(2) 268-273.

Blackorby, C., R. Russell. 1981. The Morishma Elasticity of Substitution; Symmetry, Constancy, Separability, and its Relationship to the Hicks and Allen Elasticities. *Review of Economic Studies* **48:1** 147-158.

Blackorby, C., R. Russell. 1989. Will the Real Elasticity of Substitution Please Stand Up? (A Comparison of the Allen/Uzawa and Morishima Elasticities). *The America Economic Review* **79:4** 882-888.

Brynjolfsson, E., L.M. Hitt. 2000. Beyond Computation: Information Technology, Organizational Transformation and Business Practices. *Journal of Economic Perspectives* **14**(4) 23-48.

Brynjolfsson, E., L.M. Hitt. 2003. Computing Productivity: Firm-Level Evidence. *Review of Economics and Statistics* **85**(4) 793–808.

Bureau of Labor Statistics. 2007. Technical Information About the BLS Multifactor Productivity Measures. www.bls.gov/mfp/mprtech.pdf (downloaded July 19, 2014).

Caves, D., L. Christensen. 1981. Global Properties of the FFFs. *American Economic Review* **70** 422-432.

Chambers, R. G. 1988. *Applied Production Analysis: A Dual Approach* Cambridge, Cambridge University Press.

Cheng, Z., B.R. Nault. 2007. Industry Level Supplier-Driven IT Spillovers. *Management Science* **53**(8) 1199–1216.

Cheng, Z., B.R. Nault. 2012. Relative Industry Concentration and Customer-Driven IT Spillovers. *Information Systems Research* **23**(2) 340–355.

Chib, S., E. Greenberg. 1994. Bayes Inference for Regression Models with ARMA(p,q) Errors. *Journal of Econometrics* **64** 183-206.

Chib, S., Greenberg, E. 1995. Understanding the Metropolis-Hstings Algorithm. *The American Statistician* **49**(4) 327-335.

Chun, H., S. Mun. 2006. Substitutability and Accumulation of Information Technology Capital in U.S. Industries. *Southern Economic Journal,* **72**(4) 1002-1015.

Chwelos, P., R. Ramirez, K.L. Kraemer, N. Melville. 2010. Does Technological Progress Alter the Nature of Information Technology as a Production Input? New Evidence and New Results. *Information Systems Research* **21**(2) 392–408.

Coelli, T.J., D.S. Prasada Rao, C.J. O'Donnell, G.E. Battese. 2005. *An Introduction to Efficiency and Productivity Analysis* Springer.

Dewan, S., C. Min. 1997. The substitution of Information Technology for other factors of production: A firm level-analysis. *Management Science* **43**(12) 1660–1675.

Diewert, W., T. Wales. 1987. FFFs and Global Curvature Conditions. *Econometrica* **55**(1) 43-68.

Fan, Y., S. Pastorello, E. Renault. 2007. Maximization by Parts in Extremum Estimation. Working Paper.

Farrell, D. 2003. The Real New Economy *Harvard Business Review*, October 105-112.

Gallant, A., E. Golub. 1984. Imposing Curvature Restrictions on FFFs. *Journal of Econometrics* **26** 295-321.

Hastings, W. 1970. Monte Carlo Sampling Methods Using Markov Chains and Their Applications. *Biometrika,* **57** 97-109.

Hicks, J. 1932.*The Theory of Wages, 2nd edition* London: MacMillian.

Hicks, J., R.G.B. Allen. 1934. A Reconsideration of the Theory of Value. Part I. *Economica* **1**(1) 52-76.

Hitt, L.M., E. Snir. 1999. The Role of Information Technology in Modern Production: Complement or Substitute to Other Inputs? *Working Paper.*

Jorgenson, D.W., B. M. Fraumeni. 1981. Relative Prices and Technical Change", *Modeling and Measuring Natural Resource Substitution*, MIT Press, Cambridge, MA.

Judge, G.G., R.C. Hill, W.E. Griffiths, H. Lutekpohl, T.C. Lee. 1988. *Introduction to the Theory and Practice of Economics.* New York: John Wiley and Sons.

Kudyna, S., R. Diwan. 2002. Increasing Returns to Information Technology. *Information Systems Research* **13**(1) 104-111.

Lau, L. 1978. Testing and Imposing Monotonicity, Convexity and Quasiconvexity Constraints. in Fuss, M. and McFadden (eds) Production Economics: A Dual Approach to Theory and Applications, Vol. 1, North-Holland, Amsterdam.

Mittal, N., B.R. Nault. 2009. Investments in Information Technology: Indirect Effects and Information Technology Intensity. *Information Systems Research* **20**(1) 140-154.

Morey, E. 1986. An Introduction to Checking, Testing, and Imposing Curvature Properties: the True Function and the Estimated Function. *Canadian Journal of Economics* **19**(2) 207-235.

Morishima, M. 1967. Danryokusei Rison ni kansuru Ni-san no Teian (A Few Suggestions on the Theory of Elasticity). *Keizai Hyoron (Economic Review, Tokyo)* **16** 144-150.

Moschini, G. 1999. Imposing Local Curvature Conditions in Flexible Demand System. *Journal of Business & Economic Statistics* **17** 487-490. Mun, S., D. Hong. 2004. An Analysis of Elasticity of Substitution for the Production Function with IT Capital, Labor and Non-IT Capital. *Bank of Korea Economic Papers* **7**(2) 165-176.

Mundra, K., R. Russell, R. 2004. Dual Elasticity of Substitution *Working Paper*.

Nordhaus, D. 2002. Productivity Growth and the New Economy *Brookings Paper on Economics Activity* **2** 201-217.

O'Donnell, C., T. Coelli. 2005. A Bayesian Approach to Imposing Curvature on Distance Functions. *Journal of Econometrics* **126** 493-523.

Pollak, R., Sickles, R., and T. Wales. 1984. The CES-Translog: Specification and Estimation of a New Cost Function. *Review of Economics and Statistics* **66** 602-607.

Ryan, D., T. Wales. 1998. A Simple Method for Imposing Local Curvature in Some Flexible Consumer-Demand System. *Journal of Business and Economic Statistics* **16**(3), 331-338.

Salvanes, K., S. Tjotta. 1998. A Note on the Importance of Testing for Regularities for Estimated FFFs. *Journal of Productivity Analysis* **9** 133-143.

Solow, R. 1987. We'd better watch out. *New York Times Book Review* July 12, 36.

Song, P. X.-K., Y. Fan, J. D. Kalbfleisch. 2005. Maximization by Parts in Likelihood Inference. *Journal of the American Statistical Association* **100** 1145-1157.

Stiroh, K.J. 2002. Information Technology and the U.S. Productivity Revival: What do the Industry Data Say? *American Economic Review* **92**(5) 1559–1576.

Tambe, P., L.M. Hitt. 2014. Job Hopping, Information Technology Spillovers, and Productivity Growth. *Management Science*, forthcoming.

Terrell, D. 1996. Incorporating Monotonicity and Concavity Conditions in FFFs. *Journal of Applied Econometrics* **11**(2) 179-194.

Uzawa, H. 1962. Production Functions With Constant Elasticities of Substitution. *Review of Economic Studies* **29** 291-299.

Table 1 Summary Statistics (N=708)

Variable	Mean	Std. dev.	Min	Max
VA: Value added (in millions of 2005 dollars)	175787	232158.2	-5532.266	1545957
L: Labor (in millions of hours)	3154.267	4486.429	84	25810
K: Non-IT capital stock (in millions of 2005 dollars)	233465.3	318652.7	9381	1924724
IT: IT capital stock (in milllions of 2005 dollars)	29747.91	49982.64	413	358847

Table 2 Parameter Estimates

	Cobb-Douglas	CES-Translog	Imposing regularity
		Least-squares approach	Bayesian approach
ρ		0.699***	0.692***
		(0.091)	(0.091)
IT	0.047***	0.580***	0.582***
	(0.021)	(0.052)	(0.051)
K	0.359***	0.050	0.050
	(0.031)	(0.044)	(0.043)
L	0.586**	0.369***	0.368***
	(0.029)	(0.036)	(0.036)
IT x IT		0.023	0.031***
		(0.014)	(0.011)
IT x K		0.000	0.008
		(0.017)	(0.015)
IT x L		-0.091***	-0.085***
		(0.018)	(0.017)
K x K		0.039***	0.040***
		(0.007)	(0.006)
K x L		-0.076***	-0.074***
		(0.016)	(0.015)
L x L		0.110***	0.111***
		(0.011)	(0.011)
N	708	708	708
Industries	59	59	59
Controls	Group, year	Group, year	N/A

- IT: IT capital, K: non-IT capital, L: labor
- Standard errors in parentheses
- Dummy variables are suppressed for brevity
- *** significance at 1% level, ** significance at 5% level, * significance at 10% level

Table 3 Elasticity of Substitution Estimates between IT and non-IT or Labor

Panel A: Before imposing regularity (monotonicity) conditions, least-squares approach

Industry group	NAICS (# of industries)	AES_IT_K (σ^A_{zk})	MES_IT_K (σ^M_{zk})	MES_K_IT (σ^M_{kz})	AES_IT_L (σ^A_{zl})	MES_IT_L (σ^M_{zl})	MES_L_IT (σ^M_{lz})
Manufacturing	31-33 (18)	0.73	1.21***	0.25	0.09	0.61	0.25
Transp., comm. & public utilities	22,48-49 (9)	1.31**	1.56***+	1.23***	2.05	1.89**	1.46***
Wholesale trade	42 (1)	0.84	1.30***	-0.09	-0.45	0.14	-0.05
Retail trade	44-45 (1)	0.95***	1.34***+++	0.37***+++	0.05	0.50***+++	0.32***+++
Accommod., food serv. & drinking places	72 (2)	0.96***	1.41***+++	0.74***+++	0.52***	0.88***+++	0.65***+++
FIRE	52-53 (6)	1.09***	1.33***+++	0.57	0.10	0.72***	0.54
Entertainment	51,71 (6)	1.32***	1.49***+++	0.61***+++	0.15**	0.73***+++	0.66***++
Professional services	54,56,81 (7)	1.38***	1.57***+++	1.04**	0.91**	1.07***	1.02***
Educational serv., health care & soc. assist.	61-62 (4)	1.24***	1.49***+++	0.98***	0.84***	1.06***	0.96***
Forestry	11 (1)	0.99***	1.38***+++	0.63***+++	0.31***	0.74***+++	0.52***+++
Mining	21 (3)	-0.77***	0.24** +++	0.42***+++	4.00***	3.08***+++	0.51***+++
Construction	23 (1)	0.89***	1.44***+++	0.29***+++	0.06	0.43***+++	0.25***+++
Economy wide	All above (59)	1.04***	1.37***+++	0.60***+	0.46	0.84***	0.60***++

Panel B: After imposing regularity (monotonicity) conditions, Bayesian approach

Industry group	NAICS (# of industries)	AES_IT_K (σ^A_{zk})	MES_IT_K (σ^M_{zk})	MES_K_IT (σ^M_{kz})	AES_IT_L (σ^A_{zl})	MES_IT_L (σ^M_{zl})	MES_L_IT (σ^M_{lz})
Manufacturing	31-33 (18)	0.90***	1.19***+++	0.85***+++	0.81***	1.05***+++	0.83***+++
Transp., comm. & public utilities	22,48-49 (9)	0.84***	1.15***+++	0.91***+++	1.15***	1.33***+++	0.97***
Wholesale trade	42 (1)	0.89***	1.19***+++	0.80***+++	0.72***	0.93***+++	0.77***+++
Retail trade	44-45 (1)	0.95***	1.21***+++	0.81***+++	0.66***	0.88***+++	0.74***+++
Accommod., food serv. & drinking places	72 (2)	0.95***	1.26***+++	0.87***+++	0.75***	1.00***	0.81***+++
FIRE	52-53 (6)	0.91***	1.16***+++	0.85***+++	0.75***	1.02***	0.81***+++
Entertainment	51,71 (6)	0.80***	1.15***+++	0.96***+	1.18***	1.30***+++	1.02***
Professional services	54,56,81 (7)	0.85***	1.23***+++	0.82***+++	0.79***	0.97***+++	0.80***+++
Educational serv., health care & soc. assist.	61-62 (4)	0.85***	1.22***+++	0.84***+++	0.82***	1.01***	0.83***+++
Forestry	11 (1)	0.97***	1.25***+++	0.82***+++	0.63***	0.92***+++	0.73***+++
Mining	21 (3)	0.80***	1.12***+++	0.94***+++	1.42***	1.74***+++	1.08***+++
Construction	23 (1)	0.92***	1.25***+++	0.79***+++	0.68***	0.88***+++	0.74***+++
Economy wide	All above (59)	0.88***	1.19***+++	0.85***+++	0.82***	1.04***+++	0.83***+++

- AES: Allen Elasticity of Substitution, MES: Morishima Elasticity of Substitution
- IT: IT capital, K: non-IT capital, L: labor
- ***, **, and *: significance at 1%, 5%, and 10% respectively
- +++, ++, and +: significance from 1 at 1%, 5%, and 10% respectively

Table 4 Elasticity of Substitution Estimates between non-IT and Labor

Panel A: Before imposing regularity (monotonicity) conditions, least-squares approach

Industry group	NAICS (# of industries)	AES_K_L (σ^A_{kl})	MES_K_L (σ^M_{kl})	MES_L_K (σ^M_{lk})
Manufacturing	31-33 (18)	1.60***	1.56***+++	1.56***+++
Transportation,communications and public utilities	22,48-49 (9)	2.15***	2.23***++	1.99***+++
Wholesale trade	42 (1)	1.47***	1.53***+++	1.49***+++
Retail trade	44-45 (1)	1.56***	1.47***+++	1.52***+++
Accommodation, food services and drinking places	72 (2)	1.83***	1.56***+++	1.64***+++
FIRE	52-53 (6)	1.52***	1.48***+++	1.50***+++
Entertainment	51,71 (6)	1.61***	1.61***+++	1.56***+++
Professional services	54,56,81 (7)	1.69***	1.60***+++	1.62***+++
Educational services, health care and social assistance	61-62 (4)	1.67***	1.58***+++	1.60***+++
Forestry	11 (1)	1.71***	1.49***+++	1.60***+++
Mining	21 (3)	2.89***	2.91***+++	2.81***+++
Construction	23 (1)	1.66***	1.58***+++	1.62***+++
Economy wide	All above (59)	1.66***	1.61***+++	1.61***+++

Panel B: After imposing regularity (monotonicity) conditions, Bayesian approach

Industry group	NAICS (# of industries)	AES_K_L (σ^A_{kl})	MES_K_L (σ^M_{kl})	MES_L_K (σ^M_{lk})
Manufacturing	31-33 (18)	1.55***	1.38***+++	1.41***+++
Transportation,communications and public utilities	22,48-49 (9)	1.70***	1.57***+++	1.51***+++
Wholesale trade	42 (1)	1.44***	1.31***+++	1.34***+++
Retail trade	44-45 (1)	1.50***	1.29***+++	1.36***+++
Accommodation, food services and drinking places	72 (2)	1.69***	1.38***+++	1.45***+++
FIRE	52-53 (6)	1.51***	1.34***+++	1.38***+++
Entertainment	51,71 (6)	1.60***	1.49***+++	1.43***+++
Professional services	54,56,81 (7)	1.54***	1.37***+++	1.39***+++
Educational services, health care and social assistance	61-62 (4)	1.55***	1.38***+++	1.40***+++
Forestry	11 (1)	1.63***	1.34***+++	1.44***+++
Mining	21 (3)	2.06***	1.92***+++	1.78***+++
Construction	23 (1)	1.53***	1.33***+++	1.39***+++
Economy wide	All above (59)	1.55***	1.38***+++	1.40***+++

- AES: Allen Elasticity of Substitution, MES: Morishima Elasticity of Substitution
- K: non-IT capital, L: labor
- ***, **, and *: significance at 1%, 5%, and 10% respectively
- +++, ++, and +: significance from 1 at 1%, 5%, and 10% respectively

Table 5 A Comparison of our AES Results and Previous IT Substitution Studies

Study	Data Year	AES_IT_K (σ^A_{zk})	AES_IT_L (σ^A_{zl})	AES_K_L (σ^A_{kl})
Dewan and Min (1997)	1988-1992: Firms CES-Translog	1.006	1.063	1.005
Hitt and Snir (1999)	1987-1994: Firms Translog	0.945	0.688	NA
Chwelos et al. (2010)	1987-1999: Firms CES-Translog	-3.406	3.012	2.244
Our study	1998-2009: Industries CES-Translog; Least-squares	1.04	0.46	1.66
	1998-2009: Industries CES-Translog; Bayesian	0.88	0.82	1.55

- AES: Allen Elasticity of Substitution, IT: IT capital, K: non-IT capital, L: labor